"Monopolies Life Cycle, Bureaucratization, and Schumpeterian Growth"

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Monopolies Life Cycle, Bureaucratization, and Schumpeterian Growth*

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ABSTRACT

This paper analyzes the links between the internal organization of firms and macroeconomic growth. We present a Schumpeterian growth model in which firms face dynamic agency costs. These agency costs are due to the formation of vertical collusions within the organization. To respond to the opportunity of internal collusion, firms go through a whole life cycle, getting more bureaucratized and less efficient over time. Weak creative destruction in the economy facilitates informal collusion inside firms and exacerbates bureaucratization. As bureaucratization affects the firms’ profitability and the return to innovation, stationary equilibrium growth depends in turn on the efficiency of collusive side-contracts within firms.

Keywords: Bureaucratization, Schumpeterian Growth, Dynamic Collusion, Internal Organization of the Firm.

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1 Introduction

How does the internal organization of the firm affect the rate of growth of an economy? Does the process of creative destruction exert an efficient control on organizational slacks? More generally, are agency costs within firms linked to the macroeconomic environment in which they evolve? And finally, are the social relationships which establish within firms dependent on their environment?

These questions are important ones if we want to understand better the role of organizations and, more broadly of institutions, in the growth process. Noticeably, these issues have already motivated numerous contributions in economic history (Schumpeter (1942), North (1990) and Mokyr (1990)) and economic sociology (Weber (1922) and Granovetter and Swedberg (1992)). However, almost no effort has been devoted to give formal answers to these questions. This lack of formal work is probably explained by the difficulty one faces at describing the internal structure of the firm at a relatively detailed microeconomic level and integrating these considerations into general equilibrium macroeconomic growth models.

This paper is a first attempt at giving a unified theoretical framework describing how agency costs within firms are linked to their environment. Our main motivation is to understand the links between the rate of growth of an economy and the kind of social relationships which establish within firms. In particular, we are interested in explaining how the development of bureaucratic rules in large monopoly firms involved in the growth process depends on and affects the rate of creative destruction in the economy.

The focus of this paper comes from the juxtaposition of three incomplete strands in the economic literature. First, taking a macroeconomic perspective, Olson (1982, p.42) has forcefully argued in his celebrated book The Rise and Decline of Nations that "Stable Societies with unchanged boundaries tend to accumulate more collusions and organizations for collective action over time". Starting from this casual observation, Olson emphasizes that collusion has detrimental consequences for the growth rate of the economy. However, his argument is quite informal and lacks some theoretical underpinnings. In particular, one aspect which remains unclear is why and how collusions tend to develop inside organizations.

Second, the more recent microeconomic literature on collusion in organizations (Tirole (1986)) has underscored how bureaucratic rules represent in fact optimal responses to the emergence of collusions inside the firm. However this literature remains essentially partial equilibrium and derives no consequences for the behavior of the economy as a whole. In particular it leaves open the question of how collusions may interact to reduce the macroeconomic growth.

Finally there is the so called literature on "population ecology" (Hannan and Freeman (1977)), which investigates the evolution of organizations but from a population dynamics point of view. In this approach, the emphasis is on describing how changes in the firms' external environment affect organizational behaviors through selective forces at the population level. This approach brings an interesting linkage from the macro side of the economy to the micro relationships inside firms. However, it is incomplete in the sense that it does not provide clear foundations for the nature of the social selection process by which certain types of internal organizations are selected away. Also, it does not take either into account the aggregate potential feedback effects of the distribution of organiza-
tional behaviors on the dynamics of the external environment itself. Hence, it misses the macroeconomic implications of collusion in organizations firms so forcefully illustrated by Olson and economic historians.

In this paper, we try to integrate together the insights of these three literatures. In order to do this, we consider a Schumpeterian model of economic growth (Aghion and Howitt (1992) and (1998), Grossman and Helpman (1992)) in which we embed a simple agency model with vertical collusion à la Tirole (1986) and Martimort (1998). As it turns out, this model provides a suitable and tractable framework to analyze macroeconomic growth in which the process of creative destruction associated to technological change generates also a natural endogenous economic selection mechanism across monopoly firms and organizational behaviors in the population.

More precisely, a final competitive sector uses intermediate goods produced by monopolies. The perspective of monopoly profits in these intermediate sectors creates the incentives of the R & D sectors to innovate. The profitability of those monopolies coupled with the positive externality among innovating firms due to the public good nature of knowledge generated in society is thus the fundamental engine of growth.

However, contrary to standard neoclassical growth models (both exogenous and endogenous ones), the firm is not viewed as a simple production function but the black box of its internal organization is explicitly opened. In particular we consider monopolistic firms as three-tier hierarchies involving owners, supervisors and managers and facing the two separation problems between ownership and control of productive assets and ownership and control of monitoring (Jensen and Meckling (1976)). More precisely, owners allocate resources within the firm subject to the incentive constraints which arise when managers have a better knowledge on the firm's market demand and may get informational rent from this knowledge. But also owners use monitoring structures (supervisors) to reduce the informational gap with the firms' managers.

Following then the industrial sociology literature (Dalton (1959) and Crozier (1962)), we recognize that supervisors and managers are at a nexus of commonly known information. They may take advantage of this collective informational advantage to promote their own objectives and collude against the interests of owners. ¹ In Dalton's vocabulary, supervisors and managers form active vertical cliques which affect the allocation of resources within the firm. As a response to the possibility of those collective manipulations of information, owners tilt incentive schemes towards relatively bureaucratic rules which are less sensitive to information and which leave few discretion to supervisors (Tirole (1986)).

Departing however from most of literature on collusion in organizations (Tirole (1986) and (1992), Kofman and Lawarrée (1993) and Laffont and Martimort (1995)), we recognize that the quality of the colluding relationship between supervisors and managers (ie. side contracting), is not exogenously given. Instead, transaction costs of side-contracting within the firm must be linked to the prospects that members of a collusion have to continue to exchange favors in the future if the firm goes on. i.e. if there is no innovation making obsolete the firm's technology. We build therefore on Martimort (1998) which provides a microeconomic theory of the life cycle of captured regulatory agencies.

¹The supervisor may manipulate or delay the audit he performs to assess the manager's performances. In exchange, the latter may bribe him or give him some in-kind transfers.
with *endogenous dynamic transaction costs*.\(^2\) One crucial result of this analysis is that transaction costs of side-contracting decrease over time, capturing Olson’s intuition that collusions become more efficient over time.

In this context, an important contribution of the paper is to emphasize the fact that macroeconomic growth and the quality of social relationships which establish within firms are strongly intertwined. In particular we first show the existence of a *two-way-causality* between macroeconomic performances and the microeconomic structure of firms of the economy.

Indeed, as noted by Crozier (1962), the stability of the environment is the fundamental glue of collusions inside organizations\(^3\). When the process of creative destruction in a society does not exert its clumpsering effect, *vertical cliques* within the firm form more easily. In Olson’s vocabulary, collusions “*have more opportunities to emerge*.” In “population ecology” parlance, social selection forces are biased towards higher survival rates of organizations with vertical collusion. The threat of facing more collusion over the life of a monopoly make its owners eager to move the supervisors and the managers’ compensation schemes towards being less and less powered over time. Rules become increasingly bureaucratized as time passes. This *bureaucratization* phenomenon impedes, in turn, significantly the achievement of efficiency within monopolized sectors since it implies an insufficient use of information. As a result, the profitability of the intermediate sectors falls. This reduces the R & D sectors incentives to innovate. Growth is slowed down by the agency problems faced within intermediate sectors. and, in turn, *creative destruction* remains weak in economy.

On the contrary, when the rate of innovation is greater, the process of creative destruction fully exerts its disciplinary effect. Vertical cliques have less opportunities to form. Collusion does not affect too much the allocation of resources since firms die on average before the end of the bureaucratization. From a “population ecology” point of view, selection is biased towards “less bureaucratized organizational behaviors”. In turn, the profitability of the intermediate sectors is larger and pushes up the R & D sectors incentives to innovate. Growth is reinforced.

We prove then the existence of a stationary equilibrium growth. We also discuss how the two-way complementarity between growth and the internal structure of the firm is reinforced by the macroeconomic externality in innovation due to the public good nature of social knowledge. As a matter of fact, the negative impact of the bureaucratization of intermediate sectors on the incentives to innovate is amplified. Since less innovation occurs at the level of each individual sector, there is less accumulation of social knowledge and productivity growth is less important. This effect reduces even further the profitability of intermediate sectors and the incentives to innovate in the research sectors. Multiple equilibria may then emerge as a result of this coordination failure. Some with higher growth rate, more creative destruction and less bureaucratization. others with slow growth, weak creative destruction and bureaucratized monopolitistic firms.

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\(^2\)The vertical collusion which is considered there involves a captured regulator and an interest group acting against the public interest.

\(^3\)See also Bluedorn (1982), on the negative relationship between turnover of personnel and the formation of internal collusions and manipulations inside organizations. See also Pfeffer (1983) and Granovetter (1992) for a discussion on the role of turnover of the labor force and inside efficiency and bureaucratization of firms.
Our analysis also allows us finally discussing the distortions of the growth rate with respect to a setting where transaction costs of side-contracting are not time-dependent and to provide comparative statics of the stationary equilibrium growth rate to parameters of the technology of internal organization of firms.

This paper is linked to a small literature trying to link agency considerations, the internal organization of the firm and the growth process. In a model with horizontally differentiated products, Acemoglu and Zilibotti (1997) argue that the performances of competitors may provide useful information to improve incentives within a given firm. As in our model, the equilibrium growth rate is affected by the firm's profitability which, in turn, depends on the spectrum of products already available. Aghion, Dewatripont and Rey (1997) analyze a model where the threat of bankruptcy forces conservative managers to speed up the adoption of new technologies to remain competitive on the product market. Debt contracts act as commitment devices to move towards profit maximization and this increases the growth rate. Thesmar and Thoenig (1998) endogenize the choice of firms' organizational structures in a Schumpeterian growth. However their concern is different from ours. Abstracting from an explicit discussion of agency costs, they emphasize how these structures affect the tradeoff between productivity and delay of adoption in new technologies and derive implications for wage inequality and growth.

At a broader level, since we investigate the impact of the external environment on agency costs, our paper is also related to a literature (Hart (1983), Hermalin (1992), Schmidt (1997), among others) which discusses the one-sided causality from market competition to agency costs. While in these papers, competition is viewed as a static phenomenon, our analysis of technological change and creative destruction stresses the effects of dynamic competition on firms' side-contracting.

Considering collusive side-contracting within the firm and their impact on resource allocation provides in fact a political economy view of the firm in which group interests matter. This view should be contrasted with papers in the political economy of technological change which analyze vested interests as a source of stagflation (Krussel and Rios-Rull (1996) and Jovanovic and Nyarko (1994)). In those models, decision-making is made through voting for or against a new technology. There exists a bias for stagflation when old agents benefiting from a learning effect have the control of the political process.

