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**"WELFARE COSTS OF INFLATION THE
CASE FOR INTEREST-BEARING MONEY
AND EMPIRICAL ESTIMATES FOR BRAZIL"**

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**Welfare Costs of Inflation
The Case for Interest-Bearing Money and Empirical Estimates for
Brazil**

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Abstract

We provide in this paper a closed form for the Welfare Cost of Inflation which we prove to be closer than Bailey's expression to the correct solution of the corresponding non-separable differential equation. Next, we extend this approach to an economy with interest-bearing money, once again presenting a better approximation than the one given by Bailey's approach. Finally, empirical estimates for Brazil are presented.

Keywords: Inflation, Near-Money, Welfare Cost.
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Welfare Costs of Inflation: The Case for Interest-Bearing Money and Empirical Estimates for Brazil

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1) Introduction

Starting with Martin Bailey in 1956, the literature on welfare costs of inflation has recently been fostered by contributions by Lucas (1993 and 1994), Correia and Telles (1994), Cooley and Hansen (1989), Eckstein and Leiderman (1992) and Gillman (1993). A tentative analysis of the role of the banking sector in this process has been made by Yoshino (1993), as well as by Lucas (1993).

In this paper we derive three important results. First, we depart from the basic theoretical work by McCallum and Goodfriend (1987) and Lucas (1993 and 1994) to derive an approximate solution for the non-separable differential equation only numerically solved by Lucas (1993 or 1994) in the fourth section of his work. Given Lucas' hypothesis of a constant returns to scale time-transacting technology, we prove that this approximate solution is a better one than Bailey's, and actually lies between the correct welfare cost and Bailey's approximation. An estimate of the maximum relative error of our solution is also presented.

Secondly, we provide an extension of the basic model for an economy with n different kinds of money, for an integer n . As in the previous classical case, we present an approximate solution to the underlying non-separable differential equation, proving once more that this approximate solution lies between the correct one and that which would be generated had one extended Bailey's approach to this case. We also show that the higher the banking income share in GDP, the less accurate Bailey's approximation. For countries like Brazil in the nineties, where the financial system accounts for a share over ten percent of GDP, the use of Bailey's approximation can turn out to be very misleading.

Thirdly, we re-interpret the previous model to conclude that in the basic model money should stand not only for the currency held by the public, but also for the non-interest-bearing demand deposits, that is to say, M1. Finally, we derive sufficient conditions which allow the use of the basic formulas derived in the first section of the paper even in the presence of interest-bearing near monies. This is the case when the real demand for such deposits does not depend on the inflation rate.

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2) The McCallum-Goodfriend Framework

The McCallum-Goodfriend framework assumes an economy where the representative household gains utility from the consumption of a single non-storable consumption good, its preferences being determined by:

$$\int_0^{\infty} e^{-\rho t} U(c) dt \quad (2.1)$$

where $U(c)$ is a concave function of the consumption $c = c(t)$ at instant t and $\rho > 0$. The household is endowed with one unit of time that can be used to transact or to produce the consumption good with constant returns to scale:

$$y + s = 1 \quad (2.2)$$

where y stands for the production of the consumption good and s for the fraction of the initial endowment spent as transacting time.

Households can accumulate two assets, money (M) and bonds (B), the latter yielding a nominal interest rate equal to r . Indicating by $P = P(t)$ the price of the consumption good, the household faces the budget constraint:

$$\dot{M} + \dot{B} = rB + P(y - c) + H$$

H indicating the (exogenous) flow of money transferred to the household by the government. Making $\pi = \frac{\dot{P}}{P}$ (inflation rate), $m = M/P$, $b = B/P$, $h = H/P$, the budget constraint reads:

$$\dot{m} + \dot{b} = y - c + h + (r - \pi)b - \pi m$$

or, taking into account (2.2):

$$\dot{m} + \dot{b} = 1 - (c + s) + h + (r - \pi)b - \pi m \quad (2.3)$$

Compared to money, bonds are obviously preferable from the point of view of interest yield. Yet money is useful because it saves transacting time, as can be described by the technology function:

$$c = F(m, s) \quad (2.4)$$

where F is an increasing function of both m and s . That means that, by using higher cash balances (in real terms) the household can consume the same with less transacting time.

The household is supposed to maximize (2.1) subject to the budget constraint (2.3) and to the time-transacting technology (2.4). Defining the expenditure $z = c + s$

we obtain from (2.4) that $c = F(m, z - c)$ and since $F_z \neq -1$ we can apply the implicit function theorem to define $V(z, m)$ as:

$$V(z, m) = U(c) \quad (2.5)$$

which expresses the fact that money is useful because it saves time spent on transactions. The representative household will choose the path of its money and asset holdings so as to maximize:

$$\int_0^{\infty} e^{-\rho t} V(z, m) dt$$

where, according to (2.3) :

