Do equity and foreign currency risk premiums display common patterns?

PAULO ROGERIO FAUSTINO MATOS

(EPGE/FGV)

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Coordenação:
Prof. Luis Henrique B. Braido
e-mail: lbraido@fgv.br
Do Risk Premiums in the Equity and the Foreign Currency Markets Display Common Patterns?

Carlos E. da Costa, Paulo Rogério F. Matos,
Getulio Vargas Foundation

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Abstract

In da Costa et al. (2006) we have shown how a same pricing kernel can account for the excess returns of the S&P500 over the US short term bond and of the uncovered over the covered trading of foreign government bonds. In this paper we estimate and test the overidentifying restrictions of Euler equations associated with six different versions of the Consumption Capital Asset Pricing Model. Our main finding is that the same (however often unreasonable) values for the parameters are estimated for all models in both markets. In most cases, the rejections or otherwise of overidentifying restrictions occurs for the two markets, suggesting that success and failure stories for the equity premium repeat themselves in foreign exchange markets. Our results corroborate the findings in da Costa et al. (2006) that indicate a strong similarity between the behavior of excess returns in the two markets when modeled as risk premiums, providing empirical grounds to believe that the proposed preference-based solutions to puzzles in domestic financial markets can certainly shed light on the Forward Premium Puzzle.

JEL Code: G12; G15

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1 Introduction

Two of the most famous puzzles in financial economics are the Equity Premium Puzzle—EPP—and The Forward Premium Puzzle—FPP.

The EPP, commonly associated with the works of Hansen and Singleton (1983) and Mehra and Prescott (1985), is the failure of the traditional Consumption Capital Asset Pricing—CCAPM—to account for the excess return of stock market with respect to the risk free bond in the United States, with reasonable preference parameters. The FPP, on the other hand, relates to the difference between the forward rate and the expected future value of the spot exchange rate in a world with rational
expectations and risk neutrality. More important for our purposes is the failure of the canonical CCAPM to generate a pattern of prices for risk that could explain the FPP being only related to the implicit risk neutrality assumption.

Despite the skepticism about risk based explanations for the FPP from part of the literature, in da Costa et al. (2006), we try to relate the puzzles. We are not able to reject that the same stochastic discount factor—SDF—that explains the equity premium also explains the forward premium. We do not use a model to investigate the puzzles. Instead we use statistical methods to extract a time series for the realized SDF and show how the moment restrictions on discounted excess returns on both equity and forward markets are not rejected. Excess returns in both markets are simply a premium for risk, if we accept the covariance with the SDF as the right measure of risk.

Because we do not spell out a full specified model it is hard to justify our calling the covariance of returns with the SDF as a risk measure. In this paper, we take the discussion of the previous paper one step further by evaluating the performance of different models in pricing excess returns for each market. Once again, our goal is not to find a model that solves all the problems. Rather, our concern is to verify if the same failures and successes attained by the CCAPM in its various forms in pricing the excess returns of equity over short term risk-free bonds will be manifest in the case of forward exchange markets.

We estimate and test for the two markets six different consumption-based asset pricing models using the Generalized Method of Moments—GMM: the canonical CCAPM, Abel's (1990) catching up with the Joneses, Abel's (1999) keeping up with the Joneses models, Epstein and Zin's (1991) recursive utility specification, Campbell and Cochrane's (1999) slow-moving habit formation model and a simplified version of Garcia's (2006) model which nests Campbell and Cochrane (1999) and Epstein and Zin (1991) for an appropriate choice of the state variables. For the equity market we test the excess return of the S&P500 over the short-term American government bond, while for the foreign exchange markets we consider the excess return on the uncovered over the covered trading of Canadian, German, Japanese and British government bonds jointly.

As for the instruments used to generate the over-identifying restrictions that are tested in our procedure, for both puzzles, we use the same set of macroeconomic instrumental variables proven to be useful in forecasting the SDF. We also use lags of the real returns in question as the financial variables, specific for each puzzle. This procedure allows us to verify the robustness of the empirical results and the common pattern of the equity and foreign currency premiums. In order to analyze the predictability power of the forward premium pointed as a specificity of the FPP, we also use lags of this variable instead lags of the returns in question as financial instruments.

Briefly, according to our results, i) we can evidence the importance of the absolute value of the past and current aggregate per capita consumption, when used as a reference level in the utility function for both puzzles, ii) breaking the tight link between the coefficient of relative risk aversion and the

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1See the comprehensive surveys by Hodrick (1987) and Engel (1996).
elasticity of intertemporal substitution alone can not explain none of the puzzles, but \(iii\) when besides this separation, is also considered a slow-moving external habit, then for both puzzles the agent really seems to derive utility from the level of consumption relative to this benchmark in ratio and also in difference.

Our main findings are: \(i\) the evidence of the strong similarity between the behavior of these excess returns when modeled as risk premiums, which allows us to provide empirical grounds to believe that the proposed preference-based solutions to puzzles in domestic financial markets can certainly shed light on the FPP and \(ii\) the prominent role played by the forward premium, relative to lags of real returns in question, as instrument in a external habit formation approach explanation of the FPP.

In particular, we show that the generalized version of the model developed by Campbell and Cochrane (1999)—with low interest rates, roughly i.i.d. consumption growth with small volatility and acceptably low values of risk aversion—is able to explain not only the high market Sharpe ratio that characterizes the EPP and but also the predictability of the foreign government bond returns.

The remainder of the paper is organized as follows. Section 2 gives an account of the literature that tries to relate the two puzzles. Section 2.1 discusses some characteristics of early work with consumption based model trying to emphasize the reasons why they have encountered difficulties in explaining the stylized facts. Section 2.1.1 describes in details the features shared by most external habit formation models that address the EPP. Finally, in section 3, the empirical exercise is performed, testing all these Euler equations with GMM. Section 4 analyses the results in order to support that the equity and the currency foreign risk premiums have a similar behavior in a preference-based approach. Conclusion is the last section.

2 Literature Overview

2.1 Economic Theory, and the equity and foreign currency risk premiums

2.1.1 Stochastic Discount Factor and Asset returns

It is now a well known fact that, given free portfolio formation, the law of one price is equivalent to the existence of a SDF through Riesz representation theorem.

Following Harrison and Kreps (1979) and Hansen and Richard (1987), we write the asset pricing equations,

\[ 1 = E_t \left[ M_{t+1} R^i_{t+1} \right], \]

and

\[ 0 = E_t \left[ M_{t+1} \left( R^i_{t+1} - R^j_{t+1} \right) \right]. \]

Here, \( E_t(\cdot) \) denotes the conditional expectation given the information available at time \( t \), \( R^i_{t+1} \) and \( R^j_{t+1} \) represent, respectively, the real gross return on assets \( i \) and \( j \) at time \( t + 1 \) and \( M_{t+1} \) is
the SDF, a random variable that generates unitary prices from asset returns. Under no arbitrage, we can further guarantee that there exists a strictly positive SDF that correctly prices the returns of all assets.

An immediate consequence of (2) is that excess returns are explained by the way in which returns covary with the SDF, in fact, if we take \( R^f \) to be the return on the risk-free asset, \( R^f \), equation (2) can be expanded to read

\[
\mathbb{E}_t [R_{t+1}^f] - R^f_t = -\text{cov}_t (R_{t+1}^f, M_{t+1}).
\]  

(3)

It is then, usual practice to interpret the excess expected return as a risk premium and the covariance in the right hand side of (3) as the relevant measure of risk. Using this interpretation, da Costa et al. (2006) have shown that one can use the same measure of risk to explain the equity and the forward premiums. An important question remains open, however: what does this measure of risk measure? Without a model that generates \( M_{t+1} \) from the primitives (preferences, endowments, technologies, etc.) of the economy, the meaning of this covariance term is somewhat elusive. Not surprisingly, it is the main purpose of all research in finance economics exactly to find a model capable of generating \( M_{t+1} \) from the primitives of the economy. It is also the failure of most attempts to find such a model that defines most of the puzzles in finance theory.

We shall in the rest of this section present some of the models for \( M_{t+1} \) written in the past quarter of century and discuss their main features and drawbacks. We shall refrain from discussing models that try to account for puzzles by incorporating market incompleteness. We do so not because we think that this is not an important issue, but because following this path requires a completely different consumption data set, which we leave for later work.

