The Marginal Cost of Funds from Public Sector Borrowing

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(Revised March 1, 2004)

Abstract

An expression for the welfare cost of a marginal increase in the public debt is derived using a simple AK endogenous growth model. This measure of the marginal cost of public funds (MCF) can be interpreted as the marginal benefit-cost ratio that a debt-financed public project needs in order to generate a net social gain. The model predicts an increase in the public debt ratio will have little effect on the optimal public expenditure ratio and that most of the adjustment will occur on the tax side of the budget.

I would like to thank Ergete Ferede, Doug Hostland, Liqun Liu, Tiff Macklem, Max Nikitin, Chris Ragan, Todd Smith, and Bill Watson for their comments on an earlier draft of this paper. The financial support of the Donner-Canadian Foundation for my research on the marginal cost of public funds is gratefully acknowledged.
1. Introduction

The marginal cost of public funds (MCF) is usually defined as the social cost of a tax rate increase that raises an additional dollar of tax revenue. Most models of the MCF have focused on measuring the social cost of raising revenues caused by labour market distortions. See, for example, Wildasin (1984), Browning (1987), Snow and Warren (1996), and Dahlby (1998). A few studies have measured the MCF for taxes that distort savings and investment decisions. See, for example, Fullerton and Henderson (1989). While the MCF has usually been defined for a tax rate increase, it can also be defined for other measures that allow governments to finance additional public expenditures. For example, Fortin and Lacroix (1994) and Poapongsakorn et al. (2000) have developed measures of the MCF from increased tax enforcement activity. A natural extension of the MCF concept is to calculate the marginal social cost of funds obtained by public sector borrowing. However, to my knowledge, there are no other studies that have developed a measure of the MCF for debt financing. This is an important gap in the literature because the MCF for debt financing can be interpreted as the “hurdle benefit-cost ratio” that a debt-financed public project needs in order to generate a net social gain. Thus, the MCF for debt financing has important applications in the cost-benefit analysis of debt-financed projects. The objective of this paper is to start to fill this important gap in the public finance literature by developing a measure of the MCF for public sector borrowing.

The public debt imposes a burden on an economy because higher taxes have to be levied to finance the interest payments on the public debt. The higher tax rate may reduce the incentive to save and invest, thereby lowering the long-term growth rate of the economy. In this paper, a simple AK endogenous growth model is used to explore the
connections between the public debt, distortionary taxation, and the rate of economic growth and to develop a measure of the MCF for public sector borrowing. It is a model of a closed economy, where the net savings rate (the difference between the private sector savings rate and the public sector’s deficit ratio) is equal to the investment rate. Even though individuals’ savings behaviour has the Ricardian equivalence property in this model, the net saving rate declines with an increase in the public debt because the increase in the tax rate that is required to finance the higher interest payments on public debt reduces the net rate of return on saving, making savings less attractive. This distortionary tax effect causes the investment rate, and hence the rate of economic growth, to decline.

In Section 3, this relatively simple framework is used to derive a formula for the MCF for public sector borrowing that depends on individuals’ preference parameters—the rate of time preference, the intertemporal elasticity of substitution, and the marginal rate of substitution between public and private consumption goods—the public sector’s fiscal position—its debt ratio, its program expenditure ratio, and its tax rate—as well as the pre-tax rate of return on investment, the only production parameter in the model. This measure of the MCF for public sector borrowing has two components. One component measures the responsiveness of the present value of the government’s net revenue stream (PVNR) to changes in the tax rate. The greater the distortionary effect of a tax increase, the less responsive the PVNR is to a tax rate increase because of shrinkage of the tax base, and the higher the MCF from public sector borrowing. As is well-known, the magnitude of the MCF depends on the shape of the underlying Laffer curve for tax revenues. For example, if a government is operating near the peak of its Laffer curve,
then the MCF will be very high because a tax rate increases generates relatively little additional tax revenue. In this model, the PVNR Laffer curve always has a positive slope, and the slope is increasing in the tax rate, if the intertemporal elasticity of substitution is less than one. This effect arises because the net rate of return on government debt that is used to compute the PVNR declines by more than the reduction in the economic growth rate when the tax rate increases, leading to an increase in the present value of the government’s tax base. Thus, the model yields some interesting insights into the nature of the government’s intertemporal budget constraint as well as providing a measure of the MCF for debt financing.

The other component of the MCF measures the present value of the reduction in private and publicly-provided goods and services due to changes in the growth rate caused induced by a tax rate increase. One of the key insights from this derivation of the MCF is the importance of including the value of the foregone consumption of the public good in calculating the MCF. Our calculations show that this is likely to be a very important component of the MCF from debt financing and that the MCF will be significantly underestimated if it is ignored.

In Section 4, we then derive the condition for the optimal program expenditure ratio for a consumptive public good. The optimality condition has the same form as the Atkinson-Stern condition for a public good that does not affect the revenues generated by the public sector. While some commentators have suggested that fiscal adjustments in countries with higher debt ratio takes the form of lower program spending rather than higher taxes, we show that the optimal program spending ratio is independent of the debt ratio if the intertemporal elasticity of substitution is one, and we explain why the
numerical simulations of the model indicate that the optimal program spending ratio is (slightly) increasing in the debt ratio. Thus, the model predicts that the fiscal adjustment to an increase in the debt ratio is on the tax side of the budget.