$$z = c + s = 1 + h + (r - \pi)b - \pi m - \dot{b} - \dot{m}$$

This is a standard variational problem, where Euler equations yield

$$(r - \pi - g) V_z = -\frac{d V_z}{d t}$$

$$(\pi + g) V_z = V_m + \frac{d V_z}{d t}$$

We are interested in steady-state solutions where m , b and z converge to constant figures. In this case, the equilibrium equations are:

$$r = \pi + g \quad (2.6)$$

$$r V_z = V_m \quad (2.7)$$

Equation (2.6) states that the equilibrium real interest rate $r - \pi$ equals the discount rate on future utilities. Equation (2.7) balances the marginal utility of money with the marginal utility of expenditure times the nominal interest rate, a classical law, now expressed in terms of indirect utilities. One should remember that in equilibrium, since the consumption good is non-storable and since all households are equal:

$$y = c$$

or, equivalently:

$$z = c + s = 1 \quad (2.8)$$

Equation (2.7) can be rewritten in terms of the time-transacting technology function $c = F(m, s)$. Since $U(c) = V(z, m)$, if $c = G(z, s)$, then:

$$V_z/V_m = G_z/G_m$$

Now, since $c = z - s = F(m, s) = G(z, m)$:

$$G_z/G_m = F_s/F_m$$

as can be checked by implicit differentiation. Hence, (2.7) can be replaced by:

$$r F_s = F_m \quad (2.9)$$

Moreover, (2.4) and (2.8) yield:

$$1 - s = F(m, s) \quad (2.10)$$

This completes the description of the McCallum-Goodfriend framework. In spite of its dramatic simplifications, it addresses two really important issues. First, it shows that the utility of money is to save time spent on transactions. Second, it provides a natural measure for the welfare costs of inflation, the time s spent on such transactions. For a given nominal interest rate r the demand for money $m = m(r)$ and the welfare cost $s = s(r)$ can be determined by equations (2.9) and (2.10)¹. We shall assume that such solutions yield $m = m(r)$ as a decreasing function, and consequently $s = s(r)$ as an increasing function of the nominal interest rate r . Hence, the higher the nominal interest rate, the lower the consumption $1 - s(r)$ of the representative household. This leads to Friedman's optimum monetary rule, that of keeping the nominal interest rate as low as possible, presumably at $r = 0$. Moving the equilibrium nominal interest rate from 0 to r causes a welfare loss $s(0) - s(r)$, measured in terms of reduction in individual consumption. As the nominal interest rate increases, the opportunity cost of holding cash balances increases too. As a result, individuals will reduce transaction balances and spend more time on such transactions. The overall effect will be a reduction in time employed productively, and hence in consumption.

For empirical purposes, the problem with the McCallum-Goodfriend framework is that $s(r)$, namely the time spent on transactions, is not directly measurable. The advantage of the Bailey consumer surplus construction is that it derives $s(r)$ from $m(r)$, although through an approximation rule. To do the same in the McCallum-Goodfriend framework, an additional assumption must be introduced.

This leads to Lucas' hypothesis:

$$c = F(m, s) = m\phi(s) \quad (2.11) \quad (\phi'(s) > 0 \text{ and } \phi''(s) < 0)$$

which assumes a time-transacting technology with constant returns to scale, as in the Baumol analysis, where $F(m, s) = k m s$. Lucas shows how, under hypothesis (2.11), $s(r)$, $m(r)$ and $\phi(s)$ are inter-related, and analyses in detail the time-transacting function $F(m, s) = k m s^\mu$, where $0.5 \leq \mu \leq 1$. With the Lucas assumption (2.11), equations (2.9) and (2.10) are replaced by:

¹ A sufficient condition for this determination to be possible is that the function F satisfies $-(1 + F_s)(r F_{sm} - F_{mm}) + F_m(r F_{ss} - F_{ms}) \neq 0$. This is the case, for instance, when we have $F_{sm} > 0$, $F_{mm} \leq 0$ and $F_{ss} \leq 0$.

$$\phi(s) = r m \phi' \quad (2.12)$$

$$1 - s = m \phi(s) \quad (2.13)$$

which implies $s'(r) > 0$ and $m'(r) < 0$.

If the time-transacting function $\phi(s)$ is known, the demand for money $m(r)$ and the welfare cost $s(r)$ can be determined by the above equations. For instance, $\phi(s) = ks$ implies:

$$r = \frac{1}{m(1 + km)}$$

$$s = \frac{1}{1 + km}$$

In practice, $\phi(s)$ can not be directly estimated from statistical data, what is known from empirical studies is the money demand $m = m(r)$ or its inverse function $r = r(m)$. The problem of deriving $s(r)$ from $m(r)$ without knowing $\phi(s)$ can be easily solved as follows: We first differentiate (2.13), which leads to:

$$-ds = \phi(s) dm + m \phi'(s) ds$$

We then eliminate $\phi(s)$ and $\phi'(s)$ combining the above equation with (2.12) and (2.13). The result is the differential equation:

$$-ds = \frac{r(1-s)}{r m + (1-s)} dm \quad (2.14)$$

which determines the welfare cost $s(r)$ as a function of the money demand $m(r)$. Once $s(r)$ is thus determined, the implied $\phi(s)$ can be found by equation (2.13).