### 2.1.2 The consumption capital asset pricing model

The single most important advance in asset pricing from an economist's perspective was the development of the consumption capital asset pricing model associated with the names of Lucas (1978) and Breeden (1979).²

Consider an economy endowed with an infinitely lived representative consumer whose preferences are representable by a von Neumann-Morgenstern utility function. It is not hard to show that the SDF is simply the growth of the representative agent's marginal utility of consumption.

Formally, the first order conditions for the choice of an optimal consumption and portfolio of this investor, with exogenously specified utility function who faces exogenous asset return process, yield

\[
1 = \beta \mathbb{E}_t \left[ \frac{u'(C_{t+1})}{u'(C_t)} R^f_{t+1} \right] \quad \forall i
\]  

(4)

and, consequently,

\[
0 = \mathbb{E}_t \left[ \frac{u'(C_{t+1})}{u'(C_t)} \left( R^i_{t+1} - R^j_{t+1} \right) \right] \quad \forall i, j,
\]  

(5)

²Kocherlakota (1990), for instance, argues that, from an academic economist's perspective, this model is more important than the widely used CAPM.
where \( \beta \in (0,1) \) is the subjective time discount factor in the representative agent's utility function and, in equilibrium, \( C_t \) represents aggregate consumption at time \( t \).

Equations (4) and (5) are consistent with equations (1) and (2) if we define the SDF at time \( t+1 \), as

\[
M_{t+1} \equiv \beta \frac{u'(C_{t+1})}{u'(C_t)}.
\]

At first, and considering the very restrictive conditions under which aggregation is possible, one may wonder whether such model is not too off the mark to be taken seriously. It turns out that—see Constantinides (1982)—if markets are complete and agents have von-Neumann Morgenstern preferences, aggregation is not needed for the existence of a representative agent. This is true even if individuals are heterogeneous in preferences and in levels of wealth. Moreover, the representative individual's relative risk aversion is no larger than the most risk averse individual and no smaller than the least risk averse individual. The logic is easy to grasp. Perfect risk sharing, which is what complete markets allow for, requires equalization of marginal growth of consumption across agents in all states of the world.

2.1.3 The canonical consumption-based model and the asset pricing puzzles

Once we have decided that we are going to rely on a representative agent framework, a first important issue to deal with is the specification of utility for this investor. The observation that interest rates and risk premiums have remained stationary over more than a century of US economic growth suggests the use of a few specifications among which the most popular is the time-separable power utility defined over aggregate consumption, \( C_t \),

\[
u(C_t) = \frac{C_t^{1-\alpha}}{1-\alpha}.
\]

In this case, equation (5) can be rewritten as

\[
0 = E_t \left[ \left( \frac{C_{t+1}}{C_t} \right)^{-\alpha} \left( R_{t+1} - R_t \right) \right].
\]

This utility function has some other important properties: it is scale-invariant and if agents in the economy have different levels of wealth but have the same power utility, then this will also be
the utility function of the representative agent. Although it has been extensively used in finance literature due to its empirical, analytical and intuitive convenience, it really does not work well in practice, there being many evidences of its incapability to account for important stylized facts. The equity and the forward premium puzzles are two of the most famous and reported empirical failures of this consumption-based approach.

To give a summary account of the theoretical and empirical issues related to the EPP and the FPP, let this representative agent be an American investor who can freely trade American and foreign assets, besides having access to forward and spot exchange rate markets.\(^5\)

The consumption Euler equation (8) is useful to access the EPP, if one takes assets \(i\) and \(j\) to be S&P500 index and the short-term American government bond.

When analyzing the FPP, the relevant assets are the covered and uncovered trade of a foreign government bond. The real returns on these foreign assets in terms of the representative investor’s numeraire can be written respectively as

\[
R^c_{t+1} = \frac{F_{t+1}(1 + i_t^*)P_t}{S_t P^*_t} \quad \text{and} \quad R^u_{t+1} = \frac{S_{t+1}(1 + i_t^*)P_t}{S_t P^*_t}, \tag{9}
\]

where \(F_{t+1}\) and \(S_t\) are the forward and spot prices of foreign currency in terms of domestic currency, \(P_t\) is the dollar price level and \(i_t^*\) represents nominal net return on a foreign asset in terms of the foreign investor’s preferences. It is then possible to express the consumption Euler equation of the excess returns of uncovered operation with foreign bonds as

\[
0 = \sum_{t=0}^{\infty} \left\{ \left( \frac{C_{t+1}}{C_t} \right)^{-\gamma} \frac{P_t(1 + i_t^*)[F_{t+1} - S_{t+1}]}{S_t P^*_t} \right\}, \tag{10}
\]

along the lines of Lucas’ (1978) model.

Most tests reported in the literature are based on over-identifying restrictions on estimations that use Hansen’s (1982) Generalized Method of Moments (GMM). With regards to the FPP, the results obtained in Mark (1985) and Hodrick (1980) report values of the coefficient of relative risk aversion, \(\alpha\), above to 40 and 60, respectively, while Engel (1996) reports some estimates for \(\gamma\) in excess of 100. The findings in the case of EPP are similar, in the sense that in both cases the estimated values are above 10, which is viewed as unacceptable and unreasonable, according to Krugman (1981), Mehra and Prescott (1985) and Romer (1995).

It is beyond the scope of this paper to give a full account of the large body of research produced in the area, we must however call into attention the fact that in both cases an extremely high coefficient of risk aversion is estimated in both cases. Part of our story will be to ask whether the same values for the parameters of different models are needed to account for both puzzles. This is not the whole story, however. Even if we are willing to accept the 'unrealistic' values for the relevant parameters,

\(^5\)Here, we are implicitly assuming the absence of short-sale constraints besides other frictions in the economy, despite we recognize the significance of bid-ask spreads’ impact on the profitability of currency speculation, as mentioned in Burnside et al. (2006).
the over-identifying restrictions may be rejected. We may then check whether the circumstances in which one model is accepted (or rejected) are the same for the two markets.

There is some skepticism with this regard since the risk premiums involved in these different financial markets have their specificities: the high Sharpe ratio related to the equity premium and the predictability of foreign currency excess returns based on the respective forward discount. In da Costa et al. (2006) we have shown that the same $M_{t+1}$ is able to account for risk premium in both markets. We shall now investigate in detail the relative performance of different consumption based models to explain these features. Before doing so, let us briefly describe some stylized facts regarding both markets.

Stock markets commonly show familiar patterns. Observing the US quarterly data over the period 1977:1 to 2004:3, the summary statistics reported in the table 1 are in accordance with the data widely reported in the literature. The average real return on S&P500 has been 8.67% at an annual rate, while the real return on 90-day Treasury bill has been 1.76% per year. Clearly, real stock returns are much more volatile than the US Treasury bill, which is risk-free in nominal terms. Its annualized standard deviation are, respectively, 16.02% and 1.61%.6 We then observe an annualized Sharpe ratio for the US stock market equal to 0.44, which implies this value as the lower bound of the standard deviation for the SDF. Certainly, a very large value for the SDF, a random variable whose mean must be close to one and whose lower bound is zero.

Since our representative agent can freely trade domestic and foreign assets and since the SDF relates all payoffs to market prices, which necessarily includes both uncovered and covered trading of foreign government bonds, should not be important, or even possible, to reconcile the characterization of the SDF provided by foreign government bond-market data with the evidence from US stock-market data?

Over this same period, the real return on covered trading of Canadian, German, Japanese and British government bonds show mean and standard deviation quite similar to the American 90-day Treasury bill one, which is reasonable, since it reflects the covered interest rate parity in a frictionless economy. Regarding the return on uncovered trading, the means are ranging from 2.47% to 4.70%, while the standard deviation from 5.44% to 12.91%. For these foreign bonds, we have that the Sharpe ratios for the uncovered trading of foreign bonds range from 0.04 to 0.28.

Shiller (1982), Hansen and Jagannathan (1991) and Cochrane and Hansen (1992) relate the EPP to the volatility of the SDF, or equivalently the volatility of the intertemporal marginal rate of substitution of a representative investor required to match Sharpe ratio of the stock market. Since the higher the Sharpe ratio, the tighter the lower bound on the volatility of the SDF, when we observe these stylized facts of the American domestic and foreign government bond markets, should we consider them as an evidence that if we were able of writing down a model that consistently generated a SDF that accounts for the EPP, this model would also explain the FPP? Bearing in mind the fact

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6Much of the volatility of the return on 90-day Treasury bill is certainly due to short-run inflation risk.
that an extremely high risk aversion level in the canonical approach for both markets we ask whether there would be other similarities between these risk premiums when other preferences are considered?