In general, as Triest (1990) and Hakonsen (1998), have shown the formula for the MCF depends on the set of prices that are used to measure welfare changes. This also applies to the MCF for debt financing. The main formula for the MCF that is derived in Section 3 is based on the assumption that the government discounts tax revenues and program costs using the after-tax interest rate. This is a convenient way of defining the MCF because the private sector also discounts future benefits using the after-tax interest rate.\(^1\) However, we show that if future tax revenues and program expenditure government are discounted using the pre-tax interest rate, then the MCF will be higher, but the optimal program expenditure will remain the same, because the marginal cost of increasing the program expenditure ratio will be lower. Thus, this paper also makes a (modest) contribution to the voluminous literature on the discount rate that should be used in cost-benefit analysis.

In Section 5, we use parameter values that allow the model to replicate the average growth rate and the public and private consumption ratios for the Canadian economy in the 1990s. Using these base case parameter values, the MCF is about 1.20 if the after-tax interest rate is used to discount tax revenues and costs and 1.52 if the pre-tax rate of return is used. The model predicts that eliminating the public debt ratio would only increase the growth rate by a tenth of a percentage point. Even though the public

\(^1\) Lui et al (2002) show that if the public and private goods are separable in the individuals’ utility function, which is what is assumed in our model, then the appropriate discount rate for cost-benefit analysis is the after-tax interest rate.
debt has only a very small effect on the growth rate in this model, a public project financed by debt would have to have a marginal benefit-cost ratio of 1.20 in order to improve social welfare. The final section of the paper discusses the limitations of the model and directions for future research are discussed in the final section.

2. A Model of the Effect of the Public Debt on Economic Growth

Total output at time $t$ is equal to:

$$ Y_t = AK_t $$

(1)

where $K_t$ is the accumulated factor of production (physical and human capital) and $A$ is the constant rate of return on this input. We will restrict our attention to the balanced growth path for this economy, where total output is growing at a constant rate $\gamma$. The capital stock is also growing at the constant rate $\gamma$ because it is assumed that there is no technological change and no depreciation. This implies that the annual rate of net investment is $I_t = \gamma K_t$. Substituting back into (1), we obtain:

$$ \gamma = A i $$

(2)

where $i$ is the investment rate, $I/Y$. In other words, the growth rate is proportional to the investment rate in the economy. This simple relationship between the growth rate of the economy and the investment rate is the key feature of this simple endogenous growth model, and there is considerable empirical evidence indicating that countries with higher investments rates also have higher growth rates.\(^2\)

\(^2\) See, for example, McGrattan (1998) and Durlauf and Quah (1999).
The population is normalized to equal one, so all of the stocks and flows can be interpreted as per capita variables. Individuals are identical and are represented by a single individual whose utility at time $t$ is:

$$
U_t = \left( \frac{\sigma}{\sigma - 1} \right) C_t \frac{\sigma - 1}{\sigma} + \beta \left( \frac{\sigma}{\sigma - 1} \right) G_t \frac{\sigma - 1}{\sigma} \tag{3}
$$

where $C_t$ is private consumption, $G_t$ is consumption of publicly-provided goods and services, $\sigma > 0$ is the intertemporal elasticity of substitution, and $\beta > 0$ is a parameter that reflects the relative valuation of private and public consumption. The representative individual takes as given the level of public services, $G_t$, and the tax rate, $\tau$, used to finance them. Each period, the individual chooses his level of consumption and allocates his savings between investment in new capital and purchases of government bonds, $B_t$.

The individual’s budget constraint in each time period is:

$$
C_t + \dot{K}_t + \dot{B}_t = (1 - \tau)AK_t + (1 - \tau)AB_t \tag{4}
$$

where $\dot{K}_t$ and $\dot{B}_t$ are the rates of change in capital and government bonds. The right-hand side of (4) shows the individual’s current after-tax income from production and interest payments on government bonds. This is a closed economy, and there is no external debt, i.e. the individuals owe the public debt to themselves. The representative individual discounts future utility at the rate $\rho > 0$ and makes consumption-savings decisions to maximize welfare $V$ where:

$$
V = \int_0^\infty U_t e^{-\rho t} \, dt \tag{5}
$$
To simplify the notation, we will omit the time subscript unless it is necessary for their interpretation.

With the optimal consumption plan, private consumption grows at the rate, $\gamma$, where:

$$\frac{\dot{C}}{C} = \sigma ((1 - \tau)A - \rho) = \gamma$$  \hspace{1cm} (6)

An increase in the tax rate will slow the growth rate of consumption because it reduces the net rate of return on savings. The reduction in the growth rate caused by an increase in the tax rate, $\frac{\partial \gamma}{\partial \tau} = -\sigma A$, is proportional to the intertemporal elasticity of substitution, $\sigma$, the key behaviour parameter in the model.

The growth of the public debt is equal to the public sector’s budget deficit, which is given by the right-hand side of (7):

$$\dot{B}_t = (1 - \tau)AB_t + G_t - \tau Y_t$$  \hspace{1cm} (7)

Along the balanced growth path of the economy, $C$, $B$, $K$, $G$, and $Y$ all grow at the rate $\gamma$, and the public sector’s debt ratio, $b = B/Y$, its program expenditure ratio, $g = G/Y$, and the tax rate, $\tau$, remain constant. Therefore the deficit ratio is equal to $\gamma b$ where:

$$\gamma b = (1 - \tau)Ab + g - \tau$$  \hspace{1cm} (8)

This intertemporal budget constraint can also be written as:

$$\tau - g = [(1 - \tau)A - \gamma]b = \theta b$$  \hspace{1cm} (9)

The government’s primary surplus ratio, which is the left-hand side of (9) assuming for simplicity that interest on the government’s debt is not taxed, has to equal the equilibrium
debt ratio multiplied by \( \theta \), the difference between the after-tax rate of return on capital and the growth rate of the economy, if the debt ratio is to remain constant.