From the computational point of view, the trouble with (2.14) is that it is not a separable equation. This suggests the use of some approximation formula to ease practical calculations. Bailey's graphic construction, corresponding to the formula:

$$-ds = r dm \quad (2.15)$$

is one possible approximation, as long as the interest rate r times the real stock of money m can be neglected when compared to $1 - s$. Moreover, Bailey's construction provides an upper bound to the welfare cost s , since (2.14) obviously implies $-\frac{ds}{dm} < r$. Yet a better approximation formula, based on the fact that $0 < s < 1$, is provided by the double-sided inequality:

$$-\frac{ds/dm}{1-s} > \frac{r}{r m + 1} > -\frac{ds}{dm} \quad (2.16)$$

or, since $dm = m'(r) dr < 0$,

$$\frac{ds}{dr} < -\frac{rm'(r)}{rm(r)+1} < \frac{ds/dr}{1-s}$$

Defining $F(r)$ as the integral:

$$F(r) = \int_0^r -\frac{\rho m'(\rho)}{\rho m(\rho)+1} d\rho \quad (2.17)$$

the above inequalities yield:

$$1 - e^{-F(r)} < s(r) < F(r) \quad (2.18)$$

This shows that $F(r)$ is an upper bound to the welfare cost of inflation, with a maximum relative error of approximately $F(r)/2$.

In fact:

$$\frac{F(r) - (1 - e^{-F(r)})}{s(r)} \approx \frac{F^2(r)}{2s(r)} \approx \frac{F(r)}{2}$$

Thus, $F(r) = 0.01$ means that $0.00995 < s(r) < 0.01$. $F(r) = 0.15$ implies $0.1393 < s(r) < 0.15$. For practical purposes, $F(r)$ can be taken as an adequate measure of the welfare costs of inflation, given the simplifications involved in the theoretical analysis of the problem. **One should note that $F(r)$ is a better estimate of $s(r)$ than Bailey's approximation:**

$$B(r) = \int_0^r -\rho m'(\rho) d\rho$$

In fact, since $m'(r) < 0$, $F(r) < B(r)$, as results immediately from the expression above and (2. 17). As $F(r)$ is also a upper bound to $s(r)$, this inequality shows that it is a better approximation to the welfare cost of inflation than the one provided by Bailey's formula.

As an example, for the money demand equation:

$$m(r) = K r^{-a}$$

expression (2.17) yields:

$$F(r) = \frac{a}{1-a} \log(1 + K r^{1-a}) \quad (2.19)$$

compared to Bailey's approximation:

$$B(r) = \frac{a}{1-a} \cdot K \cdot r^{1-a} \quad (2.20)$$

3) Interest-Bearing Money

Let us extend the model of the preceding section assuming the existence of two kinds of money, currency bills and interest-bearing demand deposits, the real quantities of which will be indicated by m and x , respectively. The nominal yield of currency bills is equal to zero, that of demand deposits equal to i . This implies replacing the budget constraint (2.3) by:

$$\dot{m} + \dot{x} + \dot{b} = 1 - (c + s) + h + (r - \pi)b + (i - \pi)x - \pi m$$

or, equivalently:

$$z = c + s = 1 + h + (r - \pi)b + (i - \pi)x - \pi m - \dot{b} - \dot{x} - \dot{m} \quad (3.1)$$

The time-transacting technology is now described by:

$$c = F(m, x, s) \quad (3.2)$$

and the indirect utility function by:

$$V(z, m, x) = U(c) \quad (3.3)$$

Market clearing requires:

$$z = c + s = F(m, x, s) + s = 1 \quad (3.4)$$

The marginal utility conditions (2.6) and (2.7) being replaced by:

$$r = \pi + g \quad (3.5.a)$$

$$V_m = r V_z \quad (3.5.b)$$

$$V_x = (r - i) V_z \quad (3.5.c)$$

In terms of the time-transacting technology function, the last two equations above are equivalent to:

$$r F_s = F_m \quad (3.6.a)$$

$$(r - i) F_s = F_x \quad (3.6.b)$$

With two kinds of money, Lucas' assumption (2.11) reads:

$$F(m, x, s) = G(m, x) \phi(s) \quad (3.7)$$

where $G(m, x)$ is differentiable, homogeneous of degree one, increasing in each of its variables and with decreasing marginal returns with respect to each one. This transforms equations (3.6) into:

$$\begin{aligned} G_m \phi(s) &= r G(m, x) \phi'(s) \\ G_x \phi(s) &= (r - i) G(m, x) \phi'(s) \end{aligned}$$

