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Coordenador: Prof. Pedro Cavalcante Ferreira

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Multiple Equilibria and Protectionism

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Abstract

In a normative approach, I analyze trade policies when the industrial sector generates positive externalities in production, and there are adjustments costs to changing production from one sector to the other. Protectionist trade policy can make workers internalize the benefits from moving into the industrial sector, but it is a second best policy as it also causes consumption distortions. I show that if the government is able to fully commit to its tariff schedule for the future, the welfare maximizing policy is to maintain a positive tariff forever, even after all adjustment has already taken place. However, if the government is not able to commit at all, the only time consistent policy is zero tariff at any point in time. The time inconsistency of the full commitment policy is derived from the fact that in the model only production needs interference, and the production distortion is lagged one period with respect to the tariff while the consumption distortion is simultaneous to the tariff. In the intermediary case, i.e., when the government can commit for a limited period of time, the time consistent optimal tariff will be positive but lower than the "full commitment" tariff. This result indicates that some institutions that have always been considered pure sources of inefficiency, such as protectionist lobbying, may in fact be welfare improving in some cases!
1. Introduction

The infant industry argument for protection has been in the literature since the 18th Century. The basic classical argument states that the private rate of return on investment in the industrial sector is below the social rate, therefore the investment level is less than optimal. If industrialization improves productivity in the sector so that the private and social rates of return become equal, then infant industry protection may be welfare enhancing. If industrialization does not equalize private and social marginal returns, protection would have to be permanent to ensure a welfare maximizing investment level, rather than as an infant industry one. Using a narrower argument, it is claimed that the average cost of industrial output is initially higher than the world price, but with the passage of time it falls. Hence, a tariff could ensure the development of the industrial sector until its average cost becomes low enough to be competitive on the world market.

A revival of interest in this topic occurred in the 1950's, with the introduction of two new arguments for infant industry protection. The first one is based on the idea of external economies. Each investment may affect the profitability of other investments, but this is not internalized in the investment decision process. Here, again, the level of investments will not be socially optimal, and infant industry protection could enhance welfare, provided that once the economy reaches its new equilibrium the removal of the tariffs would not have efficiency distorting effects. This can be viewed as a revision of the basic classical argument, where a structure is given to the difference between social and private rates of return on investment.

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1See Grubel (1966) for references and a description of the following arguments.
The second argument relies on a dual economy approach, in which the economy has two sectors: agriculture and industry. Initially, most labor is in the agricultural sector. Wages paid in industry would have to be greater than wages in agriculture to attract workers to the former sector, because workers live in the rural areas and would be reluctant to move to the cities. Thus, the wage differential between the sectors would not reflect the difference in their marginal value product of labor. Hagen (1958) maintains that tariffs or subsidies are necessary to ensure that the economy will not remain in the (socially inferior) agricultural equilibrium. Kenen (1963) argues that in a dynamic perspective one can show that tariffs or subsidies can lead to a faster transition to the industrial equilibrium, but the lack of them does not necessarily mean that the economy specializes in agriculture permanently.

The model presented in this paper puts together the arguments stated in the two previous paragraphs. There is an economy with two sectors, industry and agriculture, and external economies make the social and private cost of factors in industry diverge. The externality enters through workers' labor allocation decisions: each worker perceives only her private benefit and does not internalize the effects of her decision on the income of the others. Therefore, although workers are rational, the rate of labor movement between sectors will differ from the socially optimum one. The dynamics of the economy are derived from the introduction of labor adjustment costs, which make labor movement sluggish, rather than instantaneous. The first best policy would be to introduce a production subsidy that makes the workers internalizes the benefits from moving into the industrial sector. The subsidy should be paid by lump sum transfers from the consumers. In the absence of this policy instrument, trade policy can be used as a second best policy to correct the problem.

Similarly to Kenen (1963), I show that the economy may reach the long run industrial equilibrium with no government intervention, but tariffs can make the economy
move at a (welfare improving) faster rate. My main new result is that under the conditions that the earlier literature claimed would require infant industry protection, it may actually be a second best policy to maintain import tariffs or export subsidies forever, even after the industrialized equilibrium is reached. This is true in a dynamic setting where agents have free access to financial markets and trade policy announced by the government for the whole future has full credibility from the private sector. The government's commitment capability and the infinitely lived agents' free access to financial markets are crucial to this result. The combination of these assumptions allows the setting of a tariff schedule to be aimed at smoothing consumption over the whole future. In the other extreme case, when the government's announcement has no credibility at all, the only time consistent policy is a zero tariff in all periods. Only in the intermediate case, when the government can commit to its policy for a limited number of periods, does a sort of "infant industry protection" turns out to be the best policy.

The results of this paper is can also be applied to a more general situation when the government could use subsidies, but not lump sum transfers to pay for it. If the government could only use income tax, for instance, it would still face the same trade off: introduce subsidies to deal with the externality problem and bear the distorting taxes, or no subsidies and, therefore, no distorting taxes.

Section 2 introduces the model. In subsection 2.1 the production side of the economy is described, subsection 2.2 analyses the first best policy to deal with the externality problem, and subsection 2.3 presents the consumption side of the economy. In

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2Karp and Paul (1993) reaches, independently, this same conclusion that with no credibility the best time consistent policy is zero tariffs in all periods in a situation similar to the one presented in this model, with no financial markets.
section 3 the optimal tariff schedule is analyzed in full commitment, no commitment, and limited commitment environments. Section 4 presents the conclusions.

2. The Model

I consider a country that is small in the international goods and financial markets. Goods prices are exogenous, and all economic agents may borrow or lend freely at the exogenous interest rate $r$. Each individual in this economy is a producer and a consumer. Her decision problem is analyzed in two steps: as a producer she decides how to allocate her labor endowment among sectors, and then, as a consumer, she chooses how much of each good to consume each period. For simplicity, all economic agents are assumed identical, i.e., they have the same per period labor endowment $L$, and they face the same decision problems. The production side of the economy will be studied first.

2.1. Production

The basic framework of the production side of the economy is based on Krugman (1991). There is an economy with two sectors, agriculture and industry, which produce goods $A$ and $N$, respectively. The only factor used in the production of both goods is labor. The agricultural sector presents constant returns to scale, and units are chosen so that one unit of the agricultural output requires one unit of labor. The industrial sector exhibits increasing returns to scale, such that:

$$N = E(L^*)L = (\alpha + \beta L^*)L$$

(1)

where $N$ and $L$ are the industrial sector output and labor allocation, respectively, of a representative agent, $L^*$ is the total amount of labor allocated to the sector, and $\alpha$ and $\beta$

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3The externality function $E(L^*)$ is presented as being linear in total labor for simplicity.
are constants. Note that, because workers are identical, $L^* = nL$, where $n$ is the total number of individuals.

Prices are normalized so that the prices of agricultural and industrial goods are, respectively, $p_A = 1$ and $p_N = p$. It is straightforward to see that if inequalities (2) hold there will be multiple long-run equilibria in this economy.

$$pE(nL) > 1 \text{ and } pE(0) < 1$$ (2)

When workers allocate all their time to the industrialized sector, the value of the marginal product of labor in that sector will be larger than in the agricultural sector, and workers will not want to change their allocation. An analogous situation holds when everyone works only in the agricultural sector. A third possibility is that the labor allocation is such that $pE(nL) = 1$. This will also be an equilibrium, but an unstable one: any variation in the labor allocation will make the economy go to one of the other two equilibria. Note that if there were no cost of adjustment to labor, there would be another set of equilibria where the economy alternates among periods in each of the three equilibria. The only constraint is that the labor must move at the same time to the same equilibrium.