The government’s intertemporal budget constraint does not depend on whether interest payments on government debt are taxed. If interest on the public debt is not taxed, the interest rate on government bonds would be equal to the after-tax return on capital, \( (1 - \tau)A \). If interest on the public debt is taxed, the interest rate on the public debt is pre-tax return on capital, and the right-hand side of the (9) would be \( (A - \gamma)b \). However, the left-hand side would be equal to \( \tau(1 + Ab) - g \), and therefore the government’s intertemporal budget constraint would be the same as in the case where interest on the public debt is not taxed. It will be convenient to assume that interest on the public debt is not taxed because this implies that the public sector and the private sector will discount future income streams using the same discount rate. Thus the present values of tax revenues and program expenditures are based on the after-tax rate of interest, \( (1 - \tau)A \), and not the pre-tax rate of return on capital. In Section 4, we consider the case where the interest on the public debt is taxed.

Using the expression for the equilibrium growth rate of the economy in (6), \( \theta \) is equal to:

\[
\theta = (1 - \sigma)(1 - \tau)A + \sigma \rho
\]  

A condition for dynamic stability is that \( \theta > 0 \), or in other words, that the after-tax rate of return on capital exceeds the growth rate of the economy. Since \( \tau \leq 1 \), this condition will be satisfied if \( \sigma \leq 1 \), which is the relevant range of values for \( \sigma \) based on econometric studies of savings behaviour.
To derive the consumption ratio along the balanced growth path, we divide both sides of (4) by $K$.

$$\frac{C}{K} + \frac{\dot{K}}{K} + \frac{B}{B} \left( \frac{B}{K} \right) = (1 - \tau)A \left( 1 + \frac{B}{K} \right)$$  \hspace{1cm} (11)

Substituting $\gamma$ for $\frac{\dot{K}}{K}$ and $\frac{B}{B}$ in (11), and noting that $B/K$ is equal to $Ab$, we obtain:

$$c = \frac{C}{Y} = \frac{\theta(1 + Ab)}{A}$$  \hspace{1cm} (12)

The model predicts that an increase in debt ratio, holding the tax rate constant, will increase the consumption rate and that an increase in the tax rate, holding the debt ratio constant, will reduce the consumption rate if $\sigma < 1$ because $\partial\theta/\partial\tau < 0$.

As (2) indicated, the growth rate of the economy is proportional to the investment rate, which in this closed economy is equal to the net savings rate—the difference between the private sector savings rate and the public sector’s deficit ratio. Therefore $i = s - \gamma b$, where $s$ represents the private sector’s savings ratio, $S/Y$.

The model yields the following closed-form solutions for the key endogenous variables, given $g$, $b$, $A$, $\rho$, and $\sigma$ is:

$$\gamma = \sigma \frac{(1 - g - \rho b)A - \rho}{1 + (1 - \sigma)Ab} \hspace{1cm} (16)$$

$$\tau = g + \frac{[(1 - \sigma)A + \sigma \rho]b}{1 + (1 - \sigma)Ab} \hspace{1cm} (17)$$

$$s = \sigma \frac{1 + Ab}{A} \left( \frac{1 - g - \rho b)A - \rho}{1 + (1 - \sigma)Ab} \right) \hspace{1cm} (18)$$
\[ c = \left( \frac{1 + Ab}{A} \right) \left( \frac{(1 - g)(1 - \sigma)A + \sigma \rho}{1 + (1 - \sigma)Ab} \right) \]  

(19)

and \( i = s - \gamma b \).

Below, we will try to provide an intuitive explanation of the effect of an increase in the debt ratio on the growth rate of the economy. First, note that an increase in the debt ratio, holding the expenditure rate constant, leads to an increase in the tax rate, assuming that the condition for dynamic stability is satisfied:

\[ \frac{d\tau}{db} = \frac{(1 - g)(1 - \sigma)A + \sigma \rho}{(1 + (1 - \sigma)Ab)^2} > 0 \]  

(20)

The effect of an increase in the debt ratio on the private sector savings rate can be decomposed as follows:

\[ \frac{ds}{db} = \gamma - \sigma(1 + Ab)\frac{d\tau}{db} \]  

(21)

The first term on the right-hand side of (21) is the Ricardian equivalence effect. An increase in \( b \) will increase the deficit ratio, \( \gamma b \), and this prompts an individual to increase his savings rate to offset the decline in the public sector savings rate. This forward-looking response arises from our assumption that the economy is composed of infinitely-lived individuals. The second term on the right-hand side of (21) is the distortionary tax effect which arises because the higher tax rate that is required to finance additional debt reduces the net rate of return on saving. These effects push the private sector savings rate in opposite directions, and therefore an increase in the debt ratio has an ambiguous effect on the private sector savings rate.
The overall effect of an increase in b on the growth rate depends on its effect on the investment rate, which in turn depends on the change in the net savings rate $s - \gamma b$, as shown below:

$$\frac{d\gamma}{db} = A \frac{di}{db} = A \left[ \frac{ds}{db} - \left( \gamma + b \frac{d\gamma}{db} \right) \right]$$

(22)

The first term in square brackets is the effect of an increase in b on the private sector savings rate, and the second term is the effect on the deficit ratio. Substituting (21) into (22) yields:

$$\frac{d\gamma}{db} = - A \sigma \frac{d\tau}{db} < 0$$

(24)

An increase in b causes $\gamma$ to decline, even though an increase in b has an ambiguous effect on the private sector savings rate, because the Ricardian equivalence effect from the private sector savings response exactly offsets the increase in the deficit ratio. Therefore, the total net savings rate declines by the distortionary tax effect, leading to declines in the investment rate and the equilibrium growth rate. In Section 5, the model is used to calculate the impact on the growth rate of an increase on the public debt, based on parameter values that allow the model to replicate $\gamma$, and c given g and b for the Canadian economy in the 1990s.