Given the hypotheses about $G(m, x)$, the marginal rate of substitution G_m/G_x is an increasing function of the asset ratio x/m . Taking the inverse function

$$\frac{x}{m} = J(G_m / G_x) \quad J'(\cdot) > 0$$

the derivative of the log of J with respect to the log of its variable indicating the elasticity of substitution between currency bills and demand deposits. According to the preceding analysis, utility maximization leads to

$$\frac{x}{m} = J(r / (r - i)) \quad (3.8)$$

Moreover, since $G(m, x) = m G_m + x G_x$ (Euler's theorem):

$$\phi(s) = (r m + (r - i) x) \phi'(s) \quad (3.9)$$

Finally, the market clearing condition (3.4) can be rewritten as:

$$1 - s = G(m, x) \phi(s) \quad (3.10)$$

Assuming the time-transacting technology function $\phi(s)$ to be known, equations (3.8), (3.9) and (3.10) determine the asset holdings m , x and the welfare cost of inflation s as a function of r and $r - i$. Throughout the following discussion we shall assume the banking spread $r - i$ on demand deposits to be a constant (except for very small lending rates). This is to say that the real interest yield of demand deposits $i - \pi = r - \pi - (r - i) = g - (r - i)$ is a constant too. In this case, the asset ratio x/m is an increasing function of r , as well as the welfare loss $s(r)$, while the demand for currency bills declines with the increase of the interest rate r , as can be shown by differentiating (3.8), (3.9) and (3.10). As to the demand for interest-bearing deposits, it can either increase or decrease with r , depending on the sign of the sum of the elasticity of substitution between x and m with the elasticity of the demand for m with respect to r . In any case, as we show in the appendix, $r m'(r) + (r - i) x'(r) < 0$. An important reference case is where the sum of these two elasticities is equal to 0. Here the demand for interest-bearing deposits does not depend on r , but only on their real interest yield, which is a constant. This is the same as saying that the real demand x for such deposits does not depend on the inflation rate. The case will be referred to in the following discussion as that of inflation-neutral deposits.

For empirical purposes, one should note that the specification of the function

$\phi(s)$ is usually unknown. The problem can be dealt with by the same method adopted in the preceding section. We first differentiate (3.10): $rm'(r) + (r-i)x'(r) < 0$.

$$-ds = G_m \phi(s) dm + G_x \phi(s) dx + G(m, x) \phi'(s) ds$$

or equivalently:

$$-ds = G(x, m) \phi'(s) (r dm + (r-i) dx + ds)$$

We now use (3.9) and (3.10) to eliminate $\phi(s)$ and $\phi'(s)$. This leads to:

$$-ds = \frac{(1-s)(r dm + (r-i) dx)}{1-s+rm+(r-i)x} \quad (3.11)$$

which can be interpreted as a two-dimensional version of (2.14). In the denominator, rm is the seigniorage collected on currency bills held by the public, $(r-i)x$ the banking spreads on demand deposits, both measured as a proportion of GDP. The sum $rm+(r-i)x$ is the total banking income, that is, the income received by financial intermediaries for financing medium- and long-term assets with immediately callable liabilities also measured as a ratio to GDP. It corresponds to the income share of financial intermediation in GDP, measured by national accounts, with the exclusion of revenues not related to the art of borrowing short for lending long.

Differential equation (3.11) may be solved directly or, more easily, approached by a separable differential equation, by noting that, since $rm'(r) + (r-i)x'(r) < 0$,

$$\frac{ds}{dr} < -\frac{rm'(r) + (r-i)x'(r)}{1+rm+(r-i)x} < \frac{ds/dr}{1-s}$$

Defining $F(r)$ as the integral:

$$F(r) = -\int_0^r \frac{rm'(r) + (r-i)x'(r)}{1+rm+(r-i)x} dr \quad (3.12)$$

one concludes, as in the preceding section, that the welfare cost of inflation lies in the interval:

$$1 - e^{-F(r)} < s(r) < F(r) \quad (3.13)$$

Bailey's construction formula is an approximation to $F(r)$ that treats as infinitesimal the banking income share in GDP, namely, substituting 1 for the denominator of the integrand of (3.12):

$$F(r) \approx B(r) = -\int_0^r (rm'(r) + (r-i)x'(r)) dr \quad (3.14)$$

The preceding results can be more concisely restated in vector notation. For that purpose, let us introduce the cash-holdings vector $M = (m, x)$ as well as the opportunity-cost vector $R = (r, r - i)$, noting that the inner product $R \cdot M = r m + (r - i) x$ indicates total banking income as a proportion of GDP. Equation (3.11) can be rewritten as:

$$-ds = \frac{(1-s)R \cdot dM}{1-s+R \cdot M}$$

the estimating function $F(r)$ as:

$$F(r) = -\int_0^r \frac{R \cdot M'(r)}{1+R \cdot M} dr$$

and Bailey's approximation as:

$$B(r) = -\int_0^r R \cdot M'(r) dr$$

These concise expressions are quite important, less for their elegance than for the fact that they hold for a model with n different kinds of money: for any integer n . For empirical investigation we shall be interested in a model with three types of money, currency bills, non-interest-bearing demand deposits and interest-bearing deposits, which are essential to describe payment systems after the financial innovations of the late 1970's and 1980's. The three assets must be treated as imperfect substitutes, to make possible the coexistence of both types of demand deposits, interest- and non-interest-bearing (*). Yet since the first two have zero nominal interest yield, their sum can be aggregated as if they were one single asset, which brings us back to the (m, x) model with one reinterpretation: m no longer stands for currency held by the public, but for its sum with the balance of non-interest-bearing demand deposits. In short, m now means M_1 , while x stands for interest-bearing demand deposits.