Dynamics are added to the system by introducing some cost for the labor to move from one sector to the other. Labor will not relocate instantaneously, but will follow some law of motion, governed by the adjustment cost combined with the externality. Following Krugman (1991), I take the cost to be quadratic in the per period adjustment size: 4

### Footnote

4Mussa (1978) justifies this form of adjustment cost by introducing a moving industry which requires resources to move labor from one sector to the other and presents decreasing returns to scale. Here, the movement cost is motivated by "educational costs". In each sector, the individual has to perform different tasks. The longer she works in a sector, the more tasks she has to perform. There could be introduced an
where $\gamma$ is a constant and $L_{\tau}$ is the amount of labor per worker allocated to industry at time $\tau$.

Each worker chooses how to allocate her per period labor endowment between the two sectors maximizing the present value of output,

$$\max \sum_{t=0}^{\infty} \left( \frac{1}{1+r} \right)^t \left[ pE(L_{\tau})L_{\tau} + (L - L_{\tau}) - (u_{\tau})^2 / 2\gamma \right]$$

subject to $u_{\tau} = L_{\tau+1} - L_{\tau}$

$$0 \leq u_{\tau} + L_{\tau} \leq L$$

where $u_{\tau}$ is the control variable, and $L - L_{\tau}$ represents the amount of labor the worker allocates in agriculture.

When making their labor allocation decision, workers do not perceive the effect of their decision on the productivity of labor in the industrial sector, which, in turn, will affect income of every worker. Appendix A presents the solution to the maximization problem above. Let $\xi_{\tau}$ denote the Lagrange multipliers for constraints (4.2). They will assume the value zero when the constraints are not binding, and some positive value otherwise. To solve the problem we also introduce a future value co-state variable, $\lambda_{\tau}$. The expression $\frac{\lambda_{\tau}}{(1+r)^{\tau+1}}$ represents the value today of one more unit of labor in the industrial sector at time $\tau$, and corresponds to the multipliers for constraints (4.1). Along educational industry which requires resources to teach tasks to the workers and presents decreasing returns to scale.
the optimal path, the change in the co-state variable is equal to the difference between its rate of return and the private value of one more unit of labor in the industrialized sector.

\[ \lambda_{t+1} - \lambda_t = r\lambda_t - d_t \]  

(5)

where \( d_t = p(\alpha + n\beta L_t) - 1 \).

Until an equilibrium is reached, labor will move at any time \( t \) proportionally to the value of the co-state variable.

\[ L_{t+1} - L_t = \gamma \lambda_{t+1} \]  

(6)

Depending on the initial value of labor allocation and the parameters of the economy, equilibrium conditions given by the equations of motion and transversality conditions may lead the economy either to the equilibrium with specialization in industry, or the one with specialization in agriculture. Equation (7) presents the terminal condition for the co-state variable if the economy is heading to the industrialized equilibrium, and equation (8) gives the terminal condition if it is heading to the agriculture one.\(^5\)

\[ \lambda^* = \frac{p(\alpha + n\beta L_c) - 1}{r} \]  

(7)

\[ \lambda^A = \frac{p\alpha - 1}{r} \]  

(8)

It is straightforward to see that the labor movement resulting from a free market is different from that chosen by a central planner, because of the externality in production.

\(^5\)See appendix A for the derivation of equations (7) and (8). The terminal conditions here are different from those derived in Benabou and Fukao (1993) for Krugman's model. The difference is due to their interpretation of the Krugman model as having each worker produce either the industrial or the agricultural good (not both, as in the present model). The moving cost, then, depends on the total number of workers changing sectors, so that it becomes another externality source in the model.
The workers underestimate the benefit from moving into the industrial sector, because they do not account for the effect of their movement on the productivity of the sector, and therefore on the income of every worker. The central planner’s solution to the maximization of income implies labor movement as the same function of the co-state variable as in equation (6), but the movement of the co-state variable is given by equation (9) instead of (5).

\[ \lambda_{t+1} - \lambda_t = r\lambda_t - d_t^e \]  

(9)

where \( d_t^e = p(\alpha + 2n\beta L_t) - 1 \) is the difference between the value of the marginal product of labor in the two sectors as perceived by the central planner.\(^6\)

The central planner’s terminal conditions for the co-state variable if the economy is heading to the industrialized or agricultural equilibria are given by equation (10).

\[ \lambda^j = \frac{d^j}{r}, \quad j = A, N \]

\[ d^A = p(\alpha + 2n\beta L) - 1 \text{ and } d^N = p\alpha - 1 \]

(10)

To help the comparison between the free market and central planner solutions to the problem, a graphical interpretation of the dynamics is presented.

Figure 1 shows the shape of the paths leading to the long run equilibria, derived from the system of difference equations defined by the first order conditions from the workers’ decision (equations (5) and (6)), and the terminal conditions (equations (7) and (8)). The variables will follow discrete points along the paths. The continuous path is the limit as the time interval between periods (which is taken as being ‘1’) goes to zero.

\(^6\)Note that the value for the Central Planner of one more unit of labor in industry at each period \( d_t^e \) is greater than the value for the worker \( d_t \).
For an initial labor allocation, the value of the co-state variable $\lambda_*$ will determine the path the economy will follow. In other words, the value of one more unit of labor in the industrial sector ($\lambda_*$) will determine how much labor moves, and, together with the current amount of labor in this sector, it will determine how much its own value changes.
The shape of the path in the figure represent the dynamics when the roots of the system are real.7

For initial labor allocations to the right of point M the productivity in industry is high enough so that $\lambda_1$ assumes a positive value, and the economy heads to the industrialized equilibrium. The opposite is true for initial labor allocations to the left of point M.

Now the central planner's solution is compared to the free market dynamics. The $L_{x+1} - L_x = 0$ schedule is the same in the two cases, since it must coincide with $\lambda = 0$ in both cases. But the value of one more unit of labor in the industrial sector is greater for the central planner for each allocation of labor. Therefore the $\lambda_{x+1} - \lambda_x = 0$ line for the central planner is located up and to the left compared to the market one.

Figure 2

7Krugman (1991) presents an interesting interpretation of the dynamics when the roots of the system are imaginary.
In figure 2, $jP$ and $j$, for $j=A$, $N$, represent the equilibrium points where the economy specializes in sector $j$ resulting from the central planner and free market dynamics, respectively.

The marginal product of labor in industry for the central planner includes the effect of labor movement on the productivity of every worker, which is not taken into account by the individual worker. Therefore for each labor allocation the value of the marginal product of labor in industry is higher for the central planner than for the workers. If the initial labor allocation laid between points $C$ and $D$ in figure 2, then this difference would be decisive to determine which long run equilibrium were to be reached. The central planner would lead the economy to the industrialized equilibrium, whereas the free market would specialize in agriculture.

For initial labor allocation between points $B$ and $C$, there is so little labor in industry that even the central planner considers not worth moving into that sector. Both the central planner and the free market would go to the equilibrium with specialization in agriculture, however the central planner would do so at a slower rate.

Finally, if the initial labor allocation were to the right of point $D$, central planner and the free market would head to the industrialized equilibrium. The central planner would move at a faster rate due to his higher valuation of the marginal product of labor in industry for each labor allocation.