3. The Marginal Cost of Funds from Public Sector Borrowing

We begin by deriving an expression for the equilibrium level of welfare in the economy. Along the balanced growth path, $C_t = zK_0e^{it}$ and $G_t = gAK_0e^{it}$ where $K_0$ is the
economy’s capital stock at time 0 and \( z = \theta (1 + Ab) \). Substituting these values into (3) and (5), the discounted value of the representative individual’s utility stream is:

\[
V(\tau, g) = \left( \frac{\sigma}{\sigma - 1} \right) \int_0^\infty \left( zK_0 e^{\gamma t} \right)^{\sigma - 1} + \beta \left( gAK_0 e^{\gamma t} \right)^{\sigma - 1} \right) e^{-\rho t} dt
\]

\[
= \left( \frac{\sigma}{\sigma - 1} \right) \left[ \frac{zK_0}{\theta} \left( \frac{\sigma - 1}{\sigma} \right) + \frac{\beta gAK_0}{\theta} \frac{\sigma - 1}{\sigma} \right]
\]

\[
= \left( \frac{\sigma}{\sigma - 1} \right) \left[ \frac{AK_0}{\theta} \frac{\sigma - 1}{\sigma} \right] + \beta \left( \frac{g}{\sigma} \right)^{\sigma - 1} \]

since \( \gamma((\sigma - 1)/\sigma) - \rho = -\theta \). This expression indicates that the representative individual’s welfare depends on the shares of income devoted to private consumption and government services and the present value of the stream of “potential utility”, \( (AK_0)^{\sigma - 1} \), calculated at the “implicit” discount rate, \( \theta \), which is the same implicit discount rate used to calculate the present value of the government’s tax revenues and program expenditures. Welfare also depends on \( \tau \) because \( \theta \) and \( c \) are functions of the tax rate. In other words, the implicit discount rate used to calculate the representative individual’s welfare level depends on the rate of taxation because it reduces the after-tax rate of return on savings and because it lowers the rate of economic growth.

For future reference, the marginal benefit from an increase in the program expenditure ratio, \( MB_g \), will be defined as:

\[
MB_g = \frac{1}{\lambda_0} \frac{\partial V}{\partial g} = \beta \left( \frac{AK_0}{\theta} \right)^{\frac{1}{\sigma}} \left( \frac{c}{g} \right)^{\frac{1}{\sigma}} \]

(25)
where $\lambda_0 = (cAK_0)^{-1/\sigma}$ is the marginal utility of consumption at time 0. $MB_g$ is a money measure of the gain from a permanent increase in the proportion of output devoted to public program expenditures, measured at the initial marginal utility of income.

The marginal cost of public funds is the cost to a society in raising an additional dollar of tax revenue. A tax rate increase usually induces tax avoidance and evasion behaviour that causes the government’s tax base to shrink. The shrinkage of the tax base is a reflection of the loss of economic efficiency caused by the distortion in the allocation of resources in the economy, and the marginal cost of funds is usually greater than one.\(^3\)

In static models, the MCF is usually defined as $(-1/\lambda)(\partial V/\partial \tau)/(\partial R/\partial \tau)$ where $R$ is tax revenue. However, Liu (2002) has shown that when the cost of government programs is affected by the tax rate, it is more appropriate to define the MCF as $(-1/\lambda)(\partial V/\partial \tau)/(\partial NR/\partial \tau)$ were $\partial NR/\partial \tau$ is the rate of change in the government’s net revenues, i.e. the difference between its tax revenues and program expenditures. In a dynamic model, the definition of the MCF should be based on the rate of change in the present value of the government’s net revenue stream.

The present value of the government’s net revenue stream is equal to:

$$PVN R = (\tau - g) \left( \frac{AK}{\theta} \right)$$

(26)

An increase in the tax rate has two offsetting effects on the present value of the tax/expenditure base, $AK/\theta$. On the one hand, an increase in the tax rate reduces the growth rate of the economy, which lowers the present value of the tax/expenditure base. On the other hand, a higher tax rate lowers the after-tax rate of return on government
debt, which increases the present value of the tax/expenditure base. Taking the partial derivative of PVNR in (26) with respect to $\tau$, we obtain:

$$
\frac{\partial PVNR}{\partial \tau} = \frac{AK_0}{\theta} - (\tau - g) \left( \frac{AK_0}{\theta^2} \right) \frac{\partial \theta}{\partial \tau} \\
= \frac{AK_0}{\theta} \left[ 1 - b \frac{\partial \theta}{\partial \tau} \right] = \frac{AK_0}{\theta} [1 + (1 - \sigma)Ab]
$$

(27)

since $\tau - g = 0b$ along the balanced growth path and $\frac{\partial \theta}{\partial \tau} = -(1 - \sigma)A$. Consequently, the government’s PVNR Laffer curve always has a positive slope for the empirically relevant case where $\sigma \leq 1$, and it is not possible to increase the present value of the government’s net revenues by lowering the tax rate$^4$. In fact, as shown in Figure 1, the slope of this Laffer curve increases with the tax rate when $\sigma < 1$. The implications of the shape of the PVNR Laffer curve for the MCF are noted below.