The main difficulty one must face in order to answer this question is the non-existence of a widely accepted model capable of generating the observed behavior of $M_{t+1}$. On the one hand, we have the poor performance of the canonical CCAPM as evidenced by the FPP and the EPP, while on the other, we have the still incipient discussion of the underlying assumptions (and empirical consequences) of more successful consumption models as Campbell and Cochrane (1999) and Garcia (2006).

Given our purposes, of simply relating the puzzles, both the successful stories and the failures are useful. The fact that the same unacceptably high risk aversion is needed to account for the two puzzles in the case of the canonical CCAPM help us relate the two puzzles quite as much as the finding of a reasonably low parameter that accounts for the two.

Finding models that produce non-acceptable values is easy. As for the successful ones a natural route is to use the external habit formation models which, in some versions, has proven to be able to account for the EPP and check whether it works for the FPP. In the next few pages we describe some of the properties of these models.

2.1.4 External reference level consumption-based models

The habit formation literature emerges as a natural attempt to capture some features of the consumption behavior, such as the effect of today’s consumption on tomorrow’s marginal utility of consumption, or in macroeconomic terms, the perception that recessions are so feared even though the recession period may not be one of the worst periods in the history. The major theoretical papers on this subject are Ryder and Heal (1973), Sundaresan (1989), and Constantinides (1990).

The main issue to be addressed in this framework is the specification of state variables that will be incorporated in the utility function of the agent, playing the role of a reference consumption level. When the state variable, $X_t$, depends on an agent's own consumption and the agent takes it into account when choosing how much to consume, then we have a standard internal habit model, such as those in Sundaresan (1989) and also Constantinides (1990). When this habit depends on variables which are unaffected by the agent’s own choice, rather depending on what others do, we are dealing with an external habit model. The literature based on this latter framework is rather extensive, including the works of Abel (1990), Abel (1999) and Campbell and Cochrane (1999), to name a few.

We will limit our analysis to the external habit models, for simplicity, since our intent is to explore two other common features of the habit framework when dealing with equity and foreign currency risk premiums.

When modelling habit formation the first thing we must decide is whether to use ratio or differ-
ence. In the first case, with constant risk aversion, the standard time-separable power utility function becomes
\[ u(C_t, X_t) = \frac{1}{(1 - \alpha)\varphi} \left( \frac{C_t}{X_t} \right)^{1-\alpha} X_t^\varphi, \] 
where \( \alpha \) is the curvature parameter for relative consumption, while \( 1 - \varphi \) assumes this same function for the benchmark level.

In the second case, i.e., when one is considered with the difference in consumption with respect to the reference level, the same preferences yield
\[ u(C_t, X_t) = \frac{1}{(1 - \varphi)\varphi} (C_t - X_t)^{1-\alpha} X_t^\varphi. \]

The second feature is related to the speed with which habit reacts to aggregate consumption, where this habit can depend on current and/or lags of the reference consumption level or it can still react only gradually to changes in the benchmark.

In the next subsections we describe in details the modeling strategies of the consumption reference level used in this paper, proposing possible extensions of these models and establishing how the presence of this benchmark changes the respective Euler equations.

**Catching-up with the Joneses** First, we start following the Abel's (1990) catching up with the Joneses model, one of the most traditional and mentioned habit formation approaches. This ratio model, which was initially introduced in order to account for the high observed value for the equity premium, assumes that the habit level depends only on the first lag of aggregate consumption, denoted by \( \overline{C} \).

It is easy to see this formulation as a special case of the equation (11), where \( \varphi = 0 \) and the level habit is given by \( X_t = \overline{C}_{t-1}^\kappa \), which enables us to rewrite it as
\[ u(C_t) = \frac{1}{(1 - \alpha)} \left( \frac{C_t}{\overline{C}_{t-1}^\kappa} \right)^{1-\alpha} \] 
Since, in equilibrium the relation \( C_{t+1} = \overline{C}_{t+1} \) holds, we have that the SDF can be defined as
\[ M_{t+1}^{CJ} = \beta \left( \frac{C_{t+1}}{C_t} \right)^{-\alpha} \left( \frac{C_t}{\overline{C}_{t-1}^\kappa} \right)^{\kappa(\alpha-1)} \] 

Note that the special case with \( \kappa = 0 \) corresponds to the standard time-separable model, while \( \kappa = 1 \) corresponds to the catching up with the Joneses model, where only relative consumption matters to the agent. In our estimation exercise, we follow Fuhrer (2000) in not imposing any restriction to the values of this parameter. Our intent is to figure out whether the absolute reference level matters to the representative agent and whether the agent is of the jealous or patriotic kind.

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8 There might not be a clear answer to the question of which one is optimal for each case, since both models can be justified empirically.
**Keeping-up with the Joneses** Starting with the Abel (1990) model is an interesting choice for it has many possibilities of extension. To disentangle the relative risk aversion from the elasticity of intertemporal substitution and to deal with a problem that appears when this model is used, namely, the high volatility of the risk-free rate of return, Abel (1999) assumes that the agent may take into account not only the information available to him at time $t$, but also some information available at time $t+1$, when he forms his reference consumption level, $X_{t+1}$. More specifically, this framework captures the notion that the agent’s benchmark level depends on current and recent levels of consumption per capita, given by the increasing function

$$X_t = \bar{C}_t^\gamma \bar{C}_{t-1}^\eta (G_t)^\kappa,$$  

where $0 \leq \gamma \leq 1$, $0 \leq \kappa \leq 1$, $0 \leq \eta \leq 1$ and $G \geq 1$, which allows for the possibility that the benchmark level grows simply with the passage of time. The special case with $\kappa > 0$ and $\gamma = \eta = 0$ corresponds to the simple formulation of catching up with the Joneses in Abel (1990) and when $\gamma > 0$ and $\kappa = \eta = 0$, we have the Gali’s (1994) specification of consumption externalities.

Once again, we consider $\varphi = 0$, so that this formulation is a special case of equation (11) where the utility is given by

$$u(C_t) = \frac{1}{(1 - \alpha)} \left( \frac{C_t}{\bar{C}_t^\gamma \bar{C}_{t-1}^\eta (G_t)^\kappa} \right)^{1-\alpha}.$$  

Since, in equilibrium the relation $C_{t+1} = \bar{C}_{t+1}$ holds, we have that the SDF now can be defined as

$$M_{t+1}^{\bar{C}} = \beta G^{\eta(\alpha-1)} \left( \frac{C_{t+1}}{C_t} \right)^{-\alpha+\gamma(\alpha-1)} \left( \frac{C_t}{\bar{C}_{t-1}} \right)^{\kappa(\alpha-1)}$$  

With regards the parameters involved in this Abel (1999) preference specification, there are two important issues to be mentioned. First, there are six parameters $\beta, G, \eta, \alpha, \kappa$ and $\gamma$ that affect the SDF, but only three of them are independent parameters, which implies that, relative to the power utility the current framework introduces only one additional degree of freedom. Second, although this fact allows for a variety of interpretations of preferences, our analysis will consider the Abel’s (1990) estimation results.

One of the main consequences of considering a contemporaneous reference level is that it allows for disentangling risk aversion and intertemporal elasticity of substitution. We shall return to this after discussing Epstein and Zin’s formulation.

### 2.1.5 Epstein-Zin utility specification

The inclusion of extra state variables is not the only possible approach to handling the empirical failures of the canonical CCAPM. In fact, one of the important characteristics of (7) is the fact that the coefficient of relative risk aversion is the inverse of the intertemporal marginal rate of substitution. This means that, allowing for very high risk aversion to account for the equity premium implies
to accept too low an intertemporal marginal rate of substitution, the consequence of which is the appearance of a risk-free puzzle whenever one is willing to accept the high coefficient of relative risk aversion that is needed to correctly price the equity premium. To avoid this trap a successful model must disentangle the two.

This is accomplished within the framework of Epstein and Zin (1991) and Weil (1989) who, building on the work of Kreps and Porteus (1978), define more general (than von-Neumann Morgenstern) preferences which: i) preserve many of the attractive features of power utility as the scale-invariance; ii) break the tight link between the coefficient of risk aversion and the elasticity of intertemporal substitution as in Abel (1999), and; iii) provide an amplified permanent component of the SDF, satisfying the Alvarez and Jermann’s (2002) criticism of the lower bound of the size of this component term.9 The key of this current approach is the link between the reference level and the return on the market portfolio.