### 2.2 First Best Policy

The first best economic policy to implement the central planner's solution is to introduce a production subsidy paid by lump sum transfers from the consumers. The value of the subsidy at each point in time should be the one that makes the producer follow the central planner's equilibrium path, given the initial labor allocation. The subsidy would make the workers' perception of the value of shifting labor between
sectors equal to that of the central planner, and it would have no other effect in the rest of the economy. Comparing the equations that define the dynamics of the equilibrium path for the free market (equations (5), (6), (7) and (8)), and those for the central planner (equations (6), (9) and (10)), it is straightforward to check that the subsidy at some time \( t \) is such that the price faced by the producers is the one represented in equation (11).\(^8\)

\[
p' = p\left(1 + \frac{\beta L_t}{\alpha + \beta L_t}\right)
\]  

(11)

For the range of initial labor allocation within which the free market would lead the economy to the agricultural equilibrium, whereas the central planner would follow the path to the industrial equilibrium, economic policy would increase the present value of income not only by setting the first best labor movement each period, but also by \textit{reversing} the direction of labor movement and leading the economy to the better long run equilibrium. If the economy were already heading to the industrialized equilibrium without policy intervention, the subsidy would make it move at a faster rate. On the other hand, if the initial labor allocation were so far away from the industrialized equilibrium that even the central planner would prefer to move towards the agriculture equilibrium, the subsidy would cause it to happen at a (welfare improving) slower rate than under the free market.

If the initial labor allocation lies between points B and C, the value of the subsidy will decrease until it reaches zero when the agricultural equilibrium is reached. For initial labor allocations to the right of point C, the value of the subsidy increases as the industrial equilibrium approaches, and remains positive and constant at the long run equilibrium point. This last result relies on the fact that the labor allocation at any point in

\(^8\)Note that the subsidy is taken as given by each individual producer at each point in time.
time depends on the whole path of subsidy values. As shown in appendix A, the value of
one more unit of labor in industry next period, \( \lambda_{\tau+1} \), can be written as:

\[
\lambda_{\tau+1} = \sum_{i=\tau}^{\infty} \frac{d_i}{(1 + r)^{i-\tau}}
\]  

(12)

where \( d_i = p(1 + s_i)(\alpha + n'l_i) - 1 \), and \( s_i \) is the value of the subsidy at period \( i \).

Each \( d_i \) is a function of the subsidy and labor allocation during period \( i \). Hence, each \( \lambda_{\tau+1} \) can be written as a function of all subsidies and labor allocations from period \( \tau \) on. Thus, using equation (6), each period's labor movement can also be written as a function of all subsidies and labor allocations from period \( \tau \) on.

\[
L_{\tau+1} - L_\tau = \gamma \sum_{i=\tau}^{\infty} \frac{d_i}{(1 + r)^{i-\tau}}
\]  

(13)

This means that a change in the value of the subsidy even after the economy has reached the equilibrium point would imply in a change of the whole path taken (if predicted, of course). The equilibrium conditions require that at the industrialized equilibrium the subsidy is positive and constant, so as to make the terminal condition for the market's co-state variable equal to the central planner's one. However, the subsidy can be set to zero when time goes to infinity. Using the same proof in appendix B, one can show that

\[
\frac{\partial PVI}{\partial \tau_{\tau+\tau}} = (1 + r)^{-\tau} \frac{\partial PVI}{\partial \tau},
\]

where \( PVI \) is the present value of income and \( T \) is the time the industrialized equilibrium is reached, therefore \( \frac{\partial PVI}{\partial \tau_{\tau+\tau}} \to 0 \) when \( \tau \to \infty \), so that the first order condition for the maximization of income is satisfied for any value of subsidy set when time goes to infinity.

In this paper I consider the situation in which trade policies are the only instruments available to the policy makers. An import tariff and export subsidy of equal magnitude to the production subsidy described above would mimic its effect from the
producers' point of view, but it would also affect the consumption decision, causing utility diminishing distortions. Therefore, to derive the optimal trade policy, it is necessary to study its effects on the welfare of the economy as a whole, not only on its production side.

2.2. Consumption

The government will set some import tariff/export subsidy schedule, taking as given the production and consumption decisions of the individuals. Workers and consumers face the price \( p(1 + \nu) \) for the industrial good, where \( \nu \) is the value of the ad valorem import tariff/export subsidy imposed at time \( t \). Workers choose how to allocate their labor between the two sectors, and with the income they receive they decide how much to consume of each good. Each worker is taken to be identical, as they face the same decision problem, and therefore make the same decisions. They are taken to be identical consumers as well, by assuming they have the same utility function. Due to the symmetry of the problem, maximizing welfare is equivalent to maximizing utility for the representative consumer, which is represented by:

\[
U^i = U = \sum_{t \geq 0} \frac{1}{(1 + \theta)^t} \left[ a \log C^i_t + b \log C^i_{-t} \right]
\]

where \( C^i_t \) is the consumption of good \( j \) at time \( t \), and \( \theta \) is the discount rate.

The consumer has free access to the international financial market at the world interest rate \( r \). In each period the increment to her asset holdings equals the interest on assets held at the beginning of the period, plus the difference between her total income and her total expenditures, plus any transfer received from, or given to, the government. No assets are held at the start of the initial period, by assumption.

\[
f_{t+1} - f_t = rf_t + (1 + \nu_t)C^t_t - C^t_t + R_t
\]

\[
f_0 = 0
\]
where total income received from production is $I^*_t = p(1 + v_t)N_t + A_t - \frac{\gamma(\lambda_{t+1})^2}{2}$, $f_t$ is the amount of assets the consumer holds at the start of period $t$, and $R_t$ is the lump sum transfer from the government at each period. The government's balanced budget requires that $\sum_{t=0}^{\infty} (1 + r)^{-t}[p\nu_t(C_t^N - N_t) - R_t] = 0$.

The following Hamiltonian is used to solve the problem:

$$H = (a \log C_t^N + b \log C_t^A)(1 + \theta)^{-t} +$$

$$+ \overline{y}_{t+1} \left[ rf_t + I^*_t - p(1 + v_t)C_t^N - C_t^A + R_t \right]$$

where $\overline{y}_{t+1}$ is the co-state variable, which is interpreted as the value today of having one more unit of asset at time $t + 1$.

The first order conditions for the maximization above are:

$$\frac{a}{C_t^N} - y_{t+1} p(1 + v_t) = 0 \quad (17.a)$$

$$\frac{b}{C_t^A} - y_{t+1} = 0 \quad (17.b)$$

$$y_{t+1} - y_t = \left( \frac{\theta - r}{1 + r} \right) y_t \quad (17.c)$$

$$f_{t+1} - f_t = rf_t + I^*_t - p(1 + v_t)C_t^N - C_t^A + R_t \quad (17.d)$$

where $\overline{y}_{t+1} = \overline{y}_{t+1}(1 + \theta)^t$ is the future value co-state variable.

From equation (17.c) we see that the value of $y_t$ will change over time if, and only if, the interest rate is different from the rate of time preference. As having this variable changing over time would not add any insight to the analysis, $r = \theta$ is assumed from now on. Note that this assumption implies perfect consumption smoothing in both goods.
Furthermore, the transversality condition states that as time goes to infinity either the present value of one more unit of future asset (the co-state variable) goes to zero, or no assets are being held.

\[ \lim_{t \to \infty} \frac{y_t}{(1 + \theta)^t} f_t = 0 \]  

(18)

Now the model is complete: we know how workers and consumers make their decisions and are ready to derive the best tariff policy.