The marginal cost of public funds for a tax rate increase is defined as follows:

$$
MCF_{\tau} = \left( -\frac{1}{\lambda} \right) \left( \begin{array}{c}
\frac{\partial V}{\partial \tau} \\
\frac{\partial V}{\partial PVNR} \\
\frac{\partial V}{\partial \tau}
\end{array} \right)
$$

(28)

Taking the partial derivative of (24) with respect to $\tau$, the following expression for the social cost of a tax increase can be obtained:

$$
\frac{-1}{\lambda_0} \frac{\partial V}{\partial \tau} = \frac{AK_0}{\theta} \left[ 1 + \sigma \beta \left( \frac{c}{g} \right)^{1-\sigma} \right] (1 + Ab)
$$

(29)

$^3$ See Dahlby (forthcoming) for a survey on the concept and measurement of the MCF.
Combining (27) and (29), the following formula for the MCF can be obtained:

\[
MCF_{\tau} = \frac{-1}{\lambda_0} \frac{\partial V}{\partial \tau_{\text{PNR}}} = \left[1 + \sigma \beta \left(\frac{c}{g}\right)^{\frac{1-\sigma}{\sigma}}(1 + Ab)\right] \left(\frac{1}{1 + (1 - \sigma)Ab}\right)
\]  

(30)

This formula indicates that the MCF has two components. The component in round brackets is the inverse of the elasticity of the PVNR with respect to (\((\tau - g)\)). The greater the distortionary effect of a tax increase, the lower the elasticity of the PVNR, and the higher the MCF for debt financing. This component of the MCF will be higher the greater the distortionary effect of a tax increase, greater the ratio of interest payments on the public debt to total output, Ab, and the greater the intertemporal elasticity of the substitution because this makes the tax base more sensitive to tax rate increases.

The other component in square brackets is the social loss caused by the reduction in private and public service consumption. In particular, public program expenditures are assumed to be a constant proportion of output, and therefore a slower rate of economic growth, caused by a tax rate increase, means the level of public services is lower than it otherwise would be. This loss depends on the strength of the preference for the public services, \(\beta\), and the \((c/g)\) ratio. One of the key insights from this derivation of the MCF is the importance of accounting for the value of the foregone public consumption in calculating the MCF for a tax increase. As we will see in Section 5, incorporating this component of the welfare loss from taxation has an important impact on the measured MCF.

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Note also that the MCF approaches 1.00 as when $\sigma$ approaches 0, and the tax is non-distortionary. For $0 < \sigma \leq 1$, the MCF is greater than one, but decreasing in $\tau$ for $0 < \sigma < 1$ because an increase in the tax rate causes $c$ to decline. Normally, we expect the MCF to be increasing in the tax rate because the deadweight loss from tax distortions increases with the square of the tax rate. One way of explaining this anomalous feature of the MCF is that the slope of the PVNR Laffer curve is increasing in the tax rate for $0 < \sigma < 1$, and therefore marginal tax revenues (in present value terms) are increasing as the tax rate increases, thereby lowering the cost of raising additional revenues. Finally, note that the MCF is increasing in $b$, holding the $(c/g)$ ratio constant.

We have derived this expression for the MCF for a tax rate increase, but it can also be interpreted as the marginal cost of public funds from public sector borrowing as is shown below:

$$MCF_b = \frac{-1}{\lambda_0} \frac{\partial V}{\partial b} db = \frac{-1}{\lambda_0} \frac{\partial V}{\partial \tau} \frac{d \tau}{db} \frac{\partial V}{\partial \tau} = \left( \frac{-1}{\lambda_0} \frac{\partial V}{\partial \tau} \right) \frac{\theta}{1 + (1 - \sigma)Ab}$$

$$= \left[ 1 + \sigma \beta \left( \frac{c}{g} \right)^{\frac{1 - \sigma}{\sigma}} \right] \left( \frac{1 + Ab}{1 + Ab - \sigma Ab} \right)$$

(31)

Intuitively, the $MCF_b$ is the same as the $MCF_t$ because, if the government borrows an extra dollar, the present value of its net revenue stream must also increase by one dollar. In the remainder of this paper, we will simply refer to this common value as the MCF.
4. The Effect of the Public Debt on Optimal Public Expenditures

To this point, it has been assumed that the government’s program expenditure ratio, \( g \), remains constant when the debt ratio increases, and so all of the fiscal adjustment occurs on the tax side of the budget. However, some observers feel that an increase in interest payments on the public debt will crowd out program spending. In the following section, the condition determining the optimal level of public program spending is derived in order to analyze the cost-benefit criterion in this economy and to analyze the effects of an increase in the public debt on the program expenditure ratio.

To determine the government’s optimal tax and expenditure program (holding the government’s debt ratio constant), we maximize (24) with respect to \( \tau \) and \( g \) subject to the government’s intertemporal budget constraint in (9). Let the Lagrangian for this problem be:

\[
\Lambda = V(\tau, g) + \mu[\tau - g - \theta b]
\]  \hspace{1cm} (32)

where \( \mu \) is the Lagrange multiplier on the government’s intertemporal budget constraint.