Section 2 describes both the macroeconomic and the microeconomic sides of the economy. Section 3 presents an ad hoc model in which the monopolies' life cycle is postulated rather than endogenously derived. This section derives the growth rate of the economy when transaction costs of side-contracting are not time-dependent. Section 4 shows that the existence of a stationary rate of growth is ensured when transaction costs of side-contracting decreases over time. We obtain there conditions under which bureaucratization pushes up or slows down the growth process. Section 5 goes further in deriving endogenous transaction costs and in exhibiting their time-dependency. This allows us to go further in the analysis of the distortions in growth due to agency costs. In Section 6, we derive also several comparative statics results from the endogenous relationship between the rate of growth and parameters characterizing the internal organization of the firm. Section 7 concludes. Proofs are relegated to an appendix.

\footnote{Selten (1986) is a noticeable exception. However in his "X-inefficiency" model, the dynamics of slacks is exogenously specified.}
2 Transaction Costs and Schumpeterian Growth

Our framework has two building blocks. The first one is a standard Schumpeterian model along the lines of Aghion and Howitt (1992) and (1998) and Grossman and Helpman (1992). It represents the macroeconomic side of the economy. The second building block is microeconomic and discusses the internal organization of the monopolistic firms evolving in this macroeconomic environment.

2.1 The Schumpeterian Model

We abstract completely from capital accumulation. Time is indexed by $t \in \{0, +\infty\}$.

2.1.1 Preferences

The economy is populated by a continuous mass $L$ of individuals with linear intertemporal preferences:

$$u(y) = \sum_{i=0}^{\infty} \frac{1}{(1+\rho)^t} y_t,$$

where $\rho$ is the psychological discount rate and $y = \{y_t\}_{t \geq 0}$ the vector of consumptions. We assume that $\rho = r$ where $r$ is the interest rate which is exogenously given.

Each of these individuals is endowed with one unit flow of skilled labor.

2.1.2 Technologies

- **Final Sector**: There is only one final consumption good which is produced from a continuum of intermediate goods indexed on the unit interval. More precisely, date $t$ output in the final good sector writes as:

$$y_t = \int_0^1 y_{it} di,$$

where

$$y_{it} = A_{it} \beta_{it} x_{it}^\alpha \quad (\alpha \in [0, 1])$$

is the flow of final good which can be produced using a quantity $x_{it}$ of intermediate good $i$ at date $t$.

The parameter $A_{it}$ is the "fundamental" productivity of the latest generation of intermediate good $i$. However the overall productivity of this sector is also affected by the realization of some random shock $\beta_{it}$. These shocks are independantly distributed over time and over sectors according to the same common knowledge distribution on $\{\beta, \tilde{\beta}\}$ (we denote thereafter by $\Delta \beta = \tilde{\beta} - \beta > 0$ the spread of the uncertainty) with respective probabilities $1 - \nu$ and $\nu$. $\beta_{it}$ captures the intrinsic uncertainty on the quality of the match between the intermediate sector $i$ and the final good technology. More generally, $\beta_{it}$ can be viewed as a technological shock as those considered in the real business cycle literature.

- **The final sector has perfect knowledge of the realizations of $\beta_{it}$**: This sector is perfectly competitive and its demand for each intermediate good writes as:

$$p_{it} = \alpha A_{it} \beta_{it} x_{it}^{\alpha-1}$$
where $p_{it}$ is the price of good $i$ at date $t$.

- **Intermediate Sectors:** Each intermediate good is produced by a monopolistic sector $i$. The monopolistic firm $M_i$ holds a patent of the latest generation of good $i$. To produce one unit of intermediate good $i$ requires one unit of skilled labor.

  Since the productivity of the final sector is random, this monopolist faces, at each date $t$, an uncertain demand for its good which depends on the realization of $\beta_{it}$.

- **Research Sectors:** There are as many research sectors as intermediate goods. R&D firms in each sector compete to discover the next generation of good $i$. The arrival of innovations in a given sector follows a Poisson process. An innovation appears with probability $\phi(n_{it}) \in [0, 1]$ where $n_{it}$ is the amount of skilled labor devoted to research in this sector. For technical reasons, we assume that $\phi(0) = 0$ and $\phi(L) = 1$, $\phi'(n) > 0$ with the Inada conditions $\phi'(0) = \infty$ and $\phi'(L) = 0$ and $\phi''(n) < 0$ for all $n$.

  By innovating in sector $i$, a R & D firm acquires the leading-edge technology whose productivity parameter is given by $A_{it}^{\text{max}} = \max A_{it}$.

  Each time a fraction $\phi$ of firms innovate, the leading-edge technology jumps upwards by an increment $\phi q + (1 - \phi)$ where $q > 1$.

  Given that we consider a model with a continuum of sectors, the law of large numbers applies and ensures that, in a symmetric equilibrium where all research sectors use the same amount of skilled labor $n_{it}$, there is always a fraction $\phi(n_i)$ of firms innovating at each date $t$.

  Along such a symmetric path, the leading-edge technology evolves thus over time according to the following difference equation:

$$A_{i,t+1}^{\text{max}} - A_{i,t}^{\text{max}} = \phi(n) (q - 1) A_{i,t}^{\text{max}}. \quad (4)$$

When a larger fraction of firms in the research sector innovates at the same time, the upwards jump of the leading-edge technology gets larger. The evolution of the leading-edge technology is thus affected by a macroeconomic positive externality which captures spillovers between innovating firms due the public good nature of knowledge generated in society. Note also that the fundamental productivity parameter in a given sector $i$, $A_{it}$, may move discontinuously at the time of innovation if it jumps ahead several steps to reach the new leading-edge technology. Clearly, the quality of a new generation of product benefits from this technological spillover across sectors.

### 2.2 Transaction Costs and the Internal Organization of the Monopolies

We now open the black box of the internal organization of the monopolies in the intermediate sectors.

#### 2.2.1 Internal Organization of the Monopolies

- As said above, producing one unit of intermediate good $i$ requires one unit of skilled labor used in a production department. However, it requires also to establish a sales

\footnote{We consider that $q$ is large enough so that the monopolistic firm does not have to undertake limit pricing to capture its market's demand}
Following Jensen and Meckling (1976), there is separation between ownership and control of some productive assets within each monopoly. The manager of the sales department is privately informed on $\beta_i$, the quality of the match between the intermediate $i$ and the final good. Owners remain uninformed. A supervisor may thus be used to fill the informational gap between the manager and owners. There is also separation between ownership and control of the monitoring technology. This double-separation creates scope for agency costs.

A monopoly can thus be viewed as a large firm having a hierarchical structure including a set of owners, a supervisor, a production and a sales departments.

• The manager receives the sales proceeds and gives back some transfers $T_{it}$ to the owners. The manager's intertemporal utility writes as:

$$
\sum_{r=0}^{\infty} \frac{\Pi_{r=0} (1 - \phi(n_{i(t+r)}))}{(1 + r)^r} (\alpha A_{i(t+r)} \beta_{i(t+r)} x_{i(t+r)} - T_{i(t+r)}).
$$

(5)

• The supervisor may learn an informative signal $\sigma_{i(t+r)}$ on the realization of $\beta_{i(t+r)}$. More precisely, the monitoring technology is such that $\beta$ is learned with a conditional probability $\epsilon$. Otherwise, nothing is learned. This gives the following unconditional probabilities:

$$
\sigma_{i(t+r)} = \begin{cases} 
\beta & \text{with probability } \nu \epsilon \\
0 & \text{with probability } 1 - \nu \epsilon.
\end{cases}
$$

Following the methodology developed in Tirole (1986), $\sigma_{i(t+r)}$ is a hard information signal. The knowledge of $\beta$ can be concealed but can never be manipulated by the supervisor.

The supervisor receives wages $S_{i(t+r)}$ from the owners of the firm. His intertemporal utility becomes:

$$
\sum_{r=0}^{\infty} \frac{\Pi_{r=0} (1 - \phi(n_{i(t+r)}))}{(1 + r)^r} S_{i(t+r)}.
$$

(6)

• Owners of a patent for good $i$ starting at date $t$ maximize their intertemporal discounted profits from that date on taking into account the probability that there is no new innovation, i.e., the probability that their firm goes on as a valuable venture:

$$
\Pi_{i(t+r)} = T_{i(t+r)} - S_{i(t+r)} - w_{t+r} x_{i(t+r)}
$$

is date $t + \tau$ profit and $w_{t+r}$ is date $t + \tau$ wage of skilled labor.

• There is a unit mass of supervisors (resp. sales department managers) available in this economy. Upon arrival of an innovation, there are as many supervisors and managers "dying" in old obsolete firms than new supervisors and managers hired in newly created monopolies.

• Supervisors and managers have an exogenously given reservation wage fixed to zero.\(^6\)

\(^6\)This assumption implicitly requires that they are not part of the skilled labor force. For instance, becoming a manager or a supervisor requires to incur some specific investment ex ante and this investment creates a segmented labor market. Note nevertheless that introducing a non-zero reservation wage would
2.2.2 Contracts

Because monitoring is imperfect, owners rely on incentive schemes to induce information revelation from the manager and the supervisor.

>From the Revelation Principle, there is no loss of generality in considering direct revelation mechanisms. An incentive scheme in firm is a triplet where is the supervisor’s report on the signal he has observed. To make notation simpler, we denote by the output and transfer targets respectively in the states of nature where is the supervisor’s (resp. the manager’s) report on the signal he has observed. For latter use, note that these states arise with respective probabilities .

- Contractual Incompleteness: Three important features of this contract should be stressed.

First, the agent’s report is only useful following an uninformative report from the supervisor . Indeed, when has been reported (and has also been observed since information is hard), there is no reason to use the manager’s report since the state of nature is perfectly known by owners.

Second, the owners can commit to an intertemporal production plan, i.e., the sequence once they get their monopoly patent. However, owners cannot commit to the wages needed to implement this production plan. As it will become clearer later on, this latter assumption plays an important role in describing how the firm responds to the threat of internal collusion.

Finally, the contracts that we consider thereafter are not history dependent. Wages and output targets are contingent on the whole past history of reports. Only calendar time and current reports are used in date contract.

• Incentive Compatibility and Participation Constraints: We can now easily write the incentive compatibility constraint preventing a manager having observed a high realization of demand to pretend he has observed a low realization:

where denotes the manager’s informational rent at date when he has observed (resp. ). Similarly, the manager cannot be forced to not change the main results of our analysis. Since this non-zero reservation wage is fixed and does not depend on the knowledge of , it would not have any impact on allocative distortions imposed by the incentive problem within the firm. Only the profit of the owners would be translated downward by a constant term. This certainly affects the incentives to innovate of the R & D sector. However, the stream of profits would exhibit the same intertemporal path than with a zero reservation wage. This effect is thus orthogonal to our main focus which is to analyze the impact of dynamic agency costs within the monopolies on their profitability and therefore on the growth rate.

This Principle holds in the case of no-collusion but has not yet been proved with dynamic collusion. We nevertheless conjecture it still applies.

See Martimort (1998) for further discussions of this assumption.