The Epstein-Zin objective function can be written as

\[ u(C_t, E_t(U_{t+1})) = \left(1 - \beta\right)^{\frac{1}{1-\rho}} + \beta^\frac{1}{1-\rho} \left(\mathbb{E}_t\left(U_{t+1}^{1-\alpha}\right)\right)^{\frac{1}{1-\rho}}, \]

where \( \theta = (1-\alpha)/(1-\rho) \) and \( 1/\rho \) is the elasticity of intertemporal substitution. In the special case with \( \rho = \alpha \) one can recover easily the time-separable power utility.

One important fact to bear in mind is that the Epstein-Zin non-expected recursive utility specification is observationally equivalent to the ratio habit formation model given by equation (11) if one assumes that

\[ \log \left(\frac{X_{t+1}}{X_t}\right) = \frac{1}{1-\varphi} \log(R_{M,t+1}) + \omega, \]

where \( \omega \) is a constant and \( R_{M,t+1} \) is the US real gross return on the market portfolio. The proof of this equivalence as well as the intuition of this assumption can be see in details in Garcia (2006).

Using this equivalence or the assumption that the investor has no labor income and lives entirely off financial wealth, one is able to show that the SDF can now be written as

\[ M_{t+1}^{EZ} = \beta \left(\frac{C_{t+1}}{C_t}\right)^{\rho(\alpha-1)/(1-\rho)} R_{M,t+1}^{(\rho-\alpha)/(1-\rho)}, \]

2.1.6 Campbell and Cochrane habit formation

Several papers have followed Abel (1990, 1999) in proposing ratio models where the habit depends on the current or on some lags of aggregate consumption. These models however fail to account for some other "recent" stylized facts of the stock market, as the predictable variation in stock returns, pointed as an important source of stock market volatility.

An alternative and promising route to explain this variation predictability is to assume that the price of risk changes through time. Campbell and Cochrane (1999) built a model where the habit

9Alvarez and Jermann’s (2002) argues that using SDF as a function of only consumption data it is not possible to provide a permanent component term higher than the lower bound.
level responds only gradually to changes in consumption, giving emphasis on the role played by a
time-varying risk aversion. This model makes the volatility of the SDF vary in a stochastic fashion
with the business cycle pattern allowing for a understanding of stock market volatility and the high
equity premium.

Campbell and Cochrane (1999) assume that the power utility of the representative agent is a
function of the difference between his own level of consumption and a slow-moving habit, or a time­
varying subsistence level. Consequently, as consumption declines toward the habit in a business cycle
trough, the curvature of the utility function rises, so risky asset prices fall and expected returns rise.
The key aspects responsible for the empirical success of this approach will become more clear as we
describe the main features of the model.

First, the utility function is a special case of the utility given by (12) with \( \varphi = 0 \).

Second, let \( S_t \equiv (C_t - X_t)/C_t \) denote the surplus consumption ratio. This variable captures the
relation between consumption and the habit level, so the closer its value is to one, the better is the
state. Note that the curvature of the utility function, \( \eta_t \), is now time-varying and it can be related
to the surplus consumption ratio by \( \eta_t \equiv \alpha/S_t \), which implies a higher local curvature in the worst
states. Since the model adopts an external habit specification in which habit is determined by the
history of aggregate consumption, define \( \tilde{S}_t \equiv (\overline{C}_t - X_t)/\overline{C}_t \). Once again, note that in equilibrium the
relations \( C_{t+1} \equiv \overline{C}_{t+1} \) and \( S_{t+1} \equiv \tilde{S}_{t+1} \) hold, which enables us to rewrite the SDF as

\[
M_{t+1}^{CC} = \beta \left( \frac{C_{t+1}S_{t+1}}{C_tS_t} \right)^{-\alpha}.
\]

Third, with regards the consumption growth, this process is modeled as an independently and
identically distributed (i.i.d.) lognormal process, \(^{10}\)

\[
c_{t+1} - c_t = g + \nu_{t+1}, \quad \nu_{t+1} \sim \text{iidN}(0, \sigma^2),
\]

where the variables in small letters are the logs of the variables in capital letters.

Finally, in order to specify how each individual's habit \( X_t \) responds to the history of aggregate
consumption \( \overline{C} \), let the log surplus consumption ratio \( \tilde{s}_t = \ln(\tilde{S}_t) \) evolves as a heteroskedasticity
AR(1) process,

\[
\tilde{s}_t = (1 - \phi)\bar{s} + \phi\tilde{s}_t + \lambda(\tilde{s}_t)(\tilde{e}_{t+1} - \bar{e}_t - g)
\]

where \( \phi, \bar{s} \) and \( g \) are the parameters that correspond respectively to the persistence coefficient, the
surplus consumption ratio steady state and the mean of the log consumption growth. \(^{11}\) The \( \lambda(\tilde{s}_t) \)
term, labeled sensitive function, is specified following

\(^{10}\)The Campbell and Cochrane (1999) model can also accommodate more complex consumption processes, including
processes with predictability, conditional heteroskedasticity, and nonnormality.

\(^{11}\)It is convenient, but not necessarily, to use the same value \( g \) for the mean consumption growth rate and the
parameter \( g \) in the habit accumulation equation (23).
\[
\lambda(s_t) = \begin{cases} 
\frac{1}{3} \sqrt{1 - 2(s_t - \bar{s})^2} - 1, & s_t \leq s_{\text{max}} \\
0, & s_t \geq s_{\text{max}}
\end{cases}
\] 

(24)

where \( s_{\text{max}} \equiv \bar{s} + \frac{1}{2}(1 - \bar{S}) \) and \( \bar{S} = \sigma \sqrt{1 - \varphi} \)

in order to produce a constant risk-free rate and to restrict habit behavior to keep the specification close to the traditional and sensible notions on habit. One of the main purposes of the Campbell and Cochrane (1999) model, which has influenced the choice of the sensitivity function and of the parameters, is to guarantee the constancy of the risk-free rate.

2.1.7 Generalized Campbell and Cochrane model

Reference models such as that of Cochrane and Campbell (1999) assume that agents derive utility from the relationship between their private consumption and the reference level. They do not allow for the reference level to directly affect utility. In a recent paper, Garcia (2006) have found evidence that the reference level plays an independent role in the agent’s utility function. We follow them in proposing an extension of Campbell and Cochrane (1999) model in which the agent derives utility from its own level of consumption relative to the reference level both in the ratio and in difference forms. This more general framework is a special case of the setting proposed by Garcia (2006) with \( \varphi = \alpha - 1 \).

In our generalized setting, the utility function and the SDF can be written respectively as

\[
u(C_t) = \frac{1}{(1 - \alpha)} \left( \frac{C_t - X_t}{X_t} \right)^{1-\alpha},
\]

(25)

and

\[
M_{t+1}^{CGG} = \beta \left( \frac{C_{t+1}S_{t+1}}{C_tS_t} \right)^{-\alpha} \left( \frac{X_{t+1}}{X_t} \right)^{(\alpha-1)}.
\]

(26)

3 Empirical application

In this section, we estimate the Euler equations derived from the six consumption-based models described in section 2. Our intent is to either validate or better understand the reasons for the empirical failure of each preference specification, first for the excess return on the S&P500 over the short-term American government bond and, second, for the excess returns on the uncovered over the covered trading of Canadian, German, Japanese and British government bonds jointly.

We first describe the details and sources of the data that is used. Next, we discuss the estimation procedure in light of the identification issues surrounding the preference parameters. We shall also try and justify the criteria used in the choice of the instruments.

\[12\] This means that a higher GDP does not make agents happier if they do not have their incomes increase more than average.
3.1 Data and instruments

In principle, whenever econometric or statistical tests are performed, it is preferable to employ a long data set either in the time-series or in the cross-sectional dimension. There are some obvious limitations, especially when the FPP is concerned, the main one being due to the forward rate series, since a sufficiently large time-series is hardly available\(^{13}\). Regarding the EPP these limitations are less severe. In order to have a common sample we covered the period starting in 1977:1 and ending in 2004:3, with quarterly frequency. We collected foreign exchange data for the following countries: Canada, Germany, Japan, U.K. and the U.S.

When testing the EPP or the FPP we need first to compute excess returns \(R_t^e - R_t^s\). On that regard, our data set is composed of the following. To study the EPP we used the US\$ real returns on the S&P500 and on 90-day T-Bill. To analyze the FPP, we used the US\$ real returns on short-term British, Canadian, German and Japanese government bonds, where both spot and forward exchange-rate data were used to transform returns denominated in foreign currency into US\$. The forward rate series were extracted from the Chicago Mercantile of Exchange database, while the spot rate series were extracted from Bank of England database.