3. Optimal Tariff

Three different environments will be considered: the first is when the government is able to commit to its tariff schedule for the future, the second is the opposite case, when the policy maker cannot commit to any future tariff, and, finally, the third situation is when the government has limited commitment, i.e., can commit only for a certain number of periods. In each case we derive the indirect utility function that the policy maker maximizes when setting tariffs, which is the indirect utility function for the representative individual.

3.1. Full Commitment

When the policy maker is able to fully commit to his tariff plan, he will decide in the initial period on a tariff schedule for the whole future, maximizing the indirect utility function over that period of time, i.e., forever. First this indirect utility function will be derived.

From equation (17.d) and the fact that no assets are held at the initial time, we have that:

\[ f_* = \sum_{i=0}^{\infty} (1 + r)^{i-1} [I' - p(1 + v_i)C_i + R_i] \]  

(19)
Using the government’s budget constraint and substituting equations (17.a) and (17.b) into equation (18), the transversality condition (equation (18)) yields:

\[ y = \frac{S}{PVI} \]

where \( S = \sum_{i=0}^{\infty} (1+r)^{-i} \left[ \frac{a}{1+v_i} + b \right] \) can be interpreted as a measure of the present value of the consumption distortion caused by the tariff, and \( PVI = \sum_{i=0}^{\infty} (1+r)^{-i} I_i \) is the present value of income.

Substituting this value of \( y \) into equations (17.a) and (17.b) to get the levels of consumption, and then substituting these into the utility function we get the indirect utility function:

\[ V_0(L_0, v_0) = K + \frac{(1+\theta)(a+b)}{\theta} \left( \log PVI - \log S \right) - a \sum_{i=0}^{\infty} (1+\theta)^{-i} \log(1+v_i) \]

where \( K \) is a constant term.

Before proceeding with the maximization of the equation above, we need to know how tariffs affect the present value of income, which is a function of the labor allocation in each period. Equation (13) shows how each period’s labor movement depends on future tariffs and labor allocations. That leads to the following expression for each period’s labor allocation, given that the initial labor allocation is \( L_0 \):

\[ L_\tau = L_0 + \gamma \sum_{i=1}^{\infty} \left[ \frac{\delta}{\sum_{j=i}^{\infty} (1+r)^{-j-i}} \right] \quad \text{for } 1 \leq \tau < T \]
From (22), the path of future tariffs affects the labor allocation today. At time $T$ an equilibrium is reached, such that:

$$L_{T+i} = L_0 + \gamma \sum_{j=0}^{T} \left[ \sum_{i=1}^{\infty} \frac{d_i}{(1+r)^{i+j+1}} \right] = \begin{cases} L & \text{if the upper boundary is reached} \\ 0 & \text{if the lower boundary is reached} \end{cases}$$

for $s \geq 0$ (23)

Even before doing the maximization of the indirect utility function, limits can assessed on the value of the optimal tariff and the time the economy takes to reach its long run equilibrium. A positive tariff has a positive effect on welfare by increasing the present value of income, and a negative effect because of the consumption distortion it causes. We can be sure, then, that the optimal tariff will be smaller than the one that mimics the central planner's solution to the production side of the economy. Let $T_m$, $T_c$, and $T_m$ indicate the time equilibrium will be reached by the free market, the central planner, and the market under some tariff schedule, respectively. If, given the initial labor allocation, the central planner and the free market would head toward the same equilibrium but at different speeds, then under the optimal tariff schedule the economy will head toward that equilibrium at a speed intermediate between the two. That is, if agricultural equilibrium is to be reached then $T_c > T_m > T_m$, and if the economy is heading the industrial equilibrium then $T_m > T_m > T_c$. When the initial labor allocation is such that the central planner would take the economy to the industrialized equilibrium and the free market to agriculture, then $T_m > T_c$ if the tariff schedule takes the economy

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9 Labor allocation for any period after $T$ can be written as $L_{T+i} = L_0 + \gamma \sum_{j=0}^{T} \frac{d_i}{(1+r)^{i+j+1}} + \gamma \sum_{\mu=1}^{\infty} \sum_{j=0}^{T} \frac{d_i}{(1+r)^{i+j+1}} - \gamma \sum_{\mu=1}^{\infty} \xi_j (1+r)^j$ (the last term in the expression is due to the fact that those multipliers are different from zero only when the economy is at an equilibrium). However, we know that $L_{T+i} = L_0 \forall s \geq 0$ because the economy is at one of the boundaries. Therefore the sum of the two last terms of the equation must equal zero $\forall s \geq 0$, which implies $\lambda_{\mu} = \xi_j (1+r)^j \forall j \geq T$. 
to specialization in industry. On the other hand, \( T_{xw} > T_{x} \) if with the tariff schedule the economy specializes in agriculture.

Limiting the value of \( T \) as described above, we can be sure that the system of equations represented in equation (22) has a unique solution for a set of tariff schedules. The central planner will choose the one that maximize welfare.

Given the values of the initial and final labor allocations, \( L_{0} \) and \( L_{f} \), and of \( T \), the system of equations represented in equation (22) can be solved to get the entire labor allocation stream as a function of the tariffs schedule. This gives the indirect utility function as a function of the tariff schedule only.

\[
V_{o}(L_{f}, v_{f}) = V(L_{e}, \{v_{i}\}_{0}, v_{f})
\]

(24)

To set the conditions for the welfare maximizing tariff plan (given \( T \)), all we need is the tariff schedule that makes the derivative of the indirect utility function equal zero, i.e., that satisfies:

\[
\frac{(1+\theta)(a+b)}{\theta PV^{*}v_{f}} \frac{\partial PV}{\partial v_{f}} = \left[ 1 - \frac{(1+\theta)(a+b)}{\theta S^{*}(1+v_{f}^{*})} \right] \frac{a}{(1+v_{f}^{*})(1+r)} \quad \forall \tau \geq 0
\]

(25)

where \( v_{f}^{*} \) is the optimal tariff at time \( \tau \), and \( PV^{*} \) and \( S^{*} \) represent the value for those variables when the optimal tariff plan is followed.

The left hand side of the equation above represents the effect of the tariff on the present value of income. Producers' decisions determine the present value of income, but those decisions are not optimal from the central planner's point of view because the producers do not consider the effect of their decision on the productivity of industry (review the difference between equations (5) and (9)). A positive value for the tariff will bring the producers' decisions closer to the central planner's perception of the right ones, thereby increasing the present value of income.
The right hand side represents the loss in utility brought about by the tariff due to the consumption distortion.\(^{10}\) It depends not only on the current tariff, but on the whole stream of tariffs through \(s\). This means that if, for some reason, only the tariff at some particular time \(t\) affected the \(PVI\), (i.e., if the l.h.s. were different from zero only in period \(t\)), nevertheless the optimal tariff would be different from zero at all times. Because of the concavity of the utility function, consumers would prefer to spread the consumption distortion over the time, instead of having it concentrated in only one period.

The solution to the problem as stated above yields the utility maximizing tariff schedule given that the labor frontier is to be reached in \(T\) periods. The problem can then solved for all possible values of \(T\)^\(11\), and the welfare maximizing tariff schedule is the one with a value for \(T\) that achieves the higher utility.

The optimal tariff plan is stated in proposition 1.

Proposition 1: Under full commitment \(v^*_t > 0 \forall t \geq 0\), and \(v^*_{r+s} = v^*_r \forall s \geq 0\), where \(T\) is the time at which the labor boundary is reached.