It is standard to show that this incentive constraint is the only binding one at the optimum. The incentive constraint of a low demand manager is automatically satisfied.
a negative utility in any single period, so that the limited liability constraint
\[ U_{i(t+\tau)} \geq 0 \] (9)
is always satisfied.\(^{10}\)

When \( \sigma_{it(t+\tau)} = \beta \), the manager is known having observed a high demand for his intermediate good. The owners of the firm can extract all his informational rent so that he only gets his reservation value. Denoting by \( \tilde{u}_{pit(t+\tau)}, \tilde{x}_{pit(t+\tau)}, \tilde{T}_{pit(t+\tau)} \), the manager’s informational rent, the output level and the transfer target in that situation, we have:
\[ \tilde{u}_{pit(t+\tau)} = A_{pit(t+\tau)} \tilde{x}_{pit(t+\tau)} - \tilde{T}_{pit(t+\tau)} = 0. \] (10)

### 2.2.3 Collusion

To complete the description of incentive problems within the firm, we recognize the bureaucratic limits that large organizations may face.\(^{12}\) The supervisor is aware of the discretionary power associated with his ability to reveal or not the manager’s information when he has received an informative signal \( \sigma_{it} = \beta \). By hiding this information to owners, the supervisor let the manager benefit from some informational rent. By instead revealing this information, the supervisor ensures that this informational rent can be fully captured by owners. The supervisor acts opportunistically to maximize the benefits he may draw from using this discretionary power to promote his own goals rather than those of the firm.

Since they are at a nexus of commonly known information, there is scope for a collusion between the supervisor and the manager. By concealing the hard evidence \( \sigma_{it} = \beta \) to the owners, the supervisor let the manager benefit from some informational rent. In exchange, he receives some bribes. These bribes may take the form of explicit monetary transfers but may also be viewed as a reduced form for the good social relationships which may establish on the work place, for the enforcement of a norm of reciprocity of favors, or other in-kinds transfers.\(^{13}\)

Following previous literature,\(^{14}\) all bargaining power in the side-contract is given to the supervisor. A priori, the full gain from collusion accrues thus to the supervisor. This gain is the difference in informational rents \( \tilde{u}_{i(t+\tau)} - \tilde{u}_{i(t+\tau)} \) that can be pocketed by the manager when the supervisor hides an informative signal to the owners. However, as it has been suggested by Tirole (1986), the collusive activity suffers from some deadweight-loss due to transaction costs of exchanging favors within the firm. When a side-transfer is made between the supervisor and the agent, a fraction \( 1 - K \) of this transfer is lost. The true supervisor’s benefit from colluding with the manager writes therefore as \( K(\tilde{u}_{i(t+\tau)} - \tilde{u}_{i(t+\tau)}) \) where \( K < 1 \) is the efficiency of side-contracting. For the time being, we follow the earlier literature on collusion in organizations and assume that \( K \) is exogenously.

\(^{10}\)An alternative assumption is that the manager is infinitely risk-averse below zero wealth.

\(^{11}\)When (9) and (8) are binding, it is easy to show that \( U_{i(t+\tau)} \geq 0 \) so that the limited liability constraint of a \( \beta \) manager is automatically satisfied.

\(^{12}\)See Williamson (1985, chapter 6).

\(^{13}\)See Gouldner (1954) for the sociological analysis of these reciprocal exchanges and Martimort (1997) for some formal modeling explaining how these norms of reciprocity are enforced.

\(^{14}\)See Tirole (1986).
Firms may then differ with respect to the size of this parameter. For instance, when $K$ gets large, collusion is very efficient and is quite harmful to the organization.

To be consistent with the theory that we present in section 5, one can think of $1 - K$ as representing the deadweight-loss associated with the lack of enforceability of the side-contract between the supervisor and the manager. A large value of $K$ means then that collusion can be quite easily enforced.

To exhaust the scope for collusion, the supervisor must receive a wage large enough so that he prefers revealing an informative signal $\sigma_{it} = 1$ to owners rather than concealing evidence and sharing the corresponding informational rent with the manager. An incentive mechanism prevents collusion between the supervisor and the manager when the following static collusion-proofness constraint is satisfied:

$$S_{i(t+\tau)} - S^0_{i(t+\tau)} \geq K(\bar{u}_{i(t+\tau)} - \bar{u}^*_{i(t+\tau)}).$$  

(11)

$S_{i(t+\tau)}$ (resp. $S^0_{i(t+\tau)}$) is the supervisor’s wage when he reports an informative signal $\sigma_{i(t+\tau)} = 1$ to the owners, (resp. an uninformative signal $\sigma_{i(t+\tau)} = 0$). As for the manager, we protect the supervisor by limited liability so that the constraint

$$S^0_{i(t+\tau)} \geq 0$$  

(12)

must also be satisfied.

3 Exogenous Dynamics of Transaction Costs

The goal of this section is to provide a first overview of the importance of transaction costs in the characterization of the growth rate of the economy.

3.1 Optimal Collusion-Proof Contract

As a benchmark, we find the optimal collusion-proof contract implemented by the owners of monopoly $i$ when they get a patent from date $t$ on. Without loss of generality we assume that the efficiency of side-contracting is time-dependent. $K(\tau)$ denotes therefore this efficiency in a firm having already lived for $\tau$ periods.

Since incentive, collusion-proofness and participation constraints are not linked from one period to the other, the optimal collusion-proof contract is obtained by adding altogether the solutions to the following static problems (where we have used the definitions of the informational rents to express transfers $T_{i(t+\tau)}$ as a functions of $u_{i(t+\tau)}$ into the owners’ objective function):

$$\max_{\{x_{it}, u_{it}, S_{it}\}} \nu(1 - \epsilon)(\alpha A_{it} \tilde{\beta} \tilde{x}^0_{it(t+\tau)} - w_{t+\tau} \tilde{x}_{i(t+\tau)}) + \nu \epsilon(\alpha A_{it} \tilde{\beta} \tilde{x}^*_{it(t+\tau)} - w_{t+\tau} \tilde{x}^*_{i(t+\tau)})$$

$$(1 - \nu)(\alpha A_{it} \tilde{\beta} \tilde{x}^0_{it(t+\tau)} - w_{t+\tau} \tilde{x}_{i(t+\tau)})$$

$$-(\nu \epsilon \tilde{u}^*_{i(t+\tau)}) + (1 - \epsilon) \tilde{u}_{i(t+\tau)} + (1 - \nu)u_{i(t+\tau)} + \nu \epsilon S^*_{i(t+\tau)} + (1 - \nu)S^0_{i(t+\tau)}$$

$^{15}$A noticeable exception is Faure-Grimaud, Laffont and Martimort (1998) who endogenize these frictions in a model of hierarchical delegation.
subject to (8) - (9) - (10) - (11) and (12).

The following proposition describes the classic structure of the optimal collusion-proof contract

**Proposition 1**: The optimal collusion-proof contract entails:

- **No distortion with respect to the complete information monopoly output in states** 
  \(\sigma_i(t+r) = \bar{\beta}\) and \(\sigma_i(t+r) = \emptyset, \beta_i(t+r) = \bar{\beta}\).

  \[
  \bar{x}_i(t+r) = \bar{x}^*_i(t+r) = \left(\frac{w_{t+r}}{\alpha^2 \beta A_{it}}\right)^{\frac{1}{\alpha-1}}.
  \]  

- **A downward distortion with respect to the complete information monopoly output in state** 
  \(\sigma_i(t+r) = \emptyset, \beta_i(t+r) = \bar{\beta}\).

  \[
  \bar{x}_i(t+r) = \left(\frac{w_{t+r}}{\alpha^2 \beta(\tau) A_{it}}\right)^{\frac{1}{\alpha-1}}
  \]

  where

  \[
  \bar{\beta}(\tau) = \beta - \frac{\nu}{1-\nu} \Delta \beta (1 - \epsilon + \epsilon K(\tau)).
  \]

- **Average output at date \(t + \tau\) in sector \(i\) is given by**:

  \[
  \bar{X}_i(t+r) = \left(\frac{w_{t+r}}{\alpha^2 \beta(\tau) A_{it}}\right)^{\frac{1}{\alpha-1}} \left(\nu + (1 - \nu) \left(\frac{\beta(\tau)}{\bar{\beta}}\right)^{\frac{1}{\alpha-1}}\right).
  \]

- **Average profit at date \(t + \tau\) is given by**:

  \[
  \Pi_i(t+r) = \frac{1 - \alpha}{\alpha} \bar{w}_{t+r} \bar{X}_i(t+r).
  \]

Asymmetric information makes costly the implementation of the complete information monopoly output. To make the allocation of a low-demand manager less attractive to a high-demand manager, owners reduce output in state \(\sigma_i(t+r) = \emptyset, \beta_i(t+r) = \bar{\beta}\). This reduces the costly informational rent of a high demand manager and the incentive compatibility constraint (8) becomes less costly to satisfy.

Moreover, reducing informational rent also diminishes the prospects for collusion between the supervisor and the manager once the former has observed an informative signal. The collusion-proofness constraint (11) is indeed also relaxed. The benefits of reducing the size of activity increases when \(K(\cdot)\) gets larger. More efficient side-contracting calls for more output distortions, less powerful incentives and therefore less informational rents for a high demand manager. The supervisor is given less discretion as \(K(\cdot)\) increases.

The optimal contract looks more like a bureaucratic rule leaving few discretion to the supervisor.

As a result of these two forces justifying an output reduction in state \(\sigma_i(t+r) = \emptyset, \beta_i(t+r) = \bar{\beta}\), everything happens as if the true demand faced by the monopolist had been replaced by a lower *virtual demand* \(\bar{\beta}(\tau)\).\textsuperscript{10} This virtual demand decreases when \(K(\cdot)\) gets larger.

\textsuperscript{10}Myerson (1979) coined this term.
3.2 Bureaucratization

Following sociologists like Max Weber (1922) or economists like Alfred Marshall, students of bureaucracy have soon recognized that large organizations are subject to a rapid decay in their internal efficiency. As Downs (1965) has put it when he stated his celebrated Law of Increasing Conservatism "All organisations tend to become more conservative as they get older unless they experience periods of very rapid growth or internal turnover." Even if Downs' main concern is the behavior of public bureaucracy, his insight also goes through in our analysis of the private sector.

We first capture this phenomenon of bureaucratization in a completely ad hoc but very convenient model. Let us assume that \( K(\tau) \) is now an arbitrary increasing function of the age of the firm. The motivation here has been suggested by Olson (1982). As it gets older, a firm faces more collusion and gets more bureaucratized in order to respond to the increase in the number of opportunities for collusion between the supervisor and the manager. The underlying idea is that time permits the creation of reputations and the development of trust and reciprocity which facilitiates the emergence of informal links through which collusion can be more easily sustained within the firm. Transaction costs of side-contracting are thus decreasing over time.

3.3 Impact of Transaction Costs on Growth

We now bring back this ad hoc theory of bureaucratization into the framework of Schumpeterian growth.

We consider a stationary equilibrium path with a constant growth rate and we denote by \( n^* \) the size of skilled labor force used in each R&D sector along this balanced growth path.