As for macroeconomic variables, we have used the consumer price index of services and nondurable goods in the US in order to compute the US\$ real returns besides the seasonally adjusted US per capita consumption of services and nondurable goods and US per capita gross domestic product. All of these variables were extracted from FED’s FRED database.

The final ingredient for testing these two puzzles consists in the additional variables used in orthogonality tests. In choosing these instrumental variables, we follow Mark (1985), since we believe that the instruments he used, which are macroeconomic and financial variables observed at time \(t\) by all agents, are among the most important variables used in forecasting, respectively, the SDF and the returns in question. Consequently, this set may be capable of characterizing the conditional distribution of the discounted excess returns.

For the equity premium, we use the instrument sets IS\(_1\) and IS\(_2\), given respectively by

- **IS\(_1\):** Two lags of the real US\$ returns on S&P500 and on the 90-day T-Bill and three lags of real consumption and real GDP growth rates.
- **IS\(_2\):** Three lags of the real US\$ returns on S&P500 and on the 90-day T-Bill and three lags of real consumption and real GDP growth rates.

\(^{13}\)Chicago Mercantile Exchange pioneered the development of financial futures with the launch of currency futures, the world’s first financial futures contracts, in 1972.
For the foreign currency premiums, we use the instrument sets IS$_3$ and IS$_4$, given respectively by

- **IS$_3$**: Two lags of the real US$ returns on the 90-day T-Bill and on uncovered trading of short-term British, Canadian, German, and Japanese government bonds and three lags of real consumption and real GDP growth rates.
- **IS$_4$**: Three lags of the real US$ returns on the 90-day T-Bill and on uncovered trading of short-term British, Canadian, German, and Japanese government bonds and three lags of real consumption and real GDP growth rates.

This procedure allows us to verify the robustness of the empirical results and the common pattern of the equity and foreign currency premiums.$^{11}$

In order to analyze the predictability power of the forward premium pointed as a specificity of the FPP, we also use the instrument sets IS$_5$ and IS$_6$, given respectively by

- **IS$_5$**: The current and the first lag of the forward premiums on the currencies in question and three lags of real consumption and real GDP growth rates.
- **IS$_6$**: The current, the first and the second lag of the forward premiums on the currencies in question and three lags of real consumption and real GDP growth rates.

### 3.2 Estimation

In order to evidence the common pattern displayed by the equity and the foreign currency risk premiums based on a representative agent approach, we start by testing and estimating the Euler equation derived from the canonical consumption model, which works as our benchmark. To better understand the behavior of the equity premium, the pricing error on the excess return on the S&P500 over the short-term American government bond is minimized with the Hansen’s (1982) Generalized Method of Moments (GMM). For this case the Euler equation is given by

$$
\mathbb{E}_t \left( \frac{C_{t+1}}{C_t} \right)^{-\alpha} (R^{SP}_{t+1} - R^{T_{b1}}_{t+1}) = 0, \tag{27}
$$

and the sets of instruments used are IS$_1$ and IS$_2$.

If the purpose is, however, to analyze the behavior of the foreign currency risk premiums, then the pricing errors on the excess return on the uncovered over the covered trading of Canadian, German,$^{14}$

---

$^{11}$We recognize that in order to obtain a "perfect" comparison of the behaviors of the equity and the foreign currency risk premiums, the instrument sets IS$_3$ and IS$_4$ should include lags of the US$ real returns on the covered trading of foreign government bonds instead of on returns on 90-day T-bill. But, since the covered interest rate parity must hold, this our procedure provides an excellent approximation that still enables us to deal with the problem of redundant instruments.
Japanese and British government bonds are jointly minimized with the Hansen’s (1982) GMM. We, therefore, apply the same estimation procedure to the following Euler equations

\[
E_t \left[ \left( \frac{C_{t+1}}{C_t} \right)^{-\alpha} \left( \frac{C_t}{C_{t-1}} \right)^{\kappa(\alpha-1)} (R^{SP}_{t+1} - R^{Pl}_{t+1}) \right] = 0,
\]

and the sets of instruments used are IS_3, IS_4, IS_5 and IS_6.

Regarding to the instruments, when we alter the set, adding lags of the financial variables our intent is to evidence the robustness of the results, while when we change all the financial variables, which happens only with the analysis of the FPP, we intend to analyze the relative predictability power of the financial variables in question.

The typical procedure adopted to validate or to better understand the reasons of the empirical failure of each preference specification for the excess returns in question consists in verifying if the model is supported by the data, in the sense of rejecting (or not) the over-identifying restriction and of analyzing the values of the parameters estimated in the light of the literature besides their significance at the 5% level. The results for this first model are reported in table 2.

There is a complete similarity between the methodologies used to estimate the canonical Euler equations and the Euler equations derived from the catching up with the Joneses model, given by

\[
E_t \left[ \left( \frac{C_{t+1}}{C_t} \right)^{-\alpha} \left( \frac{C_t}{C_{t-1}} \right)^{\kappa(\alpha-1)} (R^{SP}_{t+1} - R^{Pl}_{t+1}) \right] = 0,
\]

and

\[
E_t \left[ \left( \frac{C_{t+1}}{C_t} \right)^{-\alpha} \left( \frac{C_t}{C_{t-1}} \right)^{\kappa(\alpha-1)} (\tilde{R}^{U}_{t+1} - \tilde{R}^{C}_{t+1}) \right] = 0.
\]

and from the keeping up with the Joneses model, given by

\[
E_t \left[ \left( \frac{C_{t+1}}{C_t} \right)^{\theta} \left( \frac{C_t}{C_{t-1}} \right)^{\theta} (R^{SP}_{t+1} - R^{Pl}_{t+1}) \right] = 0,
\]

and

\[
E_t \left[ \left( \frac{C_{t+1}}{C_t} \right)^{\theta} \left( \frac{C_t}{C_{t-1}} \right)^{\theta} (\tilde{R}^{U}_{t+1} - \tilde{R}^{C}_{t+1}) \right] = 0.
\]

The results for these models are reported respectively in tables 3 and 4.

Now, consider the Euler equations derived from the Epstein and Zin utility specification, which are given by

\[
E_t \left[ \left( \frac{C_{t+1}}{C_t} \right)^{\rho(\alpha-1)/(1-\rho)} R^{(\rho-\alpha)/(1-\rho)}_{M,(t+1)} (R^{SP}_{t+1} - R^{Pl}_{t+1}) \right] = 0,
\]

and

\[
E_t \left[ \left( \frac{C_{t+1}}{C_t} \right)^{\rho(\alpha-1)/(1-\rho)} R^{(\rho-\alpha)/(1-\rho)}_{M,(t+1)} (\tilde{R}^{U}_{t+1} - \tilde{R}^{C}_{t+1}) \right] = 0.
\]

For this case, the procedure adopted is the same one already described with an additional specificity, how to obtain a time series for the external reference level. In these terms, we use the value-weighted return to all stocks listed on the NYSE and AMEX obtained from the Center for Research in
Security Prices (CRSP) as a proxy for the unobservable gross return on market portfolio. The results for this model are reported in table 5.

Finally, we consider the Euler equations derived from the Campbell and Cochrane (1999) model, which are given by
\[
\mathbb{E}_t \left[ \left( \frac{C_{t+1}S_{t+1}}{C_t S_t} \right)^{\alpha} (R_{t+1}^{SP} - R_{t+1}^{PB}) \right] = 0, \tag{35}
\]
and
\[
\mathbb{E}_t \left[ \left( \frac{C_{t+1}S_{t+1}}{C_t S_t} \right)^{\alpha} (\bar{R}_{t+1}^{U} - \bar{R}_{t+1}^{C}) \right] = \bar{\gamma}, \tag{36}
\]
and the Euler equations derived from the generalized version of this latter model, given by
\[
\mathbb{E}_t \left[ \left( \frac{C_{t+1}S_{t+1}}{C_t S_t} \right)^{\alpha} \left( \frac{X_{t+1}}{X_t} \right)^{(\alpha-1)} (R_{t+1}^{SP} - R_{t+1}^{PB}) \right] = 0, \tag{37}
\]
and
\[
\mathbb{E}_t \left[ \left( \frac{C_{t+1}S_{t+1}}{C_t S_t} \right)^{\alpha} \left( \frac{X_{t+1}}{X_t} \right)^{(\alpha-1)} (\bar{R}_{t+1}^{U} - \bar{R}_{t+1}^{C}) \right] = \bar{\gamma}. \tag{38}
\]

When we use the Campbell and Cochrane specification and its generalized version, our main interest remains to evidence the common pattern displayed by the equity and the foreign currency risk premiums. Once again, the procedure adopted to estimate these more complex models is the typical one already described with an additional specificity related to the non-observability of the surplus consumption ratio, $S_t$. In order to compute the time series of this process, we need to set its initial value and the values of the parameters $\sigma, \gamma, \phi$ and $\alpha$.