Proof: \(\frac{\partial PVI}{\partial v_e} > 0 \forall \tau > 0\), hence equation (25) is satisfied if and only if \(v^*_t > 0 \forall \tau \geq 0\). As for the second statement of proposition 1, we show in Appendix B that:

\[
\frac{\partial PVI}{\partial v_{r+s}} = (1 + r)^{-s} \frac{\partial PVI}{\partial v_r} \quad \forall s \geq 0
\]

\(^{10}\)If the tariff is constant and equal to zero at every period, then the r.h.s. will equal zero, since in this case \(S = \frac{1+r}{r}(a+b)\).

\(^{11}\)The range of possible values for \(T\) can be restricted as explained above.
Therefore, for the optimal tariff after the labor boundary is reached, equation (25) can be rewritten as:

\[
\frac{(1 + \theta)(a + b)}{\theta PVI} \frac{\partial PVI}{\partial v_r} = \left[ 1 - \frac{(1 + \theta)(a + b)}{\theta S'(1 + v_{r+s})(1 + v_{r+s}^2)} \right] \frac{a}{(1 + v_{r+s})^2(1 + r)} \quad \forall s \geq 0 \tag{27}
\]

Equation (27) must hold for all \( v_{r+s} \), and the other variables in the equation are constant \( \forall s \geq 0 \), except for \( v_{r+s} \). Therefore, the tariff must be constant over this period of time. Moreover, the left hand side of the equation is constant and strictly greater than zero, so that the tariff must be positive for the right hand side to be also positive.\(^{12}\)

Q.E.D.

Proposition 1 states that the optimal import tariff/export subsidy will be always positive, moreover it will be constant after the labor boundary is reached.\(^{13}\)

The result that the optimal tariff will be constant after the boundary is reached relies on the fact that tariffs from that moment on will not affect the present value of income, \( PVI \), by affecting present or future labor allocations, because labor will remain static at the frontier (i.e., \( \frac{dL_{r+s}}{dv_{r+s}} = 0 \quad \forall \tau \geq 0 \) and \( \forall s \geq 0 \)). They affect the present value of income only through labor allocation decisions prior to reaching the boundary. Therefore the effect of tariffs in different periods after time \( T \) will be equal except for the different discount applied to each one, i.e., the result in equation (26). If we did not have a "corner"

\[^{12}\text{For the r.h.s. to be positive we need } 1 - \frac{(1 + \theta)(a + b)}{\theta S'(1 + v_{r+s})} > 0, \text{ which is equivalent to } 1 + v_{r+s} > \frac{(1 + \theta)(a + b)}{\theta S'}. \text{ The expression } \frac{(1 + \theta)(a + b)}{\theta S'} \text{ would equal 1 if the tariff were zero at all periods, and would be greater than 1 for positive tariffs. Therefore } v_{r+s} > 0.\]

\[^{13}\text{This result refers to the tariff schedule that maximizes welfare. If one seeks for a tariff schedule that just ensures industrialization, many others would do the job, including an infant industry protection.}\]
solution as we do here, this result would not hold, i.e., tariffs at the equilibrium point would be able to affect the position of the economy so that $\frac{\partial L_{\tau+s}}{\partial V_{\tau+s}} \neq 0 \ \forall \tau \geq 0$ and $\forall s \geq 0$. Tariffs would still be positive at equilibrium, but they would not necessarily be constant.

Note that for the initial period only the l.h.s. of equation (25) is equal to zero. The tariff each period affects past labor decisions, and past labor decisions affect present and future labor allocation. At the initial period labor allocation is given exogenously, and there are no "past" decisions. Therefore, the tariff that period does not affect the present value of income through its effect on labor allocation. However, due to the fact that for other periods the l.h.s. will be positive, requiring a positive tariff for the r.h.s. of the equation to be positive as well, a positive tariff will be necessary for the initial period.

$$v_0^* = \frac{[(1 + \theta)(a + b)\theta] - a}{b + \sum_{i=1}^{\infty} (1 + r)^{t-i} \left( \frac{a}{1 + v_i^*} + b \right)} - 1 \quad (28)$$

The value of $v_0^*$ is smaller than the tariff at any other period because the l.h.s. of equation (25) will be strictly greater than zero for all other periods.

3.2: No Commitment

When the policy maker is not able to commit to a tariff schedule for the whole future, he will choose at each moment in time the tariff that maximizes welfare from that moment on. The optimal tariff plan under full commitment is not time consistent. Equation (13) shows that at any period $\tau$ the decision on how much labor to move into industry for the next period depends only on future tariffs. On the other hand, the consumption distortion created by the tariff is contemporaneous to it. Therefore, the government will have an incentive to announce the tariff consistent with the desired rate of labor adjustment, but by the time of its implementation the labor movement would
already have taken place, and the best thing to do would be to set zero tariffs. Proposition 2 present the time consistent solution.\(^{14}\)

Proposition 2: With no commitment \(v_t^* = 0 \forall t \geq 0\).

\textbf{Proof:} First we will derive the optimal tariffs schedule after the labor boundary is reached. The indirect utility function of the policy maker will be facing at time \(T\) is:\(^{15}\)

\[
V_T(L_t, v_t) = K - \frac{(1 + \theta)(a + b)}{\theta} \log S_T - a \sum_{i=T}^{\infty} (1 + \theta)^{T-i} \log(1 + v_i)
\]

(29)

The only variable in this new indirect utility function is the tariff. There is no labor movement after the boundary is reached, and therefore tariff changes will not affect the present value of income. The first order conditions for maximization are:

\[
\left[1 - \frac{(1 + \theta)(a + b)}{\theta S_T(1 + v_{T+s})} \right] \frac{a}{(1 + v_{T+s})(1+r)^s} = 0 \quad \forall s \geq 0
\]

(30)

The first term of the product above must equal zero for the condition to be satisfied. The value of the tariff is the only variable in this term, which means that the value of the tariff that satisfies the condition is the same over time, i.e., \(v_{T+s} = v_T \forall s \geq 0\). Hence, using the assumption that \(\theta = r\), the tariff must satisfy \(\frac{(1 + \theta)b v_T^*}{\theta(1 + v_T^*)} = 0\), which will be true if and only if \(v_T^* = 0\). Thus, the tariff will be set equal to zero when the boundary of labor supply is reached, regardless the previous tariff schedule.

\(^{14}\)This is the subgame perfect Nash equilibrium strategy. An interesting extension would be to derive the trembling-hand perfect equilibrium (using the denomination in Kreps (1990)), which is the equilibrium strategy when there is a probability that the government, when setting the tariffs, makes a mistake and sets a tariff different from the optimal one to be set.

\(^{15}\)See the derivation of this equation in appendix C.
We now take one step back, and determine the optimal tariff to be set one period before the boundary is reached. At time \( T - 1 \) workers decide how much labor to move into industry based on the value of \( \lambda_T \) (see equation (6)), which will be given by equation (12), and only tariffs after time \( T - 1 \) enter this equation. It is then clear that the value of the tariff at time \( T - 1 \) will not affect labor allocation in that or any future period, and hence will not affect income either. Therefore, the condition for maximization of welfare from period's \( T - 1 \) perspective will be analogous to equation (30):

\[
\left[ 1 - \frac{(1 + \theta)(a + b)}{\theta S_{T-1}^*(1 + \nu_{T-1}^*)} \right] \frac{a}{(1 + \nu_{T-1}^*)} = 0 \tag{31}
\]

where, given the result above that \( \nu_{T_s} = 0 \) \( \forall s \geq 0 \), \( S_{T-1}^* = \frac{a}{1 + \nu_{T-1}^*} + \frac{a}{r} \left( \frac{1 + r}{r} \right) \).