The first-order conditions for this problem are:

\[
\frac{\partial V}{\partial \tau} + \mu \left[ 1 - b \frac{\partial \theta}{\partial \tau} \right] = 0
\]

\[
\frac{\partial V}{\partial g} - \mu = 0
\]  \hspace{1cm} (33)

Using (25), (27), and (29), the condition for optimal program expenditures has the form:

\[
\frac{MB_g}{MC_g} \equiv \beta \left( \frac{c}{g} \right)^{\frac{1}{\sigma}} \left[ 1 + \sigma \beta \left( \frac{c}{g} \right)^{\frac{1 - \sigma}{\sigma}} \right] \frac{(1 + Ab)}{1 + (1 - \sigma)Ab} \equiv MCF
\]  \hspace{1cm} (34)
where $MC_g = (AK_0)/\theta$ is the marginal cost of an increase in the program expenditure ratio and the value of $c$ is determined by (19). Equation (34) is the equivalent of the static Atkinson-Stern condition for optimal public expenditures financed by distortionary taxation for a public good that does not affect tax revenues.

The optimal $(g, \tau)$ combination satisfies (17) and (34). It is not possible to obtain a general closed-form solution for $\tau$ and $g$, but some insights can be gained from examining the solution for $\sigma = 1$:

$$g = \left( \frac{\beta}{1 + \beta} \right) \frac{\rho}{A} \quad (35)$$

$$\tau = \left( \frac{\beta}{1 + \beta} \right) \frac{\rho}{A} + \rho b \quad (36)$$

$$c = \frac{\rho(1 + Ab)}{A} \quad (37)$$

$$\gamma = (1 - \rho b)A - \left[ 1 + \frac{\beta}{1 + \beta} \right] \rho \quad (38)$$

With $\sigma = 1$, the optimal program expenditure ratio is independent of the level of the public debt. An increase in the debt ratio increases the tax rate, and the consumption rate by the personal rate of time preference, $d\tau/db = dc/db = \rho$. The reason why the optimal program expenditure ratio is independent of the public debt ratio when $\sigma = 1$ is shown in Figure 2. For a given level of the public debt $b_0$, the MCF is $(1 + \beta)(1 + Ab_0)$ and therefore independent of the level of the program expenditure ratio and the tax rate, while the ratio $MB_g/MC_g$ is decreasing in $g$ and independent of the tax rate. The optimal public
expenditure ratio is \( g_0 \) when the debt level is \( b_0 \). An increase in the debt ratio, increases both the MCF and the \( \text{MB}_g/\text{MC}_g \) ratio in the same proportion, and therefore has no effect on the optimal level of \( g \). Thus, the key reason why the optimal \( g \) is independent of \( b \) is that a higher debt ratio raises the marginal benefit from \( g \) in the same proportion as it increases the MCF. This effect arises with the preferences specified in (3) because private consumption and public consumption are complementary, and a higher debt ratio leads to a higher consumption ratio.

I have not been able to sign \( dg/db \) when for \( 0 < \sigma < 1 \). However, calculations using with a wide range of parameter values indicate that the optimal \( g \) is (slightly) increasing in \( b \) when \( \sigma < 1 \). (The calculations in the next section will illustrate this effect.) In Figure 3, I try to explain why the optimal \( g \) is increasing in \( b \) for \( \sigma < 1 \). The optimal \((g, \tau)\) combination is the solution to equation (34), which we will label the optimization condition (OC), and equation (17), which is the government’s intertemporal budget constraint (BC). In the absence of the public debt, the BC is the 45 degree line from the origin. The OC has a negative slope in \((g, \tau)\) for \( 0 < \sigma < 1 \).\(^5\) Initially, there is no public debt, and the optimal expenditure and tax rates are \( g_0 \) and \( \tau_0 \). An increase in the public debt to \( b_1 > 0 \), shifts the intercept of the BC to \( \tau_L \) and the maximum program expenditure ratio that can be financed is \( g_u \). Note from (17) that the slope of the BC is \( [1 + (1 - \sigma)Ab]^{-1} \) which is less than one when \( \sigma < 1 \). The slope of the BC declines when the public debt increases because the tax rate needed to finance an increase in \( g \) causes the after-tax interest rate to decrease, thereby reducing the amount of tax revenue needed to finance the public debt. Therefore, the required \( \Delta \tau \) is less than \( \Delta g \). An increase in the
public debt also causes OC to shifts up because an increase in b increases the \( \frac{MB_g}{MC_g} \) ratio more than the MCF. To restore equality, holding \( \tau \) constant, g must increase because an increase in g reduces the \( \frac{MB_g}{MC_g} \) ratio proportionately more than it reduces the MCF. The upward shift in the OC locus and the reduction in the slope of the BC locus, offset the upward shift in the BC, and the optimal g remains virtually constant. Almost all of the adjustment to the higher debt ratio occurs on the tax side of the budget.

While the model predicts that public debt does not crowd out spending on government services as a proportion of GDP, it can be shown that an increase in the public debt will crowd out program spending in the sense that the \( \frac{c}{g} \) ratio is increasing in b if \( \sigma \leq 1 \). In other words, the model predicts that a higher public debt will reduce public service consumption relative to private consumption.

As was noted in Section 2, the government’s intertemporal budget constraint is the same whether or not interest on the public debt is taxed, and therefore the optimal level of public expenditure is independent of whether interest on the public debt is taxed. It has been convenient to assume that interest on the public debt is not taxed, in deriving the formula for the MCF and the optimal program expenditure rate, because in this case both the private and public sectors use the after-tax rate of return on assets to discount future benefits, tax revenue, and costs. If the interest on the public debt is taxed and the government uses the pre-tax interest rate to discount future tax revenues and costs, the marginal cost of funds formula would be amended to equal \( MCF' = \frac{(A - \gamma)}{\theta} \cdot MCF \).