Because of stationarity, any given monopolistic firm \( i \) has the same probability of survival in any given period, namely \( 1 - \phi(n^*) \). The intertemporal profit from being a monopolist from date \( t + 1 \) on writes thus as:

\[
\Pi_{i(t+1)} = \sum_{\tau=0}^{\infty} \left( 1 - \phi(n^*) \right)^{\tau} \frac{1 - \alpha}{\alpha} \left( \frac{w_{t+1+\tau}}{\omega_{t+1+\tau}^{\max} \hat{A}_{t+1+\tau}} \right)^{\frac{1}{\alpha}} \omega_{t+1+\tau+1} \left( \nu + (1 - \nu) \left( \frac{\beta(\tau)}{\beta} \right)^{\frac{1}{\alpha}} \right).
\]

We introduce the wage-productivity adjusted parameter

\[
\omega_{t+1+\tau+1} = \frac{w_{t+1+\tau+1}}{A_{t+1+\tau+1}^{\max}} = \frac{w_{t+1+\tau+1}}{\hat{A}_{i(t+1)}} \times \frac{1}{\prod_{l=0}^{\tau-1} (\phi(n_{t+i}) q + 1 - \phi(n_{t+i}))}.
\]

which, along a balanced growth path, is also a constant that we denote by \( \omega^* \). Note also that, given the symmetry of the model, the monopoly's intertemporal profit is the same in all sectors from any date \( t + 1 \) on and we can denote \( \Pi_{t+1} = \Pi_{i(t+1)} \quad \forall i \in [0, 1] \).

The amount of skilled labor used in each R and D sector is is such that the expected marginal benefit of innovation and being a monopoly from date \( t + 1 \) on equals the wage given to one more unit of skilled labor at date \( t \). Therefore, \( n^* \) satisfies the research arbitrage condition:

\[
\phi'(n^*) \frac{\Pi_{t+1}}{1 + r} = w_t.
\]
Simplifying the expression of the intertemporal profit and using (18) yields the research arbitrage equation (R):

\[ \phi'(n^*) \frac{(\phi(n^*)q + 1 - \phi(n^*))}{1 + r} \hat{\Pi}(\omega^*, n^*, K(.)) = \omega^* \]  

where \( \hat{\Pi}(\omega^*, n^*, K(.)) \) is the intertemporal productivity adjusted-profit of a monopolist in any sector and whose exact expression is given in the appendix.

To close our general equilibrium model, we write the skilled labor market clearing equation. First, note that the average output produced by the intermediate sector in a stationary equilibrium is:

\[ \sum_{\tau=0}^{\infty} \phi(n^*)(1 - \phi(n^*))^\tau \bar{X}_\tau(\omega^*, n^*, K(.)) \]

\( \phi(n^*)(1 - \phi(n^*))^\tau \) is the number of sectors \( \tau \) steps behind the leading-edge technology. It is also the probability that a firm has already lived for \( \tau \) periods. \( \bar{X}_\tau(\omega^*, n^*, K(.)) \) is the average output of such a firm adjusted by productivity (whose exact expression is also given in the appendix).

Note that skilled labor is either used in the R & D sectors or to produce intermediate goods. The skilled labor market clearing condition in a stationary equilibrium writes thus as:

\[ L = n^* + \phi(n^*) \bar{X}(\omega^*, n^*, K(.)) \]  

where \( \bar{X}(\omega^*, n^*, K(.)) = \sum_{\tau=0}^{\infty} (1 - \phi(n^*))^\tau \bar{X}_\tau(\omega^*, n^*, K(.)) \) is average output conditional on the realization of an innovation. It is convenient to rewrite the skilled labor market clearing condition as condition (L) in the following form:

\[ L - n^* = \phi(n^*) \bar{X}(\omega^*, n^*, K(.)) \]

3.4 Average Stationary Growth Rate

The average growth rate of the economy along a balanced growth path is:

\[ g = \frac{Y_{t+1} - Y_t}{Y_t} \]

where

\[ Y_t = w_t \left( \sum_{\tau=0}^{\infty} \phi(n^*)(1 - \phi(n^*))^\tau \bar{X}_\tau \right) \]

is average GDP of the stationary economy at date \( t \).

On a balanced growth path, wages and GDP evolve according to the same dynamics than the leading-edge technology, we have \( \frac{w_{t+1}}{w_t} = (\phi(n^*)q + 1 - \phi(n^*)) \) and thus:

\[ g^* = \phi(n^*)(q - 1). \]

Hence, a higher (resp. lower) probability of innovation increases (resp. decreases) growth in the economy. Abusing language, we will thus identify \( n^* \) with the growth rate of the economy.
3.5 The Special Case of Constant Transaction Costs

As a benchmark, it is interesting to consider the special case where transaction costs are not time-dependent. Agency costs due to collusive behavior are then also constant over time. We denote by $K(\tau) = K$ the corresponding efficiency of side-contracting and we observe that the virtual demand $\beta(\tau) = \hat{\beta}$ is thus also constant over time. (R) becomes $(R)^0$ such that:

$$\phi'(n^0) \left( \phi(n^0)q + 1 - \phi(n^0) \right) \Pi(\omega^0, n^0, K) = \omega^0$$

(22)

Similarly, (L) writes as $(L)^0$ such that:

$$L - n^0 = \phi(n^0)\hat{X}(\omega^0, n^0, K)$$

where $n^0$ and $\omega^0$ are respectively the stationary growth rate and the productivity adjusted wage when agency costs are constant over time. Using $(R)^0$ and $(L)^0$, one can eliminate $\omega^0$ and then determines the stationary equilibrium growth rate as the solution of an equation of the form $H(n^0) = 1$ where $H(.)$ is a function whose precise form is described in the appendix and is independent of $K$. We have then the following proposition which characterizes the shape of $H(.)$ and the equilibrium level $n^0$:

Proposition 2: Assume that $\phi''(n)\phi(n) + \phi'(n)^2 \leq 0$ for all $n$, then:

- $H(n)$ is a strictly decreasing function of $n$.
- There always exists a unique stationary equilibrium growth rate $n^0$ which does not depend on $K$ the efficiency of side-contracting.
- $n^0$ is equal to the complete information growth rate.

- $(L)$ and $(R)$ can easily be drawn in the $(n^*, \omega^*)$ space. Under a seemingly innocuous assumption on the technology of the R and D sectors, $(R)$ always defines a downward sloping curve. Instead, $(L)$ is upward sloping (see Figure 1). These two properties ensure the uniqueness of the equilibrium growth rate when transaction costs are constant over time.
- In the standard Schumpeterian model of Aghion and Howitt (1998), the equilibrium growth rate does not depend on the scale of activity. Demand only enters into the expression of the growth rate through the value of its price elasticity. Similarly here, the growth rate is unaffected by changes in the quality of the matches between the intermediate goods and the final sectors since this quality only affects demand multiplicatively and thus does not affect demand elasticity.

Remember that agency costs within intermediate sectors replace indeed qualities by virtual qualities. Hence, agency costs play the same role as a tax on intermediate goods. Agency costs affect only the scale of activity but leave unchanged all other parameters. Therefore, we get the striking conclusion that exogenous and constant over time transaction costs do not affect equilibrium growth. In particular, the growth rate is the same as in the absence of agency costs!

- In the sequel, we focus on time-dependent transaction costs of side-contracting which not only affect the scale of activity of the intermediate sectors, but also the intertemporal
distribution of profits and the average intersectoral output of these sectors. In this case, the growth rate will depend on the dynamic path of the agency costs incurred to prevent those collusions.

- With the theory of *endogenous transaction costs* that we propose in Section 5, we will go even further. The equilibrium growth rate will depend then on organizational parameters characterizing the internal structure of a given firm. This provides an interesting link between the organization of firms and their life prospects and the rate of creative destruction in the economy.

4 The Dynamics of Transaction Costs and Schumpeterian Growth

The key step of our analysis comes from recognizing that there is a two way causality between the internal structure of firms and the growth rate of the economy.

First, the impact of collusive side-contracting depends on the prospects of colluding agents of remaining entrenched within the firm for a long time horizon. Indeed when the external pressure of creative destruction in the economy is weak, then the expected life of a monopoly is longer and vertical collusions have more opportunities to form. This way, we illustrate informal and casual observations by management scholars that the outside competitive environment has implications for the structure of power and cliques inside organizations (Pfeffer and Leblebici (1973))\textsuperscript{17}. In economic terms the macroeconomic environment affects the internal efficiency of the firm.

However, the causality goes also the other way around. The rate of technological innovation depends on the profitability of firms which itself is a function of the amount of bureaucratization emerging as a response of the threat of internal side-contracting. The internal organization of the firm has also an impact on the aggregate rate of creative destruction and growth in the economy.

4.1 Two Different Effects of Bureaucratization

- To understand how the bureaucratization of intermediate sectors affects the growth rate of the economy, it is useful to consider the following thought experiment.

Starting from the constant transaction costs benchmark described in 3.5, let us consider an upward small shift in the efficiency of internal side-contracting, so that the whole function $K(.)$ becomes now slightly increasing over time. Because such a small shift will not affect uniqueness of the equilibrium growth rate, Figure 1 can still be used to assess the impact of bureaucratization.

With this small shift, the monopoly's profitability goes down since $\dot{\Pi}_K(.) < 0$ and the locus $(R)$ is shifted downward until $(R)'$. This profitability effect is the first consequence of

\textsuperscript{17}In a case study of the effect of competition and external environment on the internal structure of thirty seven firms in Illinois, Pfeffer and Leblebici (1973), conclude that competition appeared to have an effect on the specification of decision procedures, with the greater the competition, the greater the specification of such procedures”. See also Pfeffer (1981) for a discussion of the role of external pressure on the perpetuation of power and collusion inside organizations.
the bureaucratization of a monopoly. Slightly abusing language, it is a partial equilibrium effect.

However, when the efficiency of side-contracting increases, intermediate sectors get more bureaucratized and the overall average output of these sectors contracts. The labor-market clearing equation \((L)\) is also shifted downward to \((L')\). This is the second general equilibrium effect of an increase in the amount of bureaucratization. Bureaucratization in the intermediate sectors rejects more skilled labor into the research sectors. The latter become therefore more innovative and this increases the rate of creative destruction.

- As a result of these two forces driven by bureaucratization in the intermediate sectors, the stationary equilibrium growth rate either diminishes when the profitability effect dominates or instead increases when the labor market effect dominates. Bureaucratization has thus a priori an ambiguous impact on the equilibrium growth rate.

- Using equations \((R)\) and \((L)\) to eliminate \(\omega^*\), a stationary equilibrium growth rate can be shown now to be the solution of an equation of the form:

\[
H(n^*) = G(n^*),
\]

where

\[
G(n) = \frac{\psi \left( (1 - \phi(n)) (\phi(n) q + 1 - \phi(n))^{\frac{1}{1+\alpha}} \right)}{\psi \left( (1 - \phi(n)) (\phi(n) q + 1 - \phi(n))^{\frac{\alpha}{1+\alpha}} \right)}
\]

with

\[
\psi(x) = (1 - x) \left( \sum_{\tau=0}^{\infty} x^\tau \left( \nu + (1 - \nu) \left( \frac{\beta(\tau)}{\beta} \right)^{\frac{1}{1+\alpha}} \right) \right).
\]

Importantly, \(\psi(x)\) is a decreasing function of \(x\) when transaction costs are decreasing over time.\(^{18}\) This property turns out to be quite important to derive the behavior of \(G(n)\). \(G(n)\) measures indeed the correcting distortion with must be introduced to account for the downwards dynamics of transaction costs. Indeed, as we have already seen, in the special case of constant transaction costs, \(G(n) = 1\) for all \(n\) and the growth rate is the complete information growth rate of the economy.