Again, equation (31) will be satisfied if, and only if, \( \nu_{T-1} = 0 \). If we keep going backwards in time, we will see that at each period the best trade policy from that period's perspective is a zero tariff.

Q.E.D.

The optimal tariff plan in this no commitment case yields a third best outcome. The government lacks the policy instrument that could make possible the achievement of the first best outcome: subsidies. The use of "surprise", or diverging from the pre-announced policy, works as an additional instrument to try to improve on the second best outcome. However, the agents predict this temptation, and act accordingly: they make their decisions expecting zero tariffs in the future. Therefore, the only time consistent plan is the one with zero tariffs forever.

3.3 Limited Commitment
Instead of the two extreme cases discussed above, it may be more realistic to think of the policy maker as having the ability to commit to a policy for a limited period of time. In a democracy, the government changes periodically. It can try to create rules that make it difficult for the next government to change its policies, but it cannot fully commit to an economic policy after its term in office is over.

I thus ask which would be the best time consistent trade policy under the current model if the policy maker can commit to its policy for some number of periods \( h \), where \( h > 0 \). The dynamics work as follows: at the start of the initial period the government sets a tariffs schedule for \( h \) periods, before the end of the \( h \) period, but after labor allocation decisions are made in period \( h - 1 \), he sets the tariffs for the next \( h \) periods, and so on. Proposition 3 summarizes the optimal tariff schedule for this situation.

**Proposition 3**: If the government is able to commit to its policy plan for \( h \) periods, so that each \( h \) periods it announces the policy for the next \( h \) periods, then \( v^*_i > 0 \) for \( \tau < kh, \; k \in N, \; k-1 < T/h \) and \( v^*_i = 0 \) for \( \tau \geq kh, \; k \geq T/h \). Moreover, \( v^*_i > v^*_i \) where \( v^*_i \) is the optimal tariff if the commitment interval is \( h \) periods, and \( v^*_i \) if the interval is \( h \) periods, for \( h' < h, \; \tau < \min\{kh, k' h'\} \), and \( \max\{kh, k' h'\} < T \).

**Proof**: To prove proposition 3 we will use the same strategy as in the "no commitment" case: work backwards in time in our model. Given that the long run equilibrium is reached at time \( T \), the indirect utility function which the government will maximize every \( h \) periods to choose the tariff schedule for the \( h \)-period interval is:

\[
V_i(L_i, v_i) = \bar{R} + \frac{(1+\theta)(a+b)}{\theta} (\log PV_i - \log S_i) - a \sum_{i=1}^{1+\theta} (1+\theta)^{-i} \log(1+v_i)
\]  

Equation (29) is derived similarly to equation (26). At some time \( \tau \) future utility is maximized, taken as given stocks accumulated until that time, and, here, also given the tariffs after time \( \tau + h \).
where \( PV_{i} = \sum_{i=1}^{T} (1+r)^{-(i-1)} I_{i} + \frac{I^{E}}{r(1+r)^{T-i}} \), with \( I^{E} \) the (constant) value of the per period income at one of the long run equilibria, \( S_{i} = \sum_{i=1}^{T} (1+r)^{-(i-1)} \left( \frac{a}{1+v_{i}} + b \right) \), and \( \bar{K} \) is a constant.

The first order conditions for maximization are:

\[
\frac{(1+\theta)(a+b)}{\theta PV_{i}} \frac{\partial PV_{i}}{\partial v_{r}} = 0
\]

\[
= \left[ 1 - \frac{(1+\theta)(a+b)}{\theta S_{r}(1+v_{r})} \right] \frac{a}{(1+v_{r})(1+r)^{t-r}} \forall t, r \leq \tau < t+\gamma
\]

For \( k \geq T/h \) the l.h.s. of equation (33) is equal to zero. Therefore, using the same argument as in the previous section, we will have \( v_{r}^{*} = 0 \) for \( \tau \geq kh, k \geq T/h \). For \( k-1 < T/h \) the l.h.s. of equation (33) will be positive, hence positive tariffs will be required for the equation to be satisfied. Finally, the larger \( h \), more elements will be included in the l.h.s. sum, therefore the larger will be the tariff each period over the interval to satisfy equation (33).

Q.E.D.

The welfare resulting from the limited commitment time consistent policy is higher than the no commitment one. Because of the possibility of some commitment to future policy, the government can use, at least partially, trade policy to diminish the loss from the externality problem. But the result still worse compared to the full commitment case, when trade policy can be explored fully to deal with the externality.

4. Conclusion

This paper develops a model in which there are two sectors: agriculture and industry. The industrial sector presents positive externalities in production, and there
adjustment costs to changing production from one sector to the other. The model shows that, although the equilibrium with specialization in the industrial good is strictly better than the equilibrium with specialization in agriculture, the initial labor allocation may have so little labor in industry that the present value of income is maximized with the economy following the path to the agricultural equilibrium. The externality in the production of the industrial good distorts the incentives for the workers to shift labor into that sector. Therefore, the thread point which determines whether the economy should move towards specialization in agriculture or industry is different for the market equilibrium and the central planner's solution, as is the rate of labor movement. There is a labor allocation range over which the central planner would take the economy to the industrial equilibrium, while the market on its own would head toward agriculture. Thus, economic policy could not only make workers shift their labor at a income improving rate, but could also lead the economy to a different equilibrium than it would go to on its own. The paper analyses how trade policy should be used in this setting.

Perhaps the most striking result of this paper is the one presented in proposition 1, which states that if the government can make credible, indefinite commitments, the first best trade policy is to keep protection forever, even after the long run equilibrium is reached, and even if the equilibrium reached is the agricultural one! The intuition behind this result is the following. Given any (out of equilibrium) initial labor allocation, the present value of income under the free market is not as high as it could be due to the externality. If the government were allowed to use production subsidies to correct the problem, then the best policy would be to introduce a positive subsidy that would either equal zero at the agricultural equilibrium, or, if the economy were heading to the industrial equilibrium, it would equal zero as time went to infinity.

However, if trade policy is the only instrument available, a different result arises. Although trade policy can improve incentives on the production side of the economy, it
also causes welfare diminishing distortions on the consumption side. The optimal import
tariff/export subsidy finds the best balance between the two effects. As our infinitely
lived agents can borrow and lend at a given interest rate, and they have strictly concave
utility functions, they would like to smooth their consumption. Therefore, instead of
higher tariffs in the beginning that fade away with time, as in the subsidy prescription, the
best policy here would be lower tariffs during the transition and constant positive tariffs
after equilibrium is reached. In this way the proper incentives to the workers would be
provided, but the "consumption cost" would be spread over time.

The "tariffs forever" result relies on some strong assumptions that are not
generally observed in the real world. Even if we could justify the assumption that agents
live forever by saying that each agent cares about his successor's utility, it is hard to
believe that all agents would have free access to financial markets at a given rate as
established in the paper. Moreover, who knows of a government that can fully commit to
its policy plan for the indefinite future? Even Saddam Hussein has some probability of
being out of office some day, without leaving a successor that would follow exactly his
plans. Therefore, we should not expect that consumption smoothing throughout all time is
what agents are doing, or are able to do...

Proposition 2 highlights the importance of commitment. It shows that if the
government has no commitment capability the tariff will be always zero, i.e., the
government will not be able to use trade policy to achieve a higher income level.