With regards to the initial value of the surplus consumption ratio, we set it at the steady state value, $s_0 = \bar{s}$.

In choosing the parameters $\sigma, \gamma$ and $\phi$, we follow Campbell and Cochrane (1999), calibrating them. Since $\sigma$ and $\gamma$ correspond respectively to the standard deviation and the mean of the log consumption growth, we take them to match the consumption data over the period 1977:1 to 2004:3, obtaining the values 0.005 and 0.004. With regards the persistence parameter $\phi$, we take it to match the serial correlation of log price/dividend ratio ratios, obtaining a value of 0.989.

For the risk aversion coefficient $\alpha$, we follow Garcia (2006), proceeding by grid search to obtain an initial value that is close to the value obtained from the estimation of the Euler equations by GMM, where $\alpha$ varies between 0.00 and 50.00.

4 Results

Results reported in table 2, corresponding to the Euler equations (27) and (28) derived from the canonical consumption model evidence that this model is able to price correctly neither the excess return on the equity over the 90-day T-Bill nor the excess return on the uncovered over covered trading of foreign government bonds with acceptably low values of risk aversion. We do not have
problems with the significance of the relative risk aversion $\alpha$ and we do not reject the over-identifying restrictions. Comparing the puzzles more closely, for both of them the inclusion of the third lag of the real returns in question provides a lower risk aversion.

This empirical failure widely reported in the literature is related with the smoothness of the consumption growth along with its low correlation with the excess returns in question which require extremely high values for the risk aversion to account for the equity and the foreign currency Sharpe ratios.\footnote{One could argue that this empirical failure can associated with the problem of a near nonidentification of the risk aversion parameter in the canonical model.}

In order to try to account for the puzzles we analyze the more complex preference specifications. Table 3 presents the findings associated with Euler equations (29) and (30) derived from the Abel (1990) specification where the reference level is a function of past aggregate consumption. Comparing both puzzles: \textit{i)} the EPP and the FPP are exacerbated, in the sense that, for each instrument set used, the relative risk aversion is larger then the respective one obtained with the canonical model, \textit{ii)} we have the same magnitude order of the parameter $\kappa$ and \textit{iii)} contrary to the ratio models, but in accordance with Garcia (2006), we find support for the hypothesis that the absolute value of the past aggregate consumption enters the utility function. Only for the EPP, the parameter $\kappa$ is significant at the 10% significance level but not at the 5% level.

This model is not supported by the equity market neither the foreign currency market data, since the representative agent is of a jealous sort. Even with a positive $\kappa$ — where, according to Abel (1990) it would be possible to increase the risk aversion to solve the EPP without generating the risk-free rate puzzle one should not expect to explain both puzzles with this model.

In order to deal with this problem that appears when this Abel (1990) model is used, the high volatility of the risk-free rate of return, and to allow the disentangling of the relative risk aversion from the elasticity of intertemporal substitution, we estimate the Euler equations (31) and (32) derived from the Abel (1999) model. Observing the results reported in table 4, once more the equity and the foreign currency risk premiums show common patterns.

For interpretation purposes, we analyze the results in the light of Abel’s (1990) results. First, we consider that only the parameters $\kappa$ and $\gamma$ are independent. In this case, with regards the past aggregate consumption the agent is a jealous sort, while with regards the current aggregate consumption the agent is patriotic. When we consider that $\alpha$ and $\gamma$ are the independent parameters, the agent remains patriotic with regards the current aggregate consumption and both puzzles are still more exacerbated.

Since, according to our results, the models which adopt state variables as a function only of societal levels of consumption are unable to support the data with reasonable values for the parameters, we test the Epstein and Zin (1991) preference which adopts a reference level as a function of the return on the market portfolio. The results of the estimation of the Euler equations (33) and (34), reported in
Table 5, show clearly that this attempt to break the tight link between the coefficient of risk aversion and the elasticity of intertemporal substitution does not produce a complete explanation of none of the puzzles. In fact, even when we obtaining $\rho < \alpha$, which is a necessary condition to fit the patterns of the risk premiums, both puzzles are still more exacerbated. When we are modeling the risk premiums in these different markets we obtain close values for the elasticity of intertemporal substitution, but these values are not large enough, ranging from 0.014 to 0.022.

Our evidence about the incapacity of the Epstein and Zin (1989) framework to explain the FPP corroborates Colacito and Croce (2005), while with regards this same incapacity to explain the EPP, parts way from Epstein and Zin (1991) and Garcia (2006).\footnote{The strategy adopted in these papers is different of ours, since the Euler equations derived from the Epstein and Zin (1989) specification for the excess return on equity over 90-day T-bill and on the return on 90-day T-bill are estimated jointly.}

Even though several papers in this literature have followed Abel (1990, 1999) in the sense of proposing ratio models where the habit depends on the current or on some lags of aggregate consumption, this path seems to be doomed, since it cannot account for the predictable variation in stock returns which is an important source of stock market volatility.

An alternative and promising route is to assume that the price of risk changes through time. With this in mind, and intending to account for the most stock market stylized facts and to better understand asset pricing puzzles, Campbell and Cochrane (1999) built a model where the habit level responds only gradually to changes in consumption, giving emphasis on the role played by a time-varying risk aversion. This model is able to make the volatility of the SDF stochastically time-varying with the business cycle pattern and to better understand the stock market volatility.

Although this model can not be considered as a complete or even as a definite model of stock market behavior, it has been pointed as the main contender to explain it. This success is certainly due to its refined and pragmatic theoretical conception besides its excellent performance in the calibration exercise proposed by Campbell and Cochrane (1999). According to the results of this exercise, the artificial data generated from the model display the patterns found in the empirical literature, but they do not estimate the coefficient of relative risk aversion and set it equal to 2.

Here, our strategy to evidence if the risk premiums in the equity and the foreign currency markets have a similar behavior is different. We propose a GMM estimation procedure to see if these versions of this habit formation model are supported by the data, instead of calibrating them.

The estimation and test results of the Euler equations (35) and (36) derived from the Campbell and Cochrane (1999) model are reported in Table 6. For the equity and for the foreign currency risk premiums, using the lags of the returns in question as the financial instruments, the values of $\alpha$ are acceptably low but not significant at the 10% significance level. When we test the Euler equation (36) using the current and the past levels of the forward premium as the financial instruments, the values of $\alpha$ remain acceptably low but now we do not have problems with the significance. This result gives
us support to emphasize the prominent role played by the forward premium, relative to lags of real returns in question, as instrument in a external habit formation approach explanation of the FPP, which may be related to the strong predictability power of the forward premium.

Comparing our results for the EPP with the result reported in the literature, while our insignificant values of \( \alpha \) range from 8.01 to 8.21, Tallarini and Zhang (2005) obtains a significant \( \alpha \) equal to 6.30.

Our last attempt to model the risk premiums in these different markets relies on an extension of the Campbell and Cochrane (1999) setting, considering the case where the agent derives utility from its own level of consumption relative to the reference level both in the ratio and in difference forms. Besides allowing for a time-varying risk-free interest rate and disentangling the relative risk aversion coefficient and the elasticity of intertemporal substitution, according to the results of the Euler equations (37) and (38) reported in table 7, we can say that this extended model — with low interest rates, roughly i.i.d. consumption growth with small volatility and acceptably low values of risk aversion — is able to explain the high market Sharpe ratio that characterizes the EPP and also the strong predictability of the foreign government bond returns which characterizes the FPP.

It is hard to compare accurately our results with others reported in this literature, since the estimation strategies used and the models tested are different. At any rate, with regards the EPP, our values of \( \alpha \) range from 7.64 to 8.09, quite larger than 0.31 obtained in García (2006), where the Euler equations for the excess market portfolio return and for the risk free rate are jointly estimated. About the FPP, we could compare our results with Verdelhan (2006), where the model proposed allows for a time-varying risk-free interest rate but does not break the tight link between the relative risk aversion coefficient and the elasticity of intertemporal substitution. Our values for of \( \alpha \) range from 1.68 to 8.81, while his values range from 5.60 to 8.20.