The numerator (resp. the denominator) of \(G(n)\) measures the impact of the bureaucratization on average output (resp. average profit). As there is more innovation in the economy, i.e., as \(\phi(n)\) increases, this numerator increases capturing the fact that average output (resp. average profit) moves up since bureaucratization in old monopolies has not yet settled down and these firms are still producing relatively efficiently. Increasing the probability of an innovation has thus, a priori, an ambiguous effect on the monotonicity of \(G(n)\).

For further reference, we define also \(\hat{n}\) as the unique solution to:

\[(q - 1)\phi(\hat{n}) = r\]

when it exists, i.e. when \((q - 1)\phi(L) > r\). Otherwise, i.e., when \((q - 1)\phi(L) < r\) we write \(\hat{n} = L\).

- \(H(n)\) and \(G(n)\) have be drawn on Figure 2. We have already seen that, under a seemingly innocuous assumption on the technology of the R & D sectors, \(H(n)\) always defines

\(^{18}\)This assertion is proved in the Appendix.
a downward sloping curve. Instead, $G(n)$ may be either upward or downward sloping when transaction costs are decreasing. The next Lemma characterizes more precisely the behavior of $G(n)$.

**Lemma 1**: $G(n)$ has the following behavior:

- $G(n)$ is increasing on an interval $[0, \bar{n}_1]$ where $L > \bar{n}_1 > \hat{n}$.
- $G(n)$ is decreasing on an interval $[\bar{n}_2, L]$ where $L > \bar{n}_2 > \hat{n}$.
- $G(n)$ is strictly lower than 1 on the interval $[0, \hat{n}]$ with $G(0) < 1$ and $G(\hat{n}) = 1$.
- $G(n)$ is strictly greater than 1 on the interval $[\hat{n}, L]$ with $G(L) = 1$.

From Lemma 1, $G(n)$ is a priori a hump-shaped function of $n$. These properties of $G(n)$ help in fact to characterize the stationary equilibrium growth rates of this economy.

### 4.2 Stationary Equilibrium Growth Rates

**Proposition 3**: Assume that $\phi''(n)\phi(n) + \phi'(n)^2 \leq 0$ for all $n$ and that transaction costs of side-contracting are decreasing over time, then:

- If $n^0 < \hat{n}$, there exists a unique stationary equilibrium growth rate $n^*$ such that $n^0 < n^* < \hat{n}$.
- If $n^0 > \hat{n}$, there exists at least one stationary equilibria growth rate $n^*$. There may nevertheless exist multiple equilibria. In any case, all these equilibria are such that $n^0 > n^* > \hat{n}$.
- If $n^0 = \hat{n}$, there exists a unique stationary equilibrium growth rate $n^*$ such that $n^0 = n^* = \hat{n}$.

**Growth Rate Distortions**:

In the case of a relatively drastic innovation or in the case of a small interest rate ($\hat{n}$ is small when $q - 1$ is large and when $r$ is small), the profitability effect of bureaucratization always dominates the labor market effect. Profitability diminishes with the development of internal collusions and this reduces significantly the R & D sectors incentives to innovate. The incentives of the R & D sectors to innovate are indeed provided by the whole sequence of per-period profits made over the whole life of a monopoly since there is little discounting. In particular, these incentives are mainly driven by the level of profits achieved when the organization is fully bureaucratized.

In this case, the contracting path of activities of the intermediate sectors affects relatively more their intertemporal profit than their average output. The equilibrium growth rate is always lower than under complete information.

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19To simplify exposition, we will represent graphically on Figure 2 the case where it has only one hump. However, our analytical results hold more generally.
When innovations are not so drastic or when the interest rate $r$ is relatively large ($\hat{n}$ is large when $q - 1$ is small and when $r$ is large), the reverse phenomenon happens. The profitability of intermediate sectors is less affected by a declining path of activity than their average output. The incentives of the R & D sectors to innovate are only provided by the profits made in the very first periods of the monopoly’s life when it is still not much affected by the bureaucratization. The general equilibrium effect dominates. A larger share of the skilled labor pool becomes available to work in the R & D sectors and the rate of innovation increases, resulting in more creative destruction. The equilibrium growth rate is always higher than under complete information.

The possibility that the equilibrium growth rate may be either above or below its value under complete information suggests that economies may follow very different behaviors depending on the size of innovation and the interest rate. Some countries may be stuck with very low growth rates and heavily bureaucratized. Other countries may exhibit less bureaucratic structures and at the same time face a higher rate of creative destruction.

- **Multiple Equilibria:**
  In the case of a quite drastic innovation, less innovation in the R & D sectors makes monopolies in the intermediate sectors enjoy a relatively easy life. As their expected life time gets longer, collusive relationships find more opportunities to form and have enough time to develop and to affect the allocation of resources in the economy. This in turn decreases the profitability of the intermediate sectors and thus reduces the R & D sectors' incentives to innovate. There is thus less innovations in each research sector. Because of the macroeconomic externality embodied in social knowledge, productivity growth is also less important. This effect decreases even further the profitability of the intermediate sectors. Finally, these depressed incentives of the R & D sectors ensure an even easier life to monopolies. Thanks to this strategic complementarity between the micro and the macro sides of the model, multiple stationary equilibria may therefore arise.

## 5 Endogenous Transaction Costs

To go further, and in particular, to assess the relationship between the macroeconomic growth rate and the internal organizational structure of the firm, we need to *endogenize* transaction costs of side-contracting.

### 5.1 The Life Cycle of Monopolies

The key to generate endogenous transaction costs is to let time plays a crucial role in the formation of coalitions within monopolies of the intermediate sectors. Following Martinort (1997) and (1998), we assume from now on that collusions are not enforceable through binding side-agreements. Instead, any collusive agreement must be *self-enforcing*. Both the supervisor and the manager must find profitable to continue to collude if the benefits from doing so are larger than the current gains obtained by any of them if he breaks the cooperative behavior.

- **Self-Enforcing Collusion:**
  Let us consider a monopoly in sector $i$ which starts its life at date $t$. The supervisor and the manager decide to collude from any date $t + \tau$ on if an informative signal $\sigma_{i(t+\tau)} = \bar{3}$
has been observed by the supervisor. Such a collusion is an implicit contract stipulating bribes $b_{i(t+\tau')} = \bar{\sigma}(t+\tau')$ at all future dates $\tau' \geq \tau + 1$. These bribes are paid by the manager to the supervisor when the latter gets informative signals at these dates.

This implicit contract is sustained with trigger strategies. There is reversion to a non-cooperative behavior from any date $t + \tau''$ on ($\tau'' \geq \tau'$) if either the supervisor reports $\bar{\sigma}(t+\tau'') = \bar{\sigma}$ to the owners or if the manager fails to give any bribe at date $t + \tau'' - 1$. In such a non-cooperative behavior, the supervisor reports always any informative signal he may have observed and the manager never bribes.

**Dynamic Collusion-Proofness Constraints:**

We now derive sufficient conditions such that colluding from date $t + \tau$ is an equilibrium strategy for both the supervisor and the manager. To simplify notations, we analyze an environment characterized by a stationary growth rate.

First, the supervisor must prefer hiding the informative signal $\sigma_{i(t+\tau)} = \bar{\sigma}$ rather than immediately reporting this information to the owners of the firm and then behaving always non-cooperatively in the future. The supervisor’s incentive constraint in the repeated game starting at date $t + \tau$ writes as:

$$S^*_{i(t+\tau)} + \nu \epsilon \sum_{i=1}^{\infty} \left( \frac{1 - \phi(n^*)}{1 + r} \right)^i S^*_{i(t+\tau+i)} \leq b_{i(t+\tau)} + \nu \epsilon \sum_{i=1}^{\infty} \left( \frac{1 - \phi(n^*)}{1 + r} \right)^i b_{i(t+\tau+i)}. \quad (25)$$

Similarly, the manager must prefer giving up a bribe $b_{i(t+\tau)}$ when the supervisor has hidden information to owners rather than gaining today by not bribing a lenient supervisor and then being always denounced in the future. The manager’s incentive constraint in the repeated game starting at date $t + \tau$ writes as:

$$\bar{u}_{i(t+\tau)} \leq \bar{u}_{i(t+\tau)} - b_{i(t+\tau)} + \nu \epsilon \sum_{i=1}^{\infty} \left( \frac{1 - \phi(n^*)}{1 + r} \right)^i \left( \bar{u}_{i(t+\tau)} - b_{i(t+\tau+i)} \right). \quad (26)$$

A contract is collusion-proof when it exhausts any scope for a self-enforceable collusion between the supervisor and the sales managers from date $t + \tau$ on. There must exist no possible stream of bribes such that (25) and (26) simultaneously hold. This leads to the condition:

$$S^*_{i(t+\tau)} + \nu \epsilon \sum_{i=1}^{\infty} \left( \frac{1 - \phi(n^*)}{1 + r} \right)^i S^*_{i(t+\tau+i)} \geq \nu \epsilon \sum_{i=1}^{\infty} \left( \frac{1 - \phi(n^*)}{1 + r} \right)^i \bar{u}_{i(t+\tau+i)} \quad (27)$$

(27) is the dynamic collusion-proofness constraint which must be satisfied to prevent agents from starting to collude from date $t + \tau$ on. When owners have no ability to commit to future wages, Martimort (1998) shows that this constraint must be binding to prevent this type of collusive behavior since owners have then no ability to break collusion by inducing future deviations from these collusive behaviors at some date $t + \tau$.

The important thing to note is that, over time, there are more and more dynamic collusion-proofness constraints which must be satisfied by the optimal collusion-proof contract. The reason is that a new self-enforceable collusion may start each period when $\sigma_{i(t+\tau)} = \bar{\sigma}$.

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In our simple game, these trigger strategies turn out to be also optimal in the sense of Abreu (1986).
Moreover, the informational rent at date \( t + \tau \), \( \bar{u}(t + \tau) \), enters exactly into \( \tau - 1 \) different dynamic collusion-proofness constraints, namely the constraints preventing the collusive agreements that could have started at all previous dates \( t + \tau' \) with \( \tau' < \tau \). Therefore, there are more and more reasons to reduce these informational rents when \( \tau \) gets larger.

Over the monopoly's life, the manager's informational rent is thus decreasing and the supervisor's discretionary power diminishes. Rules become more bureaucratic as the monopoly gets older.

**Proposition 4**: When collusion within the monopolistic firm is self-enforcing, average output and average profit at date \( t + \tau \) are respectively given by (16) and (17) provided that transaction costs are now defined as \( 1 - K(\tau, \nu \epsilon) \) where

\[
K(\tau, \nu \epsilon) = 1 - (1 - \nu \epsilon)^\tau.
\]

Moreover, per-period profit \( \Pi \) and average output \( X \) decrease with the age \( \tau \) of the monopoly.