The result from proposition 3 seems to be more realistic. It states that when the
government has limited commitment to its future policy tariffs will be lower than with
full commitment, and it will eventually be zero after equilibrium is reached. The result is
still worse for our economy, compared to the case the government has full credibility.
An interesting development of these results is that institutions that give credibility to a government's long term policy plan could be welfare improving. Take, for instance, protectionist lobbying. Given that it may be a credible guarantor of protection over some period of time, its presence may be welfare improving if the government does not have much policy credibility. A word of caution is necessary here: if the lobbying is not believed to be effective over the medium term, but is able to raise trade barriers, then we have the worse result of all. Workers will make their labor allocation decisions given that they expect no protection in the future, so that the (unexpected) protection does not improve labor allocation, but still causes consumption distortions.

What about a developing country that inherits trade barriers on industrial goods, and is on its way to the industrialized equilibrium? If the government is to restructure its trade policy, the first best alternative is to choose import tariffs/export subsidies so as to maximize the indirect utility function, as we did in section 3.1. Given that the government can commit to its policy for some number of periods $h$, this would imply positive tariffs, even for some time after complete industrialization. However, one has to inquire the effect of changing current trade policy on the government's credibility vis-a-vis the new tariff plan it announces. A result like preposition 2 may arise.
Appendix A

In this appendix the producer's problem is solved, which is the maximization the present value of income (equation (4)) subject to the constraint on the movement of the state variable (equation (4.1)), and the per period constraints on the state variables (equation (4.2)). Equation (4.2) yields two possible cases: the lower boundary is binding, in which case the constraint is \( \bar{G}(L,u) = -u_e - L_e \leq 0 \); or the upper boundary is binding, in which case it is \( \bar{G}(L,u) = u_e + L_e - \bar{L} \leq 0 \).

The following Hamiltonian is used to solve the producer's maximization problem:

\[
H(L,u,\lambda, \tau) = \left[ d_e L_e + \bar{L} - u_e^2 / 2 \gamma \right] (1 + r)^{\tau} + \bar{\lambda}_e u_e
\]  

(A1)

where \( d_e = p(\alpha + n\beta L_e) - 1 \), and \( \lambda_{e+1} = \bar{\lambda}_e (1 + r)^{\tau} \).

The first order conditions necessary, and which will also be sufficient\(^{17}\), for maximization are:

(a) the control variable \( (u_e) \) at each period must be chosen to maximize the function \( H(L,u,\lambda, \tau) \), subject to constraint \( \bar{G}(L,u) \) or \( \bar{G}(L,u) \), depending on which one is binding;

(b) the state and co-state variables must change over time according to the following equations:

\[
\begin{align*}
\bar{\lambda}_{e+1} - \bar{\lambda}_e &= -H'_{L}(L_e,\lambda_{e+1}, \tau) \\
L_{e+1} - L_e &= H'_{\bar{L}}(L_e,\lambda_{e+1}, \tau)
\end{align*}
\]  

(A2)

\(^{17}\)The following conditions are sufficient as well as necessary because the Hamiltonian maximized with respect to the control variable is a concave function of the state variable. (See Intrilligator (1971), p.366, fn. 5)
where \( H^*(L_{\tau}, \lambda_{\tau+1}, \tau) \) is the value of the Hamiltonian after \( u_{\tau} \) is optimally chosen.

There are two different sets of conditions for part (a), depending on which constraint is binding, i.e., whether the economy will reach the agricultural or industrialized equilibrium in the long run.

If the economy is heading to the agricultural equilibrium, we use the Lagrangian \( \mathcal{S} = H(L, u, \lambda, \tau) + \xi \bar{G}(L, u) \), and derive the following condition from part (a):

\[
\frac{u_{\tau}}{\gamma} - \lambda_{\tau+1} - (1+r)^{\xi} \xi_{\tau} \leq 0, \quad -u_{\tau} \geq 0 \quad \text{with complementary slackness} \quad (A3)
\]

\[ u_{\tau} + L_{\tau} \geq 0, \quad \xi_{\tau} \geq 0 \quad \text{with complementary slackness} \]

And if the economy is heading to the industrialized equilibrium, the Lagrangian used is \( \mathcal{S} = H(L, u, \lambda, \tau) + \xi \bar{G}(L, u) \), and the conditions:

\[
\frac{-u_{\tau}}{\gamma} + \lambda_{\tau+1} - (1+r)^{\xi} \xi_{\tau} \leq 0, \quad u_{\tau} \geq 0 \quad \text{with complementary slackness} \quad (A4)
\]

\[ u_{\tau} + L_{\tau} - \bar{L} \geq 0, \quad \xi_{\tau} \geq 0 \quad \text{with complementary slackness} \]

Finally, the equations for part (b) are:

\[
\lambda_{\tau+1} - (1+r) \lambda_{\tau} = -d_{\tau} \quad (A5)
\]

\[ L_{\tau+1} - L_{\tau} = u_{\tau} \quad (A6) \]

The transversality conditions also need to be satisfied. This condition states that as time goes to infinity either the value of moving one more unit of labor into a sector is zero, or there is no labor in that sector. Equation (A7) presents the transversality conditions for the industrialized and agricultural equilibria, respectively.
Equations (A3)-(A7) completely determine the solution to the problem. From equations (A3) and (A4) we know that when the labor frontier is not binding, labor movement will follow:

\[ L_{e+1} - L_e = \gamma \lambda_{e+1} \]  

(A8)

When one of the frontiers is reached the control variable is set to zero due to equation (A6), \( u_e = 0 \), and either \((1+r)^T \xi_e \geq -\lambda_{e+1} \) or \((1+r)^T \xi_e \geq \lambda_{e+1} \), depending on whether the lower or upper boundary is reached, respectively.

From equation (A5), and using the transversality conditions, we have that:

\[ \lambda_{e+1} = \sum_{i=e+1}^{N} \frac{d_i}{(1+r)^{-i}} \]  

(A9)

Using the equation above we derive the value of the co-state variable when the economy is at the long run equilibrium, given that \( d_e \) is constant at the boundaries:

\[ \lambda^* = \frac{d^*}{r} \text{ for } k = N, A \]  

(A10)

where \( d^* = p(\alpha + n\beta L) - 1 \) and \( d^* = p\alpha - 1 \).
Appendix B

In this appendix the validity of equation (26) will be proved. We want the value of:

\[
\frac{\partial PVI}{\partial \nu_{s+1}} = \sum_{i=1}^{l} \frac{\partial PVI}{\partial L_i} \frac{\partial L_i}{\partial \nu_{s+1}} + \sum_{i=1}^{l} \frac{\partial PVI}{\partial \lambda_i} \frac{\partial \lambda_i}{\partial \nu_{s+1}} \quad (B1)
\]

First we focus on the second term of the sum of the equation above. Combining equations (4.1) and (6) we get that 

\[ u_{s+1} = \gamma \lambda_{s+1}, \] and substituting this into equation (4), it is straightforward to check that \( \frac{\partial PVI}{\partial \lambda_i} \) is the same for all \( \nu_{s+1}, s \geq 0 \):

\[
\frac{\partial PVI}{\partial \lambda_i} = (1 + r)^{-(i-1)} \gamma \lambda_i \quad (B2)
\]

Using equation (A9) from appendix A, we have that:

\[
\frac{\partial \lambda_{s+1}}{\partial \nu_{s+1}} = p(\alpha + n \beta L_r (1 + r)^{-(T-s-1)}) = (1 + r)^{-s} \frac{\partial \lambda_{s+1}}{\partial \nu_r} \quad \text{for } 1 \leq i < T \quad (B3)
\]

where \( L_r \) is the (constant) value of labor at one of the long run equilibria.