With this understanding, the problem of estimating the welfare cost of inflation is reduced to solving differential equation (3.11), or more simply to calculate either $F(r)$ in (3.12) or $B(r)$ in (3.14).

For that purpose, besides the knowledge of the banking spread $r - i$, estimates should be available for the demand $m(r)$ of M_1 and $x(r)$ of interest-bearing demand deposits, as a function of the nominal interest rate.

Estimations are considerably simplified in the case of inflation-neutral interest-bearing deposits. Here $dx = 0$, and since the total bank spreads $(r - i)x$ on such deposits can be neglected when compared to GDP plus the inflationary tax rm , $F(r)$ can be approximated by:

(*) A more promising analysis would perhaps treat the two types of deposits as two parts of a same product supplied under a technology involving fixed costs. In fact, interest is only paid for demand deposits in excess of a contractual minimum balance.

$$F(r) = - \int_0^r \frac{r m'(r)}{1 + r m} d m$$

which is equivalent to equation (2.17). In short, with inflation-neutral interest-bearing deposits the welfare waste $s(r)$ can be calculated as if there were no other kind of money besides M_1 .

Additional information on the welfare cost of inflation results from the assumption that the time-transactions technology function can be specified as:

$$\phi(s) = k s^\mu \quad (3.15)$$

where $0.5 < \mu < 1$, as suggested by Lucas. This parametrization emerges in the inventory-theoretical literature on money demand, following Baumol's seminal work. The latter implies $\mu = 1$, while the Miller and Orr stochastic version leads to economies of scale that make $\mu = 0.5$. Introducing $\phi(s) = k s^\mu$ in (3.9) yields:

$$s = \mu(r m + (r - i) x) \quad (3.16)$$

which is to say that the welfare cost of inflation is μ times the total banking income, measured as a proportion of GDP. This conclusion is very much in line with the perception that inflation leads to excess financial intermediation.

In the particular case where interest-bearing demand deposits are inflation-neutral, one may establish a simple approximate relation between μ and the interest rate elasticity $-a$ of the money (M_1) demand. Differentiating (3.16), since $(r - i)x$ is a constant:

$$d s = \mu(r d m + m d r)$$

Since the interest rate elasticity of the demand for M_1 is equal to $-a$ as in $m(r) = K r^{-a}$:

$$d m = -a \frac{m}{r} d r$$

Now, since $dx = 0$, Bailey's approximation formula implies:

$$d s = -r d m$$

Combining the last three relations:

$$a = \frac{\mu}{\mu + 1} \quad (3.17)$$

a very important relation that connects the Baumol case $\mu = 1$, where the welfare cost of inflation equals the total banking income, with an interest rate elasticity of the money demand equal to -0.5 , as empirically supported in many countries. An elasticity

with absolute value $a = 0.33$ corresponds to the case described by Miller and Orr.

4) Some Empirical Estimates for the Welfare Cost of Inflation in Brazil

Empirical estimates of the welfare cost of inflation in Brazil have been provided by Dias (1993), Cysne (1994a and 1994b) and Pastore (1993). Dias' work unfortunately presents strong evidence of miscalibration of the parameters. Indeed, the values obtained for the elasticity of the money demand, the inflation tax, as well as for the welfare costs of inflation, are very far from those which one would theoretically expect. The figures presented by Cysne (1994a and 1994b) are preliminary to those obtained here, and were still based on Bailey's approximation, which we prove here to be a worse approximation to the true welfare cost than those provided by the approximation we suggest. Pastore's work, also based on Bailey's approximation, presents a welfare waste of about 8.5% of GDP for a 40% monthly inflation rate. This figure, however, is overestimated. First because of the use of Bailey's formula, whose inaccuracy is relatively high for a country like Brazil, which presents a bloated financial sector of around 10.5% of GDP. Secondly, because Pastore used the periodic values of the interest rate, when the correct ones would be the logarithmic. This is to say that Pastore's estimate of a welfare cost of 8.5% of GDP actually refers to a 48.18% $((\exp 0,4) - 1)$ monthly inflation rate, and not to a 40% monthly inflation. This correction makes Pastore's estimates very close to those presented in table 1.

The empirical results here derived were based on monthly and annual data on the means of payment, overnight interest rate, GDP, industrial production index and the IGP-FGV price index. The original sources were, respectively, Central Bank (for M_1 and for the overnight rate), FIBGE (for GDP and the industrial production index) and Getulio Vargas Foundation (for the IGP price index).