Transaction costs diminish over time to capture the fact that there exists an increasing number of opportunities for a vertical collusion to form. In the limit of \( \tau = \infty \), \( K(\cdot) \) converges to 1, capturing the fact that supervision is useless in old fully bureaucratized monopolies.

These transaction costs also depend on the quality of the monitoring technology. The efficiency of side-contracting increases with \( \epsilon \). Indeed, better supervisory information increases the continuation value of keeping on colluding since the state of nature in which collusion occurs is more likely.

### 5.2 Stationary Equilibrium Growth Rates

To derive further consequences of our model, we now assume that the spread of uncertainty \( \Delta \beta \) is small enough\(^{21}\). Using simple Taylor expansions, we obtain the following approximation of the equilibrium growth rate of the economy:

**Proposition 5**: Assume that conditions of Proposition 1 are satisfied so that \( H(n) \) is strictly decreasing and assume that \( \Delta \beta \) is small enough. Then there exists a unique stationary equilibrium growth rate \( n^* \) such that (up to terms of order \( \Delta \beta^2 \)):

\[
n^* - n^0 = -\Delta \beta W(n^0, \nu \epsilon, q, r)
\]

where \( W(.) \) is a function such that \( W(n^0, \nu \epsilon, q, r) > 0 \) (resp. \( < 0 \)) whenever \( n^0 > \hat{n} \) (resp. \( n^0 < \hat{n} \)).

Note immediately that \( n^* < n^0 \) (resp. \( n^* > n^0 \)) when \( n^0 > \hat{n} \) (resp. \( n^0 < \hat{n} \)) as already shown in Proposition 3.

\(^{21}\)This simplification is made for tractability but we feel confident on the robustness of the insights that we derive thereafter when this spread gets in fact larger.
6 Comparative Statics

6.1 Spread of Uncertainty

An increase in $\Delta \beta$ enlarges the difference between $n^*$ and $n^0$. Agency costs due to bureaucratization become more important when uncertainty is larger. Large adverse selection problems which are very likely to call for the implementation of monitoring devices have thus a significant impact in contracting or expanding the equilibrium growth rate depending on the size of the innovation.

6.2 Monitoring Structures

As it has been forcefully argued by Chandler (1962), most of organizational reforms within the firm concern changes in the monitoring technologies. A more efficient monitoring technology improves the probability that the supervisor gets an informative signal on the manager. We now investigate the consequences of such an improvement on the equilibrium growth rate.

**Corollary 1** : A more informative monitoring technology increases the distortion in the equilibrium growth rate: $|n^* - n^0|$ is increasing with $\epsilon$.

The key to understand this corollary is again to come back to the origins of transaction costs of side-contracting. When the supervisor gets more often informative signals on the manager, their continuation values of keeping on colluding increase. Transaction costs of side-contracting diminish and the organizational response, i.e., the firm's bureaucratization, must be more pronounced. As a result, when the partial equilibrium (resp. general equilibrium) effect dominates, the growth rate is further decreased (resp. increased) compared to the complete information growth rate. Summarizing the equilibrium growth rate is more distorted away from its complete information value.

This shows also that changes in the internal structure of the firm may have strong macroeconomic consequences.

6.3 Norms of Collusive Behavior

So far, we have assumed that collusion occurs whenever possible within the firm. Each time a supervisor observes an informative signal on the manager, he may refuse to reveal this information if he is bribed. It is immediate to extend our framework to the case where, conditionally on having observed this signal, the supervisor and the manager only collude with probability $x$. Everything happens as if $\epsilon$ is replaced by $\epsilon' = \epsilon x$ in (29).

$x$ can then be viewed as an index of the norm for collusive behavior which establishes in society. Large values of $x$ correspond to efficient collusive technologies which are thus more costly to prevent.

**Corollary 2** : A more collusive norm increases the distortion in the equilibrium growth rate; $|n^* - n^0|$ is increasing with $x$. 
When the norm of collusive behavior is more efficient, the continuation values of keeping on colluding increase. Transaction costs of side-contracting diminish and the organizational response, i.e., the firm's bureaucratization, must be more pronounced. As a result, when the partial equilibrium (resp. general equilibrium) effect dominates, the growth rate is further decreased (resp. increased) compared to the complete information growth rate.

7 Conclusion

In the above model, the macroeconomic environment and the microeconomic conditions strongly interact to determine the equilibrium growth rate. A lower (resp. higher) rate of creative destruction increases (resp. decreases) the expected life of agents within organizations. It therefore increases (resp. decreases) their ability to collude efficiently and to undermine the firm's profitability. This, in turn, reduces (resp. increases) the R and D sectors incentives to innovate. The growth rate of the economy and the transaction costs of side-contracting are therefore characterized altogether.

This result was elaborated from the analysis of a simple stationary equilibrium. However, as it has been shown by Olson (1982), the sclerosis phenomena faced by societies subject to the formation of collusions may be better analyzed as non-stationary phenomena. Even if moving to a non-stationary environment certainly involves lots of technical difficulties, this extension is also necessary to understand how organizational forms affect convergence towards a given stationary equilibrium growth rate.

The methodology developed in this paper provides a mapping between the endogenous transaction costs of side-contracting within the firm and the equilibrium growth rate of the economy. Within this framework, private incentives to create organizational innovations to prevent those collusions and their consequences on the equilibrium growth rate could certainly be analyzed. Therefore, our framework may also be an important building block towards the theory of endogenous institutional changes that North (1990) has called for. We leave these extensions of our theory for further research.
REFERENCES


Appendix

Proof of Proposition 1:
It is standard to show that all constraints (8) to (12) are in fact binding at the optimum. Inverting the corresponding value \( u_{i(t+\tau)} = 0, \bar{u}_{i(t+\tau)} = \alpha A_u \Delta \beta z_{i(t+\tau)}, u_{i(t+\tau)} = 0, S^o_{i(t+\tau)} = 0, S^\omega_{i(t+\tau)} = K(\tau) \alpha A_u \Delta \beta z_{i(t+\tau)} \) into the owners' objective function and optimizing with respect to \( \bar{x}_{i(t+\tau)}, \bar{z}_{i(t+\tau)} \) and \( \bar{x}_{i(t+\tau)}^\omega \), we obtain (13) and (14).

Explicit expressions of \( \Pi(\omega^*, n^*, K(.)) \). \( \bar{X}(\omega^*, n^*, K(.)) \) and \( \bar{X}(\omega^*, n^*, K(.)) \).

The intertemporal profit from being a monopolist from date \( t+1 \) on writes as:

\[
\Pi_{t(t+1)} = \sum_{\tau=0}^{\infty} \left( 1 - \phi(n^*) \right)^\tau \frac{1 - \alpha}{\alpha} \left( \frac{w_{i(t+\tau)} + 1}{\alpha^2 \beta A_i(t+1)} \right)^{\frac{1}{\alpha - 1}} \nu + (1 - \nu) \left( \frac{\beta(\tau)}{\beta} \right)^{\frac{1}{\alpha - 1}}.
\]

We introduce the wage-productivity adjusted parameter

\[
\phi_{i(t+\tau+1)} = \frac{w_{i(t+\tau+1)}^{\alpha^2 \beta}}{A^\alpha_{i(t+1)}^{\alpha - 1}} \frac{1}{\Pi_{t(t+1)}^{\alpha - 1} (\phi(n_{t+i})q + 1 - \phi(n_{t-i}))}.
\]

which, along a balanced growth path, is constant and denoted it by \( \omega^\alpha \). Substituting one obtains

\[
\Pi_{t(t+1)} = \frac{1 - \alpha}{\alpha} A^\alpha_{i(t+1)} \sum_{\tau=0}^{\infty} \left( 1 - \phi(n^*) \right)^\tau \phi(n^*)q + 1 - \phi(n^*)^{\frac{1}{\alpha - 1}} \times \frac{\omega^\alpha}{(\alpha^2 \beta)^{\frac{1}{\alpha - 1}}} \nu + (1 - \nu) \left( \frac{\beta(\tau)}{\beta} \right)^{\frac{1}{\alpha - 1}}.
\]

Given the symmetry of the model, the monopoly's intertemporal profit is the same in all sectors from any date \( t+1 \) on and we can denote \( \Pi_{t(t+1)} = \Pi_{t+1} \forall t \in [0, 1] \). Hence, we obtain the intertemporal productivity adjusted-profit of a monopolist in any sector as:

\[
\Pi(\omega^*, n^*, K(.)) = \Pi_{t+1}/A^\alpha_{i(t+1)}^{\alpha - 1} \text{ given by:}
\]

\[
\Pi(\omega^*, n^*, K(.)) = \sum_{\tau=0}^{\infty} \left( 1 - \phi(n^*) \right)^\tau \phi(n^*)q + 1 - \phi(n^*)^{\frac{1}{\alpha - 1}} \times \frac{\omega^\alpha}{(\alpha^2 \beta)^{\frac{1}{\alpha - 1}}} \nu + (1 - \nu) \left( \frac{\beta(\tau)}{\beta} \right)^{\frac{1}{\alpha - 1}} \frac{1 - \alpha}{\alpha}.
\]

We write then the skilled labor market clearing equation. First the average output produced by the intermediate sector in a stationary equilibrium is:

\[
\sum_{\tau=0}^{\infty} \phi(n^*) (1 - \phi(n^*))^\tau \bar{X}_\tau.
\]

\( \phi(n^*) (1 - \phi(n^*))^\tau \) is the number of sectors \( \tau \) steps behind the leading-edge technology. It is also the probability that a firm has already lived for \( \tau \) periods. \( \bar{X}_\tau(\omega^*, n^*, K(.)) \) is the average output of such a firm adjusted by productivity given by:

\[
\bar{X}_\tau(\omega^*, n^*, K(.)) = \left( \frac{\omega^*}{\alpha^2 \beta} \right)^{\frac{1}{\alpha - 1}} (\phi(n^*)q + 1 - \phi(n^*))^{\frac{1}{\alpha - 1}} \nu + (1 - \nu) \left( \frac{\beta(\tau)}{\beta} \right)^{\frac{1}{\alpha - 1}}.
\]

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Note that skilled labor is either used in the R & D sectors or to produce intermediate goods. The skilled labor market clearing condition in a stationary equilibrium writes thus as:

\[ L = n^* + \phi(n^*)\hat{X}(\omega^*, n^*, K(.)) \tag{32} \]

where \( \hat{X}(\omega^*, n^*, K(.)) = \sum_{\tau=0}^{\infty}(1 - \phi(n^*))^\tau \hat{X}(\omega^*, n^*, K(.)) \) is average output conditional on the realization of an innovation and given by:

\[
\hat{X}(\omega^*, n^*, K(.)) = \left( \sum_{\tau=0}^{\infty} \left( (1 - \phi(n^*))((\phi(n^*)q + 1 - \phi(n^*))^{\frac{1}{\alpha-1}}) \right)^\tau \left( \nu + (1 - \nu) \left( \frac{3(\tau)}{3} \right)^{\frac{1}{\alpha}} \right) \right)
\]

is average output conditional on the realization of an innovation. Substitution provides an explicit labor market clearing condition as:

\[ L = n^* + \left( \frac{\omega^*}{\alpha^2 \beta} \right)^{\frac{1}{\alpha-1}} \left( \sum_{\tau=0}^{\infty} \phi(n^*) \left( (1 - \phi(n^*))((\phi(n^*)q + 1 - \phi(n^*))^{\frac{1}{\alpha-1}}) \right)^\tau \left( \nu + (1 - \nu) \left( \frac{3(\tau)}{3} \right)^{\frac{1}{\alpha}} \right) \right)
\]