Now we turn to the first term of the sum in equation (B1). Equation (22) can be rewritten as:

\[
L_s - \gamma \sum_{j=1}^{s} (1 + r)^{j} \left[ \sum_{i=1}^{l} \frac{d_{i}}{(1 + r)^{i-1}} \right] = L_o + \gamma \sum_{j=1}^{s} (1 + r)^{j} \left[ \sum_{i=1}^{l} \frac{d_{i}}{(1 + r)^{i-1}} \right] \quad (B4)
\]

for \( 1 \leq s < T \).

Equation (B4) represents a system of \( T - 1 \) linear equations, which can be written in matrix form as \( A \bar{L} = \bar{b} \):
For this system of equations to have a solution, matrix A has to be non-singular. The system was constructed imposing the condition that a specific long run equilibrium will be reached at period T. If matrix A turns out to be singular for any tariff schedule, it means the equilibrium is not achievable in that time frame from the initial position. Limiting the values for T as suggested in section 3.1 overcomes this problem.

Using Cramer’s rule to solve for the value of each labor allocation, we have:

$$L_s = \frac{D(A_s^\tau)}{D(A)}$$

(B5)

where $b_i = L_0 + K_i([v_s]_i^{T-1}) + \gamma(1+r)^T \sum_{t=T}^{T-1} \frac{d_t}{(1+r)^{t+1}}$, with $K_i$ a function of tariffs previous to period T, $a_u = 1 - \frac{\gamma p\beta v_i}{(1+r)^T} \left( \sum_{t=0}^{i-1} (1+r)^t \right)$, and $a_{ij} = -\frac{\gamma p\beta v_i}{(1+r)^T} \left( \sum_{t=0}^{i-1} (1+r)^t \right)$.

For this system of equations to have a solution, matrix A has to be non-singular. The system was constructed imposing the condition that a specific long run equilibrium will be reached at period T. If matrix A turns out to be singular for any tariff schedule, it means the equilibrium is not achievable in that time frame from the initial position. Limiting the values for T as suggested in section 3.1 overcomes this problem.

Using Cramer’s rule to solve for the value of each labor allocation, we have:

$$L_s = \frac{D(A_s^\tau)}{D(A)}$$

(B6)

where $D(A)$ represents the determinant of matrix $A$, and $A_s^\tau$ is a matrix obtained from $A$ by replacing its $\tau$th column by the vector $\bar{b}$.

We want to know the value of the derivative $\frac{\partial L_s}{\partial \nu_{s+1}} \forall s \geq 0$. The denominator of the function that determines $L_s$ (equation (B6)) does not depend on the value of the tariffs after time T. Therefore, we have:

$$\frac{\partial L_s}{\partial \nu_{s+1}} = \frac{1}{D(A)} \frac{\partial D(A_s^\tau)}{\partial \nu_{s+1}}$$

(B7)

We can expand $D(A_s^\tau)$ using the $\tau$th column, obtaining:
\[ D(A^i) = (-1)^{i+\tau} b_1 D(A_{\tau}) + \ldots + (-1)^{(T-1)+\tau} b_{T-1} D(A_{T-1, \tau}) \]

where \( A_{\tau} \) is a matrix obtained from \( A^i \) by excluding row \( i \) and column \( \tau \) from it.

Now equation (B7) can be rewritten as:

\[ \frac{\partial L_\tau}{\partial v_{\tau+v}} = \frac{\partial b_i}{\partial v_{\tau+v}} \frac{D(A_{\tau})}{D(A)} \ldots + \frac{\partial b_{T-1}}{\partial v_{\tau+v}} \frac{D(A_{T-1, \tau})}{D(A)} \]  

(B8)

We know that:

\[ \frac{\partial b_i}{\partial v_{\tau+v}} = \gamma (1 + r)^{i} \frac{p(\alpha + \eta \beta \gamma)}{(1 + r)^{\gamma+1}} = (1 + r)^{-z} \frac{\partial b_i}{\partial r} \]  

(B9)

Combining equations (B8) and (B9), we finally get:

\[ \frac{\partial L_\tau}{\partial v_{\tau+v}} = (1 + r)^{-z} \frac{\partial L_\tau}{\partial r} \]  

(B10)

Substituting equation (B3) and (B10) into (B1) we finish our proof:

\[ \frac{\partial PVL}{\partial v_{\tau+v}} = \sum_{i=1}^{T} \frac{\partial PVL}{\partial L_i} (1 + r)^{-z} \frac{\partial L_\tau}{\partial r} + \sum_{i=1}^{T} \frac{\partial PVL}{\partial L_i} (1 + r)^{-z} \frac{\partial L_\tau}{\partial r} \]  

(B11)
Appendix C

The problem for the policy maker after the long run equilibrium is reached is to maximize the indirect utility function from that moment on. First we need to derive the function for this time frame.

In period $T$ consumers will maximize:

$$\sum_{t=T}^{\infty} \frac{1}{(1 + \theta)^t} [a \log C^N_t + b \log C^A_t]$$  \hspace{1cm} (C1)

subject to the following budget constraints, observing that the quantity of assets held at the beginning of period $T$ is exogenously given from that period's point of view:

$$f_{t+1} - f_t = r f_t + l - R_t$$  \hspace{1cm} (C2)

$$f_T = \sum_{t=0}^{T} (1 + r)^{T-t} (l_t - R_t)$$

The first order conditions for the maximization above are analogous to those derived previously, i.e., equations (17.a-d); and the transversality condition in equation (18) can be rewritten as:

$$\lim_{T \to \infty} \frac{-y}{(1 + \theta)^T} f_{T+1} = 0$$  \hspace{1cm} (C3)

Using equation (17.d) and the initial value for $f_T$, it follows that:

$$f_{T+1} = (1 + r)^T f_T + \sum_{i=0}^{T} (1 + r)^{T-i} [l_{T+i} - R_{T+i}]$$  \hspace{1cm} (C4)

Using the government's budget constraint, and substituting equations (17.a) and (17.b) into equation (C4), the transversality condition yields:

$$y = \frac{S_T}{PVI_T + \phi_T}$$  \hspace{1cm} (C5)
where

\[ S_T = \sum_{i=0}^{T} (1 + r)^{-i} \left[ \frac{a}{(1 + \nu_{T+i})} + b \right] \]

\[ PV_T = \sum_{i=0}^{T} (1 + r)^{-i} I_{T+i} \]

\[ \Phi_T = \sum_{i=0}^{T} (1 + r)^{-i} [I_i - pC_i - C_i^N]. \]

Substituting equation (C5) into (17.a) and (17.b), and then substituting these optimal consumption decisions into the utility function, gives the following indirect utility function for time \( T \).

\[ V_T(L_T, \nu_T) = K - \frac{(1 + \theta)(a + b)}{\theta} \log S_T - a \sum_{i=T}^{\infty} (1 + \theta)^{-i} \log(1 + \nu_i) \]  

(C6)

where \( K \) is a constant term.

Note that \( PV_T \) is now included in the constant term due to the fact that the labor allocation is constant from period \( T \) on. We have that \( PV_T = I^E \frac{(1 + r)}{r} \), where \( I^E \) is the per period income at the equilibrium point.
Bibliography


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