According to tests presented in Cysne and Issler (1993), where this hypothesis proved not to be rejectable, we worked with an unitary income elasticity of the money demand function.

The choice between a semi-log and a log-log money demand specification was decided upon by empirical criteria, in favor of the latter. As can be concluded from figures 1 and 2, the log-log demand function, as was also the case in Lucas' work for U.S. data, presents a much better fit.

Our first goal was to use the results derived in section 3 of the paper to estimate the welfare costs of inflation, since Brazil is a country which very easily falls into the category of interest-bearing demand deposits.² In order to do so, however, we should have empirical estimates of the function $x(r)$. But this is certainly not an easy task in an economy like Brazil's, which in many cases presents interest controls as well

²Actually, demand deposits in Brazil are not allowed to pay interest, but we are referring here to short-run funds which are automatically transferred to the demand-deposit accounts in case of overdraw.

as heavy taxation on some financial transactions.³ We then decided to assume the hypotheses here derived which provide sufficient conditions for the estimation of welfare costs based only on the knowledge of $m(r)$. In doing so, the only benefit we could get from the theoretical results here derived was related to the use of our approximation $F(r)$, besides Bailey's $B(r)$, in order to evaluate the welfare figures.

We then applied the formulas (2.19) and (2.20) derived in the text using the parameters obtained from the empirical estimation of the money demand specification:

$$m(r) = k r^{-a}$$

The parameters for Brazil, where R is selected so the curve passes the geometric means of the last five-year data pairs, are:

$$k = 0.0368 \text{ and } a = 0.525$$

from which we obtain

$$\begin{aligned} F(r) &= 1.105 \log(1 + 0.0368 \cdot r^{0.475}) \\ B(r) &= 0.0407 \cdot r^{0.475} \end{aligned}$$

The following table compares the figures for the welfare cost of Brazilian inflation using our approximation $F(r)$ and Bailey's $B(r)$. As we mentioned before in this section, the interest rate to be used in these calculations is the logarithmic rather than the periodic. In table 1, columns two (M.INT) and three (A.INT) stand, respectively, for the monthly and yearly periodic inflation rates. Column 4 (LA.INT) presents the logarithmic annual inflation, to be used in the above formulas. Column 5 shows the lower bound $X(r) = (1 - \exp(-F(r)))$ for the welfare cost which is presented in inequality 2.18. Column 6 shows the numerical solution of (2.14) with the initial condition $s(0.01) = 0.01465$, which is the average of $X(0.01)$ and $F(0.01)$. Finally, columns 7 and 8 present the welfare cost of inflation as a fraction (not percentage) of GDP, the first one using the formula here derived $F(r)$, and the other Bailey's expression $B(r)$.

As one can observe, the estimations using Bailey's formula always lead to higher welfare costs. This is also visible in figure 3, which shows the lower bound for the welfare cost $(1 - \exp(-F(r)))$ as well as our approximation $F(r)$ and Bailey's $B(r)$. Figure 4 shows the four solutions presented in table 1. One can observe that $F(r)$ lies very close to $s(r)$. Finally, figure 5 presents the differences $F(r) - s(r)$ and $B(r) - s(r)$. It becomes clear that $F(r)$ presents a better approximation to $s(r)$ than $B(r)$.

Theoretically, the differences between $F(r)$ and $B(r)$ are particularly high for high inflation rates, when the value added by the financial system is not insignificant compared to the GDP. It is important to notice, however, that given the formulas we are using, deduced in section 2 of the paper, the differences between $F(r)$ and $B(r)$ arise

³Lucas reported to us that he had the same difficulty in trying to estimate such a function for the American economy.

only from part of the value added by the financial system (the total inflationary transfers on the means of payment $m(r).r$), the remaining part $((r-i).m)$ not being considered.

An inference on the value added by the financial institutions issuing interest-bearing deposits can be made from our estimates. The elasticity of the money demand we are dealing with, 0.525, is very close to Baumol's theoretical prediction. As we show in section 3 of the paper (equation (3.16)), in this case we have a transacting technology of the type $\phi(s) = ks$, and the welfare cost is equal to the value added by the part of the financial system which issues means of payment and interest-bearing deposits $(mr + (r-i)x)$. The first part of this total, mr , standing for the total inflationary transfers on the means of payment, represents something around 6% of GDP. The remaining 1.98% can be consequently accrued to the value added by the financial institutions issuing interest-bearing demand deposits.

It is also worthwhile noticing that for the level of inflation around 45% per month existing at the time when this paper was being written, the evaluated welfare cost (7.98% of GDP) corresponds very closely to the difference between the size of the financial system in Brazil (around 10.5% of GDP) and the size of a financial system one would expect for a low-inflation country (around 2.5% of GDP, to take an average of the German and American parameters). By this time, a steady inflation in this level would have cost the country, along the years, something around 7.98% of its GDP, that is to say, something in the area of 33.9 billion dollars per year.