Explicit expression of the function \( H(.) \):

Using the explicit expression of \( \Pi(\omega^0, n^0, K) \), with constant transaction costs \( K(.) = K \). \((R)\) becomes \((R)^0\) as:

\[
\phi'(n^0) \frac{(\phi(n^0)q + 1 - \phi(n^0))}{1 + r} \left( \nu + (1 - \nu) \left( \frac{3}{\beta} \right)^{\frac{1}{\alpha}} \right)^{\frac{1 - \alpha}{\alpha}} \left( 1 - \left( 1 - \phi(n^0) \right) \frac{\phi(n^0)q + 1 - \phi(n^0))^{\frac{\alpha}{\alpha-1}}}{1 + r} \right) = \left( \frac{\omega^0}{\alpha^2 \beta} \right)^{\frac{1}{\alpha-1}}.
\]

Similarly, using the explicit expression of \( \hat{X}(\omega^0, n^0, K) \), \((L)\) writes as \((L)^0\) such that:

\[ L - n^0 = \frac{\phi(n^0)}{1 - (1 - \phi(n^0))(\phi(n^0)q + 1 - \phi(n^0))^{\frac{1}{\alpha-1}}} \left( \nu + (1 - \nu) \left( \frac{3}{\beta} \right)^{\frac{1}{\alpha}} \right)^{\frac{1 - \alpha}{\alpha}} \left( \frac{\omega^0}{\alpha^2 \beta} \right)^{\frac{1}{\alpha-1}}
\]

where \( n^0 \) and \( \omega^0 \) are respectively the stationary growth rate and the productivity adjusted wage when agency costs are constant over time.

Using \((R)^0\) and \((L)^0\) to eliminate \( \omega^0 \), the stationary equilibrium growth rate solves the following equation:

\[
H(n^0) = \frac{\phi'(n^0) \frac{1 - \alpha}{\alpha} (L - n^0) \left( (\phi(n^0)q + 1 - \phi(n^0)) - (1 - \phi(n^0))(\phi(n^0)q + 1 - \phi(n^0))^{\frac{\alpha}{\alpha-1}} \right)}{1 + r - (1 - \phi(n^0))(\phi(n^0)q + 1 - \phi(n^0))^{\frac{\alpha}{\alpha-1}}} \tag{33}
\]

\[
= 1 \tag{34}
\]

Proof of Proposition 2:
• $(R)^0$ defines a downward sloping curve. Note that $(R)^0$ rewrites as:

$$
\frac{1}{\left(\nu + (1 - \nu) \left(\frac{3(\tau)}{3}\right)^{\frac{1}{1 - \alpha}}\right)} \left(\frac{\omega_0^{1 - \alpha}}{\alpha^2 \beta}\right) = \frac{\phi(n^0)q + 1 - \phi(n^0)}{1 + r} \frac{\phi'(n^0)}{\left(1 - \frac{1 - \phi(n^0)}{1 + r} \left(\phi(n^0)q + 1 - \phi(n^0)\right)^{\frac{1}{\alpha - 1}}\right)} \frac{1 - \alpha}{\alpha} \tag{35}
$$

We observe that

$$
d\frac{d}{dn} ((\phi(n)q + 1 - \phi(n))\phi'(n)) = \phi''(n) + (q - 1)(\phi''(n)\phi(n) + \phi'(n)^2) < 0
$$

under the assumption of Proposition 2. Similarly, we have

$$
d\frac{d}{dn} \left(1 - \frac{(1 - \phi(n))}{1 + r} \left(\phi(n)q + 1 - \phi(n)\right)^{\frac{1}{\alpha}}\right) =
\frac{\phi'(n)(\phi(n)q + 1 - \phi(n))^{\frac{1}{\alpha - 1}}}{1 + r} \left(\phi(n)q + 1 - \phi(n) + \frac{\alpha}{1 - \alpha} (q - 1)(1 - \phi(n))\right) > 0
$$

since $q > 1$ and $\alpha < 1$. Hence, the right-hand-side of (35) is decreasing with $n^0$ and $(R)^0$ defines a downward sloping curve.

• $(L)^0$ defines an upward sloping curve. Note that $(L)^0$ rewrites as:

$$
\left(\nu + (1 - \nu) \left(\frac{3(\tau)}{3}\right)^{\frac{1}{1 - \alpha}}\right) \left(\frac{\omega_0^{1 - \alpha}}{\alpha^2 \beta}\right) = \frac{(L - n^0) \left(1 - (1 - \phi(n^0)) \left(\phi(n^0)q + 1 - \phi(n^0)\right)^{\frac{1}{\alpha - 1}}\right)}{\phi(n^0)} \tag{36}
$$

We observe that

$$
d\frac{d}{dn} (L - n) < 0.
$$

Similarly,

$$
d\frac{d}{dn} \left(\frac{(1 - (1 - \phi(n)) \left(\phi(n)q + 1 - \phi(n)\right)^{\frac{1}{\alpha}}}{\phi(n)}\right) =
\frac{\phi'(n)}{\phi^2(n)} \left(-1 + \phi(n)q + 1 - \phi(n)\right)^{\frac{1}{\alpha - 1}} \left(\phi(n)q + 1 - \phi(n) + \frac{\phi(n)(1 - \phi(n))(q - 1)}{(1 - \alpha)}\right).
$$

This function is decreasing when

$$
(\phi(n)q + 1 - \phi(n))^{\frac{1}{1 - \alpha}} > 1 + \frac{\phi(n)(1 - \phi(n))(q - 1)}{(1 - \alpha)(\phi(n)q + 1 - \phi(n))}.
$$

The right-hand-side above can be bounded below by

$$
1 + \frac{(1 - \phi(n))(q - 1)}{(1 - \alpha)(\phi(n)q + 1 - \phi(n))}
$$
since $\phi(n) \leq 1$. Setting $z = \phi(n)(q - 1)$

$$(\phi(n)q + 1 - \phi(n))^\frac{1}{1-a} > 1 + \frac{(1 - \phi(n))(q - 1)}{(1 - \alpha)(\phi(n)q + 1 - \phi(n))}$$

holds when $f(z) = (1 + z)^\frac{1}{1-a} - 1 - \frac{z}{(1-a)(1+z)} > 0$ when $z > 0$. Note that $f(0) = 0$ and that $f'(z) = \frac{1}{1-a} \left( (1 + z)^\frac{1}{1-a+1} - 1 \right) > 0$ for $z > 0$. Hence, $f(z) > 0$ for $z > 0$.

We deduce from that that the right-hand-side of (36) is decreasing with $n^0$ and $(L)^0$ defines therefore an upward sloping curve.

Note that $\tilde{H}(L) = 0$ and $\lim_{n \to 0} \tilde{H}(n) = +\infty$. Hence, if $H(n)$ is strictly decreasing there always exists a unique solution to $H(n) = 1$. However, under the condition $\phi''(n)\phi(n) + \phi'(n)^2 \leq \psi \forall \nu$, shows that $H(n)$ is in fact decreasing.

**Proof of Lemma 1:**

- Define first:
  $$\psi(x) = (1 - x) \left( \sum_{\tau=0}^{\infty} x^\tau \tilde{H}(\tau) \right)$$

with

$$\tilde{H}(\tau) = \nu + (1 - \nu) \left( \frac{\beta(\tau)}{\beta} \right)^{\frac{1}{1-a}}.$$

We observe that:

$$\psi'(x) = \sum_{\tau=1}^{\infty} \tau x^{\tau-1} \tilde{H}(\tau) - \sum_{\tau=0}^{\infty} (\tau + 1)x^\tau \tilde{H}(\tau) = \sum_{\tau=1}^{\infty} \tau x^{\tau-1} \left( \tilde{H}(\tau) - \tilde{H}(\tau - 1) \right).$$

Since $\beta(\tau)$ is decreasing with $\tau$, $\tilde{H}(\tau) < \tilde{H}(\tau - 1)$ and $\psi'(x) < 0$.

- Define now $\tilde{G}(x, \phi) = \frac{\psi \left( \frac{x}{\phi q + 1 - \phi} \right)}{\psi \left( \frac{x}{\phi q + 1 - \phi} \right)}$.

Note first that $\tilde{G}((1 - \phi(n))(\phi(n)q + 1 - \phi(n))^{\frac{1}{1-a}}, \phi(n)) = G(n)$.

We have

$$\frac{\partial \tilde{G}(x, \phi)}{\partial x} = \frac{1}{(\phi q + 1 - \phi)} \left( \frac{(1-x) \psi' \left( \frac{x}{\phi q + 1 - \phi} \right) - (\phi q + 1 - \phi) \psi' \left( \frac{x}{\phi q + 1 - \phi} \right)}{1 + r} \right).$$

But

$$\frac{d}{dx} \left( \frac{\psi'(x)}{\psi(x)} \right) = \sum_{\tau=0}^{\infty} (\tau + 1)(\tau + 2)x^\tau \left( \tilde{H}(\tau + 2) - \tilde{H}(\tau + 1) \right) - \frac{\psi'(x)}{\psi(x)}.$$

Since $\tilde{H}(\tau + 1) < \tilde{H}(\tau)$ and $\psi'(x) < 0$, we have thus

$$\frac{d}{dx} \left( \frac{\psi'(x)}{\psi(x)} \right) < 0.$$
Hence, for $\phi(q - 1) > r$

$$0 > \frac{\psi'\left(\frac{x}{\phi q + 1 - \phi}\right)}{\psi\left(\frac{x}{\phi q + 1 - \phi}\right)} > \frac{\psi'\left(\frac{x}{1 + r}\right)}{\psi\left(\frac{x}{1 + r}\right)} > \frac{(\phi q + 1 - \phi) \psi'\left(\frac{x}{1 + r}\right)}{1 + r} \psi'\left(\frac{x}{1 + r}\right).$$

and thus $\frac{\partial \tilde{G}(x, \phi)}{\partial x} > 0$.

Similarly, for $\phi(q - 1) < r$

$$0 > \frac{(\phi q + 1 - \phi) \psi'\left(\frac{x}{1 + r}\right)}{1 + r} > \frac{\psi'\left(\frac{x}{1 + r}\right)}{\psi\left(\frac{x}{1 + r}\right)} > \frac{\psi'\left(\frac{x}{\phi q + 1 - \phi}\right)}{\psi\left(\frac{x}{\phi q + 1 - \phi}\right)}.$$

and $\frac{\partial \tilde{G}(x, \phi)}{\partial \phi} < 0$.