Since \( \frac{(A - \gamma)}{\theta} > 1 \), the \( MCF' > MCF \). However, the definition of the marginal cost of increasing the program expenditure ratio would also change if the government uses the

It can be shown that the slope of \( OC \) is \( \frac{d\tau}{dg} = -c[(1 - \sigma)Abg]^{-1} \).
pre-tax rate of return to discount future costs and it would equal \( MC_g' = AK_0/(A - \gamma) < MC_g = AK_0/\theta \). Therefore, \( MCF' \cdot MC_g' = MCF \cdot MC_g \) and the optimal \( g \) is independent of whether the interest payments on the public debt are taxed or not, and whether the government uses the pre-tax or the post-tax rate of return to calculate the present value of future tax revenues and costs. Note however, that the \( MB_g \) would be calculated using the after-tax rate of return that the private sector receives on savings in either case.

5. Calculations

Table 1 shows the calculation of the marginal cost of public funds using parameter values that replicate the average values of key variables for the Canadian economy in the 1990s. In particular, for Canada in the 1990s, \( \gamma = 0.016, b = 0.728, c = 0.589, \) and \( g = 0.213 \).\(^6\) Given these values, \( A = \gamma/(1 - c - g) = 0.081 \). That leaves the preference parameters—\( \rho, \sigma, \) and \( \beta \)—to be determined. In the base case scenario, we have used a conventional value for \( \rho = 0.02 \). We then computed the values of \( \sigma = 0.391 \) and \( \beta = 0.088 \) that generate values of \( c = 0.589 \) and \( g = 0.213 \) using equations (19) and (34). All of these parameter values are plausible. (One of the attractive features of this simple model is that it can replicate key features of the Canadian economy in the 1990s with a few “reasonable” parameter values.)

With the base case parameters, the MCF is 1.195. In other words, the “hurdle benefit-cost ratio” that a debt-financed public project needs in order to generate a net social gain is about 1.2. Alternatively, these calculations indicate that reducing the public
debt by $1.00 has a long-term payoff, through lower taxes and slightly higher rates of economic growth, of $1.20.

As noted above, one of the most important features of our derivation of the formula for the MCF was showing that its value depends on the strength of the preference for the public good. If the private sector did not value the public good and \( \beta = 0 \), then the
\[
MCF = \frac{1 + Ab}{1 + (1 - \sigma)Ab} = 1.022.
\]
This shows that the MCF would be significantly underestimated if we ignored the social loss that arises from the reduction in public good consumption as a result of a slower rate of economic growth. Finally, the table shows that \( MCF' \), the marginal cost of public funds when tax revenues and program costs are discounted using the pre-tax rate of return, is significantly higher than the MCF value based on the after-tax rate of return. We want to stress, however, that either value of the MCF can be used as long as \( MC_g \) is defined in a consistent manner.

Table 1 also shows how the growth rate, the MCF, and the optimal program expenditure ratio vary with the public debt ratio. If the public debt were eliminated, the model predicts that the growth rate of the economy would increase by a tenth of a percentage point. If the debt ratio doubled to 1.456, the growth rate of the economy would decline by a tenth of a percentage point. The relatively modest impact of the public debt on the growth rate in this model occurs because, as noted earlier, the Ricardian equivalence effect offsets the increase in the deficit ratio and therefore the reduction in the net savings rate is very modest.

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6 All data were taken from the IMF, International Financial Statistics. The program expenditure ratio is the ratio of consumptive government spending to GDP.
The MCF would decline only modestly from 1.195 to 1.166 if the public debt were eliminated, and it would increase to 1.221 if the public debt ratio doubled. The reason why the MCF is so unresponsive to the debt ratio is that the tax rate only falls by 3.5 percentage points with the elimination of the public debt, and therefore a fairly large distortionary tax exists even if the debt is eliminated. Thus, the social cost of the first dollar of public debt is almost as high as an additional dollar of debt when the debt ratio is over 100 percent.

Finally, the model predicts that with the elimination of the public debt, the optimal program expenditure ratio would decrease very slightly to 0.211. Doubling the public debt would increase the program expenditure ratio slightly to 0.215. Thus the calculations confirms the analysis in Figure 3, which showed that an increase in the public debt can increase the optimal program expenditure ratio if $0 < \sigma < 1$.

These calculations are based on the parameter values $\rho = 0.02$ and $\sigma = 0.391$. While the value for $\rho = 0.02$ is commonly used in simulations of endogenous growth models, the value for $\sigma$ is lower than the values used in calculating the effects of tax policies in endogenous growth models. For example, in the four endogenous growth models surveyed by McGrattan and Schmitz (1999), the elasticity of substitution varied between 0.5 and 1.0. In order to see how sensitive the predictions of this model are to the value of $\sigma$, we performed a second set of calculations using $\sigma = 0.75$. We then computed the values of $\rho = 0.0396$ and $\beta = 0.375$ that generate values of $c = 0.589$ and $g = 0.213$ using equations (19) and (34). With the higher values for $\rho$, $\sigma$, and $\beta$, the two alternative definitions of the marginal cost of public funds are higher—$\text{MCF} = 1.455$ and the $\text{MCF}' = 1.616$—which is not surprising because this makes the growth rate more
sensitive to increases in the tax rate and the social loss from reduced program expenditures is also higher. These calculations indicate that the MCF is somewhat sensitive to the assumed value of the elasticity of substitution and the implied values of the other parameters.

Both sets of calculations predict that the optimal g and (c/g) ratios will be relatively insensitive to variations in the debt ratio. To my knowledge, there are no empirical studies of the extent to which public debt crowds out government spending on goods and services. Some evidence concerning the impact of public debt on the size of the public sector, are contained in Figures 4 and 5, which plots the average ratio of consumptive government spending to GDP and the average ratio of public to private consumption spending to the average debt ratio for 22 industrialized countries over the period 1990-98. In either case, a country’s debt ratio does not have a statistically significant effect on its g or g/c, results that are broadly consistent with the prediction of this model. This is of course only a very superficial analysis, and a more detailed empirically analysis is required to test the hypotheses regarding the effect of debt on government spending.