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Table 1

Obs	M.Int	A.Int	LA.Int	X(r)	s(r)	F(r)	B(r)
1	0.0100	.1268	0.1194	0.0146116	0.0146500	0.0147194	0.0148310
2	0.0200	.2682	0.2376	0.0201520	0.0202859	0.0203578	0.0205647
3	0.0300	.4258	0.3547	0.0242789	0.0245051	0.0245785	0.0248759
4	0.0400	.6010	0.4706	0.0276766	0.0279915	0.0280668	0.0284514
5	0.0500	.7959	0.5855	0.0306145	0.0310155	0.0310929	0.0315624
6	0.1000	2.1384	1.1437	0.0416225	0.0424234	0.0425135	0.0433803
7	0.1500	4.0350	1.6771	0.0495303	0.0506944	0.0507989	0.0520307
8	0.2000	7.9161	2.1879	0.0558424	0.0573414	0.0574622	0.0590347
9	0.2500	13.5519	2.6777	0.0611383	0.0629488	0.0630871	0.0649802
10	0.3000	22.2981	3.1484	0.0657200	0.0678224	0.0679791	0.0701758
11	0.3500	35.6442	3.6013	0.0697639	0.0721414	0.0723169	0.0748019
12	0.4000	55.6939	4.0377	0.0733862	0.0760238	0.0762184	0.0789784
13	0.4500	85.3806	4.4588	0.0766679	0.0795523	0.0797663	0.0827891
14	0.5000	128.7463	4.8656	0.0796673	0.0827867	0.0830201	0.0862947

Appendix

Here we prove that $r m'(r) + (r - i) x'(r) < 0$. Differentiating (3.9) one gets

$$s'(r) \cdot [r m'(r) + m + (r - i) x'(r)] > 0 \quad (1)$$

From (3.10),

$$s'(r) \cdot [r m'(r) + (r - i) x'(r)] < 0 \quad (2)$$

Subtracting (2) from (1),

$$s'(r) m > 0 \rightarrow s'(r) > 0 \quad (3)$$

It follows from (2) and (3) that $r m'(r) + (r - i) x'(r) < 0$. ■

FIGURE 1

Semi-log Specification: Plot on Interest Rates, Actual x Fitted

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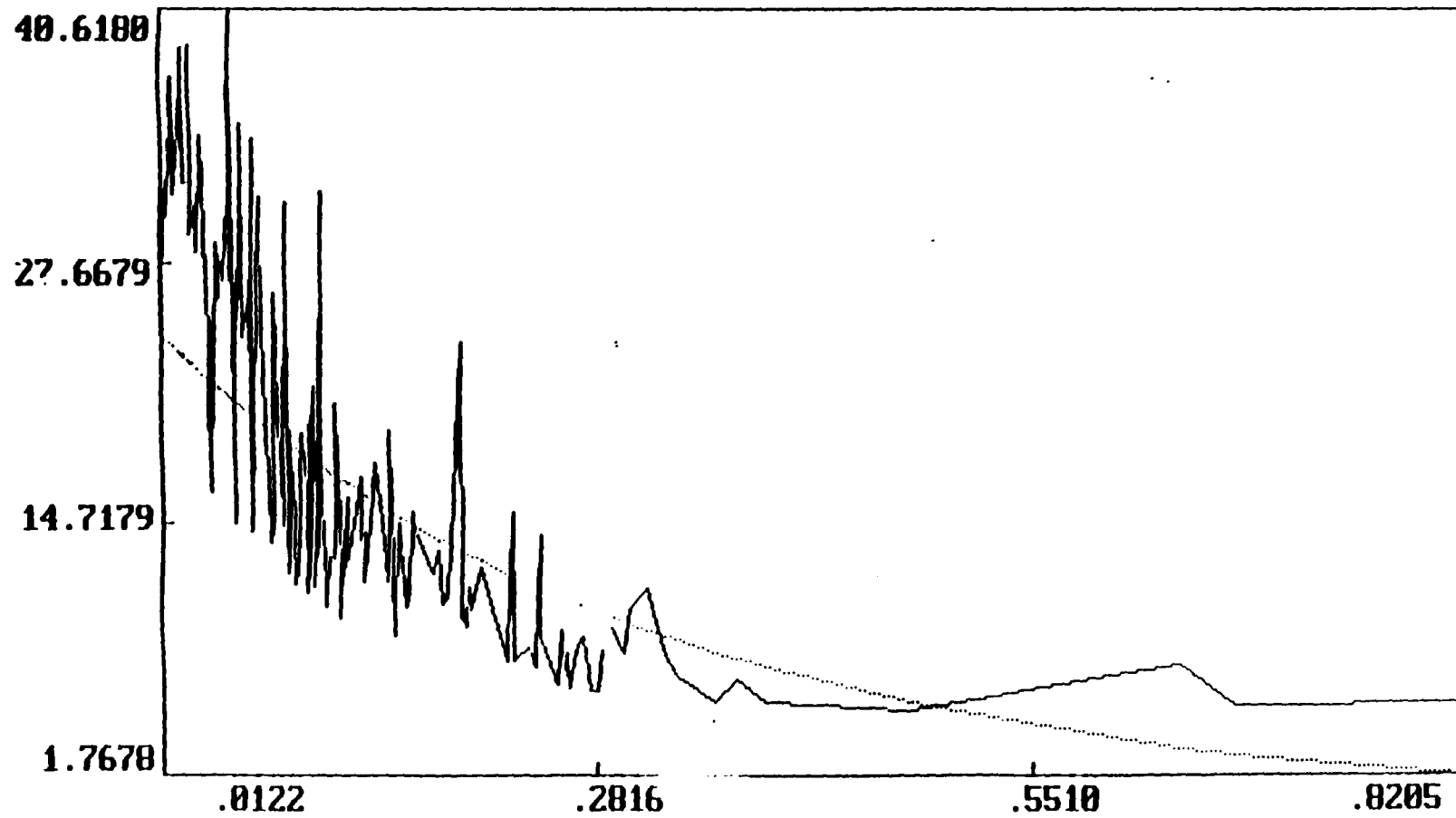