* We have also:

$$\frac{\partial \tilde{G}(x, \phi)}{\partial \phi} = -\frac{x(q - 1)}{(\phi q + 1 - \phi)^2} \psi'\left(\frac{x}{\phi q + 1 - \phi}\right) > 0$$

since $\psi'(\cdot) < 0$.

* We denote $x(n) = (1 - \phi(n))(\phi(n)q + 1 - \phi(n))^\frac{1}{-\alpha}$, and we observe that:

$$x'(n) = -\phi'(n)(\phi(n)q + 1 - \phi(n))^{-\alpha} - 1\left(\phi(n)q + 1 - \phi(n) + \frac{\alpha}{1 - \alpha}(1 - \phi(n))(q - 1)\right) < 0.$$

We can thus write:

$$G'(n) = \frac{\partial \tilde{G}(x(n), \phi(n))}{\partial x} x'(n) + \frac{\partial \tilde{G}(x(n), \phi(n))}{\partial \phi} \phi'(n).$$

Assume first that $n \leq \hat{n}$, so that $\phi(n)(q - 1) \leq r$. Then both $\frac{\partial \tilde{G}(x(n), \phi(n))}{\partial x} x'(n)$ and $\frac{\partial \tilde{G}(x(n), \phi(n))}{\partial \phi} \phi'(n)$ are positive and $G(\cdot)$ is increasing on $[0, \hat{n}]$.

Since $\frac{\partial \tilde{G}(x(n), \phi(n))}{\partial x} x'(n) = 0$ for $n = \hat{n}$ and $\frac{\partial \tilde{G}(x(n), \phi(n))}{\partial \phi} \phi'(n) > 0$ at this point, $G(\cdot)$ is also increasing on $[\hat{n}, \tilde{n}_1]$ for some $\tilde{n}_1 > \hat{n}$.

Assume now that $n > \hat{n}$, so that $\phi(n)(q - 1) > r$. Then $\frac{\partial \tilde{G}(x(n), \phi(n))}{\partial x} x'(n) \leq 0$ and $\frac{\partial \tilde{G}(x(n), \phi(n))}{\partial \phi} \phi'(n) \geq 0$.

Since $\phi'(L) = 0$ from Inada conditions, $\frac{\partial \tilde{G}(x(n), \phi(n))}{\partial \phi} \phi'(n) = \frac{\partial \tilde{G}(x(n), \phi(n))}{\partial x} x'(n) = 0$ for $n = L$. Hence, $G'(L) = 0$.

Moreover, when $\phi(L) = 1$, $x(L) = 0$ and thus $G(L) = 1$. Similarly, $\phi(0) = 0$, $x(0) = 1$, $\psi(1) = 0$ and thus $G(0) = 0$. Finally, $\frac{x(h)}{\phi(q + 1 - \phi(n))} = \frac{x(h)}{1 + r}$, and thus $G(\hat{n}) = 1$.

* Using that $\psi(\cdot)$ is decreasing,

$$\frac{\psi'\left(\frac{x}{\phi q + 1 - \phi}\right)}{\psi\left(\frac{x}{\phi q + 1 - \phi}\right)} > 1$$

(resp. $< 1$) when $\phi(q - 1) > r$ (resp. $\phi(q - 1) < r$) and $G(n) > 1$ (resp. $< 1$) for $n > \hat{n}$ (resp. $n < \hat{n}$).

* Therefore, $G(\cdot)$ is necessarily decreasing on $[\hat{n}_2, L]$ for some $\hat{n}_2 > \hat{n}$.

**Proof of Proposition 3:**
• Assume first that \( n^0 \leq \hat{n} \). \( H(\cdot) \) is decreasing and \( G(\cdot) \) is increasing on \([n^0, \hat{n}]\). \( H(n^0) = 1 \geq G(n^0) \) and \( H(\hat{n}) < 1 = G(\hat{n}) \), hence, there exists a unique solution \( n^* \) to \( H(n^*) = G(n^*) \) on \([n^0, \hat{n}]\). Moreover, for \( n > \hat{n} \), \( H(n) < 1 < G(n) \) and \( n^* \) is thus unique.

• Assume now that \( n^0 > \hat{n} \). \( H(\cdot) \) is decreasing and \( G(\cdot) \) is hump-shaped (with possibly several humps) on \([\hat{n}, n^0]\). \( H(n^0) = 1 < G(n^0) \) and \( H(\hat{n}) > 1 = G(\hat{n}) \), hence, there exists at least one solution \( n^* \) to \( H(n) = G(n) \) on \([\hat{n}, n^0]\).

**Proof of Proposition 4:** The proof is similar to that of Proposition 1 except that the dynamic collusion-proofness constraint (27) replaces (12) in the optimization of the owners’ problem. Let denote by \( \mu_{t+T} \), the multiplier of the collusion-proofness constraint preventing the collusion starting at date \( t + \tau \). Because of symmetry, let also omit index \( i \). Since we are interested in a stationary equilibrium \( n_{t+\tau} = n^* \) for all \( t \) and \( \tau \).

• The Lagrangean \( L \) of the owners’ problem writes as:

\[
L = \sum_{t=0}^{\infty} \left( \frac{1 - \phi(n^*)}{1 + r} \right)^t \left[ \nu(1 - \varepsilon)(\alpha A_t \beta \bar{x}_{t+\tau} - \omega_{t+\tau} \bar{x}_{t+\tau}) + \nu \varepsilon (\alpha A_t \beta \bar{x}_{t+\tau} - \omega_{t+\tau} \bar{x}_{t+\tau}) \right]

+(1 - \nu)(\alpha A_t \beta \bar{x}_{t+\tau} - \omega_{t+\tau} \bar{x}_{t+\tau}) - \nu(1 - \varepsilon) \bar{u}_{t+\tau} - \nu \varepsilon \bar{S}_{t+\tau}^*

+\mu_k \left( S_{t+\tau}^* + \nu \varepsilon \sum_{\ell=1}^{\infty} \left( \frac{1 - \phi(n^*)}{1 + r} \right)^\ell S_{t+\tau+\ell}^* - \nu \varepsilon \sum_{\ell=1}^{\infty} \left( \frac{1 - \phi(n^*)}{1 + r} \right)^\ell \bar{S}_{t+\tau+\ell} \right).
\]

where we have already noted that \( \bar{u}_{t+\tau}^* = S_{t+\tau}^* = \bar{y}_{t+\tau} = 0 \) in the expression above.

• Optimizing with respect to \( \bar{x}_{t+\tau} \) and \( \bar{x}_{t+\tau}^* \) yields:

\[
\bar{x}_{t+\tau} = \bar{x}_{t+\tau}^* = \left( \frac{\omega_{t+\tau}}{\alpha^2 \beta A_t} \right)^{\frac{1}{\alpha - 1}}
\]

• Optimizing with respect to \( \bar{x}_{t+\tau} \) yields:

\[
\bar{x}_{t+\tau} = \left( \frac{\omega_{t+\tau}}{\alpha^2 \beta(\tau) A_t} \right)^{\frac{1}{\alpha - 1}}
\]

where

\[
\beta(\tau) = \beta - \frac{\nu}{1 - \nu} \Delta \beta \left( 1 - \varepsilon + \varepsilon \sum_{t=0}^{\tau-1} \mu_t \left( \frac{1 + r}{1 - \phi(n^*)} \right) \right).
\]

• Optimizing with respect to \( S_{t+\tau}^* \) and taking into account that \( S_{t+\tau}^* \) is finite at the optimum

\[
\mu_{t+\tau} + \nu \varepsilon \sum_{t=0}^{\tau-1} \mu_{t+\tau} \left( \frac{1 - \phi(n^*)}{1 + r} \right)^{\tau - \ell} = \nu \varepsilon \left( \frac{1 - \phi(n^*)}{1 + r} \right)^\tau.
\]

We denote thereafter \( M_{\tau} = \sum_{t=0}^{\tau-1} \mu_{t+\tau} \left( \frac{1 - \phi(n^*)}{1 + r} \right)^{\tau - \ell} \). Taking into account that \( M_0 = 0 \) \( (\mu_0 = 0) \) and that \( M_{\tau} \) solves the difference equation

\[
M_{\tau+1} - (1 - \nu \varepsilon) M_{\tau} = 1 - \nu \varepsilon.
\]

we find

\[
M_{\tau} = \left( \frac{\nu \varepsilon}{1 + r} \right) (1 - (1 - \nu \varepsilon)^\tau).
\]

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Inserting this expression into (38) yields
\[ \beta(\tau) = \beta - \frac{\nu}{1 - \nu} \Delta \beta (1 - \varepsilon + \varepsilon (1 - (1 - \nu \varepsilon)^r)). \]

Note that \( M_r \) is strictly increasing so that all multipliers are positive except \( \mu_0 \).

**Proof of Proposition 5:**

When \( \Delta \beta \) is small enough, the following Taylor expansions hold:

\[ \psi(x) = (1 - x) \sum_{r=0}^{\infty} x^r \left( 1 - \frac{\Delta \beta}{\beta(1 - x)} (1 - \nu \varepsilon) - \frac{\nu^2 \varepsilon^2 \Delta \beta}{\beta(1 - \alpha)} \left( \frac{1}{z} - 1 + \nu \varepsilon \right) \right). \]

Hence
\[ G(n^*) = \frac{\psi \left( (1 - \phi(n^*)) (\phi(n^*) q + 1 - \phi(n^*))^{\frac{1}{1 - \alpha}} \right)}{\psi \left( (1 - \phi(n^*)) (\phi(n^*) q + 1 - \phi(n^*))^{\frac{1}{1 - \alpha}} \right)} = \frac{1}{1 + \nu \varepsilon} - \frac{\nu^2 \varepsilon^2 \Delta \beta}{\beta(1 - \alpha)} \left( \frac{1}{1 - \phi(n^*)} \right)^{\frac{1}{1 - \alpha}} - 1 + \nu \varepsilon. \]

Therefore, we get the following approximation up to terms of higher order:

\[ H(n^*) - H(n^0) = -\frac{\nu^2 \varepsilon^2 \Delta \beta}{\beta(1 - \alpha)} \left( \frac{1}{(1 - \phi(n^0) q + 1 - \phi(n^0))^{\frac{1}{1 - \alpha}}} - 1 + \nu \varepsilon \right) - \frac{1}{(1 - \phi(n^0))^{\frac{1}{1 - \alpha}}} - 1 + \nu \varepsilon. \]

From this, (??) obtains immediately when:

\[ W(n^0, \nu \varepsilon, q, r) = \frac{\nu^2 \varepsilon^2}{\beta(1 - \alpha) H'(n^0)} \left( \frac{1}{(1 - \phi(n^0) q + 1 - \phi(n^0))^{\frac{1}{1 - \alpha}}} - 1 + \nu \varepsilon \right) - \frac{1}{(1 - \phi(n^0))^{\frac{1}{1 - \alpha}}} - 1 + \nu \varepsilon. \]

**Proofs of Corollaries 2 and 3:** It is immediate from (??).