In general terms: i) we can evidence the robustness of our estimation results for the EPP and the FPP when the instrument set is altered, except for the Euler equations derived from the Abel (1999) and the Epstein and Zin (1989) models and ii) according the Hansen's test of over-identifying restrictions, neither of the six models is rejected statistically.\(^{17}\)

Up to this point, we believe to have provided evidence in support of the fact that, under a preference-based approach, the equity and the foreign currency risk premiums display common patterns.

5 Conclusion

It was never clear that solving puzzles in domestic financial markets would help explaining puzzles in foreign currency markets. In fact, traditional consumption-based models do not seem to be able to account for the high equity Sharpe ratio nor to capture the non-shared characteristic of the foreign

\(^{17}\)Not necessarily these non-rejections are much informative, since one could attribute them to the poor properties of conventional asymptotics for some of the specifications used.
exchange market the question ceased to be posed for a long time.

Recent research, however, has lead many in the profession to believe that is a tight association between puzzles in domestic and foreign markets. We join the recent crowd in this paper adopting the following procedure: we analyze the behavior of the risk premiums in these different markets using different models and seeking not only for successes but mostly by the relative performance of different models in the two markets.

We model the excess returns of the S&P500 over the US short term bond and of the uncovered over the covered trading of foreign government bonds, using the canonical and some external habit formation consumption approaches. Based on estimates of these consumption Euler equations and on the test of its over-identifying restrictions using Hansen's (1982) Generalized Method of Moments (GMM), we intend to validate or to better understand the reasons of the empirical failure of each preference specification for both excess returns in question.

We present an empirical evidence of the strong similarity between the behavior of the equity and the uncovered excess returns when modeled as risk premiums, which allows us to argue that, under a preference-based approach, the equity and the foreign currency risk premiums display common patterns. We find empirical grounds to believe that the proposed preference-based solutions to puzzles in domestic financial markets can certainly shed light on the FPP.

In particular, our results offers some support to the idea that a generalized version of the slow-moving external habit formation model proposed by Campbell and Cochrane (1999) which disentangles the relative risk aversion coefficient and the elasticity of intertemporal substitution is able to explain both puzzles.

References


<table>
<thead>
<tr>
<th>Table 1: Data summary statistics*</th>
</tr>
</thead>
<tbody>
<tr>
<td>real consumption (of nondurable and services) growth**</td>
</tr>
<tr>
<td>Mean</td>
</tr>
<tr>
<td>United States</td>
</tr>
<tr>
<td>--------------------------------------------------</td>
</tr>
<tr>
<td>real return on 90-day Treasury bill**</td>
</tr>
<tr>
<td>Mean</td>
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<tr>
<td>--------------------------------------------------</td>
</tr>
<tr>
<td>real return on covered trading**</td>
</tr>
<tr>
<td>Mean</td>
</tr>
<tr>
<td>Germany</td>
</tr>
<tr>
<td>Japan</td>
</tr>
<tr>
<td>United Kingdom</td>
</tr>
</tbody>
</table>

* Quarterly data. Observations from 1977:I to 2004:III - 111 observations. ** Means and standard deviations are given in annualized percentage points.

To annualize the raw quarterly numbers, means are multiplied by 400 while standard deviations are multiplied by 200.
Table 2: Testing CRRA preference

\[ \mathbb{E}_t \left[ \left( \frac{C_{t+1}}{C_t} \right)^{-\alpha} R^{x,x}_{t+1} \right] = 0 \]

<table>
<thead>
<tr>
<th>Equity Premium Puzzle</th>
<th>Forward Premium Puzzle</th>
</tr>
</thead>
<tbody>
<tr>
<td>( R^{x,ex}<em>{t+1} ) = ( R^{SP}</em>{t+1} - R^{Th}_{t+1} )</td>
<td>( \tilde{R}^{x,ex}<em>{t+1} = \tilde{R}^U</em>{t+1} - \tilde{R}^C_{t+1} )</td>
</tr>
<tr>
<td>IS1 ( \alpha )</td>
<td>IS2 ( \alpha )</td>
</tr>
<tr>
<td>115.52</td>
<td>101.63</td>
</tr>
<tr>
<td>(0.004)</td>
<td>(0.004)</td>
</tr>
<tr>
<td>J-statistic ( \alpha )</td>
<td>J-statistic ( \alpha )</td>
</tr>
<tr>
<td>0.0932</td>
<td>0.1022</td>
</tr>
<tr>
<td>(0.443)</td>
<td>(0.523)</td>
</tr>
</tbody>
</table>

Notes: * Coefficient not significant at the 5% significance level. ** Reject the overidentifying restrictions at the 5% significance level. \(^1\) Hansen’s (1982) Generalized Method of Moments (GMM) technique is used to test Euler equations and estimate the model parameters.

Instrument sets:
- IS1: Two lags of the real US$ returns on S&P500 and on the 90-day T-Bill and three lags of real consumption and real GDP growth rates.
- IS2: Three lags of the real US$ returns on S&P500 and on the 90-day T-Bill and three lags of real consumption and real GDP growth rates.
- IS3: Two lags of the real US$ returns on the 90-day T-Bill and on uncovered trading of short-term British, Canadian, German, and Japanese government bonds and three lags of real consumption and real GDP growth rates.
- IS4: Three lags of the real US$ returns on the 90-day T-Bill and on uncovered trading of short-term British, Canadian, German, and Japanese government bonds and three lags of real consumption and real GDP growth rates.
- IS5: The current and the first lag of the forward premia on the currencies in question and three lags of real consumption and real GDP growth rates.
- IS6: The current, the first and the second lags of the forward premia on the currencies in question and three lags of real consumption and real GDP growth rates.
Table 3: Testing Abel (1990) catching up with the Joneses preference

\[
E_t \left[ \left( \frac{C_{t+1}}{C_t} \right)^{-\alpha} \left( \frac{C_t}{C_{t-1}} \right)^{\alpha-1} R^{exc}_{t+1} \right] = 0
\]

<table>
<thead>
<tr>
<th>Equity Premium Puzzle</th>
<th>Forward Premium Puzzle</th>
</tr>
</thead>
<tbody>
<tr>
<td>( R^{exc}<em>{t+1} = R^{SP}</em>{t+1} - R^{TB}_{t+1} )</td>
<td>( \tilde{R}^{exc}<em>{t+1} = \tilde{R}^{U}</em>{t+1} - \tilde{R}^{C}_{t+1} )</td>
</tr>
<tr>
<td><strong>IS</strong></td>
<td>IS2</td>
</tr>
<tr>
<td>( \alpha ) (p-value)</td>
<td>117.61 (0.008)</td>
</tr>
<tr>
<td>( k ) (p-value)</td>
<td>-0.70* (0.065)</td>
</tr>
<tr>
<td>J-statistic (p-value)</td>
<td>0.9966 (0.329)</td>
</tr>
</tbody>
</table>

Notes: * Coefficient not significant at the 5% significance level. ** Reject the overidentifying restrictions at the 5% significance level. 1 Hansen’s (1982) Generalized Method of Moments (GMM) technique is used to test Euler equations and estimate the model parameters.

Instrument sets:

IS1: Two lags of the real US$ returns on S&P500 and on the 90-day T-Bill and three lags of real consumption and real GDP growth rates.

IS2: Three lags of the real US$ returns on S&P500 and on the 90-day T-Bill and three lags of real consumption and real GDP growth rates.

IS3: Two lags of the real US$ returns on the 90-day T-Bill and on uncovered trading of short-term British, Canadian, German, and Japanese government bonds and three lags of real consumption and real GDP growth rates.

IS4: Three lags of the real US$ returns on the 90-day T-Bill and on uncovered trading of short-term British, Canadian, German, and Japanese government bonds and three lags of real consumption and real GDP growth rates.

IS5: The current and the first lag of the forward premia on the currencies in question and three lags of real consumption and real GDP growth rates.