6. Conclusion

In this paper, a simple AK endogenous growth model has been used to illustrate the inter-relationships between the public debt, distortionary taxation and economic growth. The higher tax rate that is required to finance the interest payments on a higher public debt reduces the growth rate of the economy by lowering the net savings and investment rate. Although the predicted reduction in the growth rate appears to be quite
modest—doubling the debt ratio only reduces the growth rate by about a tenth of a percentage point—it represents a significant social loss because of the cumulative forgone public and private consumption. Calculations indicate that the marginal cost of funds from public sector borrowing is around 1.20 for the base case parameter values that replicate the key variables for the Canadian economy in the 1990s. The MCF is around 1.45 if we use an elasticity of substitution that is comparable to the values used by other authors in simulating their other endogenous growth models. The model predicts that the optimal program expenditure ratio will be relatively insensitive to variations in the debt ratio, and therefore most of the fiscal adjustment that occurs to an increase in the public debt will be on the tax side of the budget. Some very simple cross-country comparisons of debt and consumptive program expenditure ratios are consistent with this prediction.

The model has the merit of providing a simple, intuitive framework for analyzing the impact of the public debt on the rate of economic growth. It allows us to obtain closed-form solutions for the key endogenous variables, such as the growth rate, and a formula for the MCF, so that we do not have to rely on the “black box” simulations that are necessary for more complex endogenous growth models. However, the simplicity of the model also imposes a number of limitations. One of the most important limitations is that the model only incorporates the aggregate tax rate, and it treats all taxes as if they taxed the return to financial and human capital. In practice, the tax mix may be more important than the level of taxation in determining the rate of economic growth, with taxes on the return to savings having a bigger impact on the growth rate than consumption taxes. It clearly would be very useful to incorporate a wider range of tax instruments in the model. It should be noted, however, that even if payroll and
consumption taxes do not affect the rate of economic growth, they could affect the level of economic activity insofar as they reduce people’s incentive to supply labour. These “level effects” are also a burden of the public debt. It would be very useful to extend the model to include a wider range of tax instruments and to include both the growth and the level effects of higher taxes in the measuring the burden of the public debt.

Second, the model represents a closed economy, and there is no foreign-held debt. Many countries borrow abroad, either directly or indirectly, in order to finance a public sector deficit. A higher public debt can impose a burden on the economy either by increasing the interest rate that foreigners require in order to finance the debt or by putting downward pressure on the exchange rate. Van der Ploeg (1996) and Turnovsky (1997) have developed open economy endogenous growth models with foreign borrowing. In these models, a higher level of foreign indebtedness increases the interest rate charged by foreign lenders which reduces investment and the rate of economic growth. Thus, the predicted effects of an increase in debt are similar, in qualitative terms, in open and closed endogenous growth models, but it would be interesting to have an analysis of the relative costs of public debt in open and closed economies.

A third limitation of the model is that it assumes that private sector savings behaviour is based on identical, forward-looking, infinitely-lived individuals, and this gives rise to the Ricardian equivalence effect. While I have reservations about the empirical importance of the Ricardian equivalence, I have adopted it in this model for two reasons. First, it greatly simplifies the modeling of savings behaviour and aggregate social welfare. Second, as Elmendorf and Mankiw (1999) note, Ricardian equivalence provides a useful benchmark, or a “natural starting point,” in constructing a model of
government debt. In the current context, it shows how a single departure from the conditions for strict Ricardian equivalence—in this case the use of distortionary taxes—will affect the results of the model. Our model suggests that distortionary taxes do not push the growth rate very far from its equilibrium value under strict Ricardian equivalence, although they have a significant effect on the MCF from debt financing. It obviously would be useful to study the effects of the public debt on economic growth in models that do not assume Ricardian equivalence behaviour. Some steps have been made in this direction by Saint-Paul (1992), van der Ploeg (1996), and Scarth (forthcoming), but in these models taxes are non-distortionary. Incorporating public debt, financed by distortionary taxes, in a non-Ricardian endogenous growth model would be a very useful direction for future research.

References


### Table 1
Calculations

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Notes: Parameter values based on averages for the Canadian economy in the 1990s.
Figure 1

The PVNR Laffer Curve for $\sigma < 1$

Present Value of the Net Revenue Stream

$\left( \tau - g \right) \left( \frac{AK_0}{\theta} \right)$

$- g \left( \frac{AK_0}{\theta} \right)$

PVNR ($\sigma < 1$)

Tax Rate, $\tau$
Figure 2
The Optimal Program Spending Ratio when $\sigma = 1$

\[ MCF = (1 + Ab_0)(1 + \beta) \]

\[ \frac{MB_{\sigma}}{MC_{\sigma}} = (1 + Ab_0)\left(\frac{\rho \beta}{\beta + \rho}\right) \]

\( g_0 \) program expenditure ratio

Figure 3
The Optimal Program Spending Ratio and Tax Rate when $0 < \sigma < 1$

\[ g_v = 1 - \sigma \rho b \quad \tau_L = \frac{\rho \beta + (1 - \sigma)Ab}{1 + (1 - \sigma)Ab} \]
Figure 4
The Government Consumption Expenditure vs Debt in 22 Industrialized Countries

Figure 5
The Ratio of Private to Public Consumption Expenditures in 22 Industrialized Countries