IS6: The current, the first and the second lags of the forward premia on the currencies in question and three lags of real consumption and real GDP growth rates.
Table 4: Testing Abel (1999) *keeping up with the Joneses* preference

\[ E_t \left[ \left( \frac{C_{t+1}}{C_t} \right)^A \left( \frac{C_t}{C_{t-1}} \right)^\theta R_{t+1}^{xc} \right] = 0 \]

where, \( A = -\alpha + \gamma(\alpha - 1) \) and \( \theta = (\alpha - 1)\kappa \)

<table>
<thead>
<tr>
<th></th>
<th>Equity Premium Puzzle</th>
<th>Forward Premium Puzzle</th>
</tr>
</thead>
<tbody>
<tr>
<td>( R_{t+1}^{xc} = R_{t+1}^{SP} - R_{t+1}^{TB} )</td>
<td>IS_1</td>
<td>IS_2</td>
</tr>
<tr>
<td>( A )</td>
<td>-90.44</td>
<td>-93.89</td>
</tr>
<tr>
<td>(p-value)</td>
<td>(0.016)</td>
<td>(0.001)</td>
</tr>
<tr>
<td>( \theta )</td>
<td>-55.95</td>
<td>-57.78</td>
</tr>
<tr>
<td>(p-value)</td>
<td>(0.020)</td>
<td>(0.036)</td>
</tr>
<tr>
<td>J-statistic</td>
<td>0.1086</td>
<td>0.1099</td>
</tr>
<tr>
<td>(p-value)</td>
<td>(0.235)</td>
<td>(0.384)</td>
</tr>
</tbody>
</table>

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Instrument sets:

IS1: Two lags of the real US$ returns on S&P500 and on the 90-day T-Bill and three lags of real consumption and real GDP growth rates.

IS2: Three lags of the real US$ returns on S&P500 and on the 90-day T-Bill and three lags of real consumption and real GDP growth rates.

IS3: Two lags of the real US$ returns on the 90-day T-Bill and uncovered trading of short-term British, Canadian, German, and Japanese government bonds and three lags of real consumption and real GDP growth rates.

IS4: Three lags of the real US$ returns on the 90-day T-Bill and uncovered trading of short-term British, Canadian, German, and Japanese government bonds and three lags of real consumption and real GDP growth rates.

IS5: The current and the first lag of the forward premia on the currencies in question and three lags of real consumption and real GDP growth rates.

IS6: The current, the first and the second lags of the forward premia on the currencies in question and three lags of real consumption and real GDP growth rates.
Table 5: Testing Epstein and Zin (1989, 1991) preference

\[ \mathbb{E}_t \left[ \left( \frac{C_{t+1}}{C_t} \right)^{\alpha(1-\rho)} \frac{R^u_{t+1}}{R^c_{t+1}} \right] = 0 \]

<table>
<thead>
<tr>
<th>Equity Premium Puzzle</th>
<th>Forward Premium Puzzle</th>
</tr>
</thead>
<tbody>
<tr>
<td>( R^e_{t+1} = R^{sp}<em>{t+1} - R^{th}</em>{t+1} )</td>
<td>( \tilde{R}^e_{t+1} = \tilde{R}^u_{t+1} - \tilde{R}^c_{t+1} )</td>
</tr>
<tr>
<td>IS(_1)</td>
<td>IS(_2)</td>
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<tr>
<td>( \alpha )</td>
<td>244.44</td>
</tr>
<tr>
<td>(p-value)</td>
<td>(0.000)</td>
</tr>
<tr>
<td>( \rho )</td>
<td>69.24</td>
</tr>
<tr>
<td>(p-value)</td>
<td>(0.005)</td>
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<tr>
<td>J-statistic</td>
<td>0.0955</td>
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<tr>
<td>(p-value)</td>
<td>(0.338)</td>
</tr>
</tbody>
</table>

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Instrument sets:
IS\(_1\): Two lags of the real US$ returns on S&P500 and on the 90-day T-Bill and three lags of real consumption and real GDP growth rates.
IS\(_2\): Three lags of the real US$ returns on S&P500 and on the 90-day T-Bill and three lags of real consumption and real GDP growth rates.
IS\(_3\): Two lags of the real US$ returns on the 90-day T-Bill and on uncovered trading of short-term British, Canadian, German, and Japanese government bonds and three lags of real consumption and real GDP growth rates.
IS\(_4\): Three lags of the real US$ returns on the 90-day T-Bill and on uncovered trading of short-term British, Canadian, German, and Japanese government bonds and three lags of real consumption and real GDP growth rates.
IS\(_5\): The current and the first lag of the forward premium on the currencies in question and three lags of real consumption and real GDP growth rates.
IS\(_6\): The current, the first and the second lags of the forward premium on the currencies in question and three lags of real consumption and real GDP growth rates.
Table 6: Testing Campbell and Cochrane (1990) habit formation preference

\[ \mathbb{E}_t \left[ \left( \frac{C_{t+1}S_{t+1}}{C_tS_t} \right)^{-\alpha} R_{t+1}^{ex} \right] = 0 \]

<table>
<thead>
<tr>
<th>Equity Premium Puzzle</th>
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<tbody>
<tr>
<td>( R_{t+1}^{ex} = R_t^{SP} - R_t^{Tb} )</td>
<td>( \vec{R}_{t+1}^{ex} = \vec{R}_U^{t+1} - \vec{R}_C^{t+1} )</td>
</tr>
<tr>
<td>IS_1</td>
<td>IS_2</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>$\alpha$</td>
</tr>
<tr>
<td>( (p\text{-value}) )</td>
<td>( (p\text{-value}) )</td>
</tr>
<tr>
<td>8.21$^*$</td>
<td>8.01$^*$</td>
</tr>
<tr>
<td>(0.212)</td>
<td>(0.200)</td>
</tr>
<tr>
<td>J-statistic</td>
<td>0.1201</td>
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<tr>
<td>(p-value)</td>
<td>(0.230)</td>
</tr>
</tbody>
</table>

Notes: $^*$ Coefficient not significant at the 5% significance level. $^{**}$ Reject the overidentifying restrictions at the 5% significance level. 1 Hansen’s (1982) Generalized Method of Moments (GMM) technique is used to test Euler equations and estimate the model parameters.

Instrument sets:
- IS\_1: Two lags of the real US\$ returns on S&P500 and on the 90-day T-Bill and three lags of real consumption and real GDP growth rates.
- IS\_2: Three lags of the real US\$ returns on S&P500 and on the 90-day T-Bill and three lags of real consumption and real GDP growth rates.
- IS\_3: Two lags of the real US\$ returns on the 90-day T-Bill and on uncovered trading of short-term British, Canadian, German, and Japanese government bonds and three lags of real consumption and real GDP growth rates.
- IS\_4: Three lags of the real US\$ returns on the 90-day T-Bill and on uncovered trading of short-term British, Canadian, German, and Japanese government bonds and three lags of real consumption and real GDP growth rates.
- IS\_5: The current and the first lag of the forward premia on the currencies in question and three lags of real consumption and real GDP growth rates.
- IS\_6: The current, the first and the second lags of the forward premia on the currencies in question and three lags of real consumption and real GDP growth rates.
Table 7: Testing a generalized Campbell and Cochrane (1990) habit formation preference

\[ E_t \left( \left( \frac{C_{t+1} S_{t+1}}{C_t S_t} \right) - \left( \frac{X_{t+1}}{X_t} \right)^{(\alpha-1)} \right) R_{t+1}^{exp} = 0 \]

<table>
<thead>
<tr>
<th>Equity Premium Puzzle</th>
<th>Forward Premium Puzzle</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>IS_1</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>8.09</td>
</tr>
<tr>
<td>(p-value)</td>
<td>(0.049)</td>
</tr>
<tr>
<td>J-statistic</td>
<td>0.1060</td>
</tr>
<tr>
<td>(p-value)</td>
<td>(0.334)</td>
</tr>
</tbody>
</table>

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Instrument sets:
IS_1: Two lags of the real US$ returns on S&P500 and on the 90-day T-Bill and three lags of real consumption and real GDP growth rates.
IS_2: Three lags of the real US$ returns on S&P500 and on the 90-day T-Bill and three lags of real consumption and real GDP growth rates.
IS_3: Two lags of the real US$ returns on the 90-day T-Bill and on uncovered trading of short-term British, Canadian, German, and Japanese government bonds and three lags of real consumption and real GDP growth rates.
IS_4: Three lags of the real US$ returns on the 90-day T-Bill and on uncovered trading of short-term British, Canadian, German, and Japanese government bonds and three lags of real consumption and real GDP growth rates.
IS_5: The current and the first lag of the forward premia on the currencies in question and three lags of real consumption and real GDP growth rates.
IS_6: The current, the first and the second lags of the forward premia on the currencies in question and three lags of real consumption and real GDP growth rates.
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N.Cham. P/EPGE SA M433d
Autor: Matos, Paulo Rogerio Faustino
Título: Do equity and foreign currency risk premiums

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