FIGURE 2

Log Log Specification: Plot on Interest Rates, Actual x Fitted

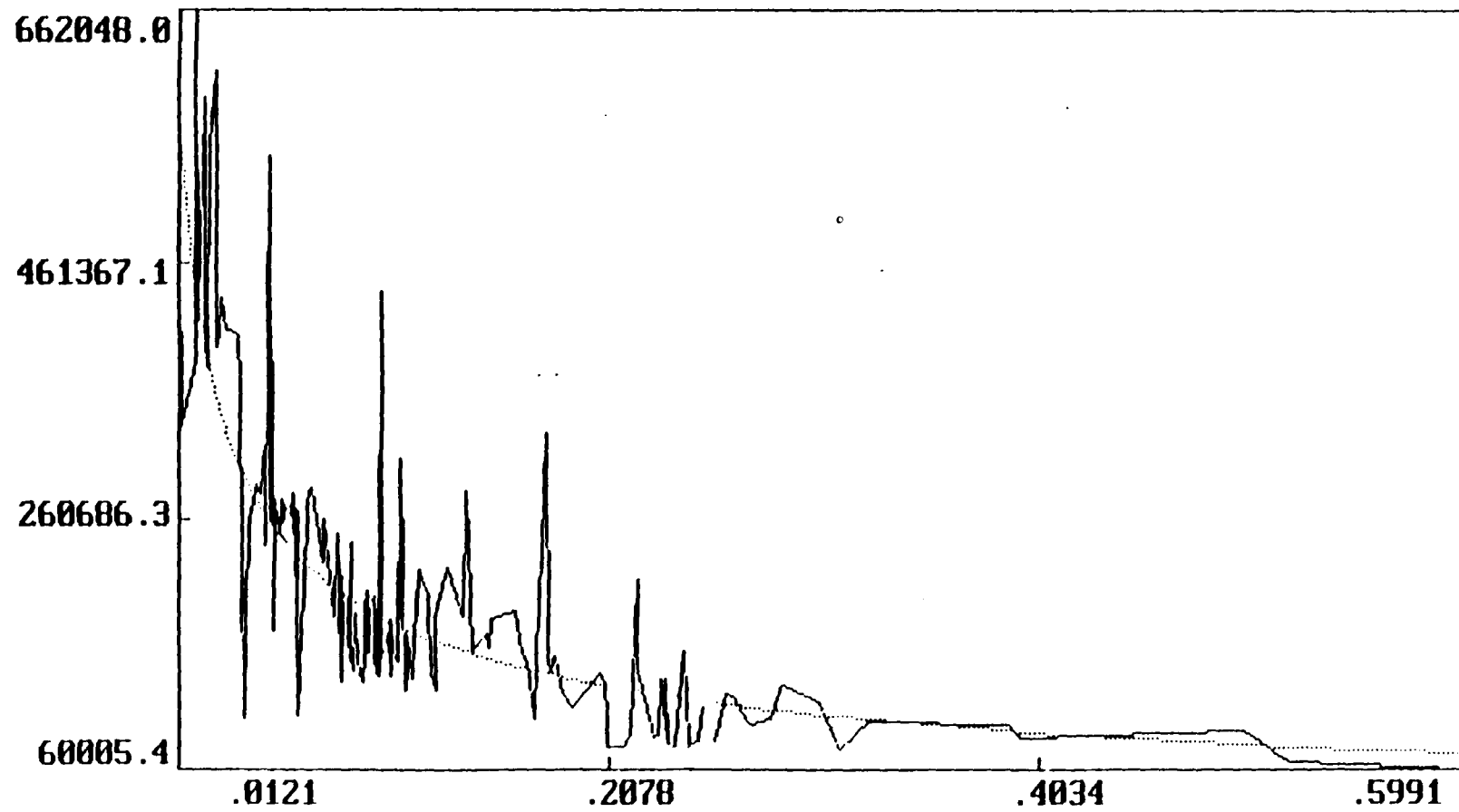
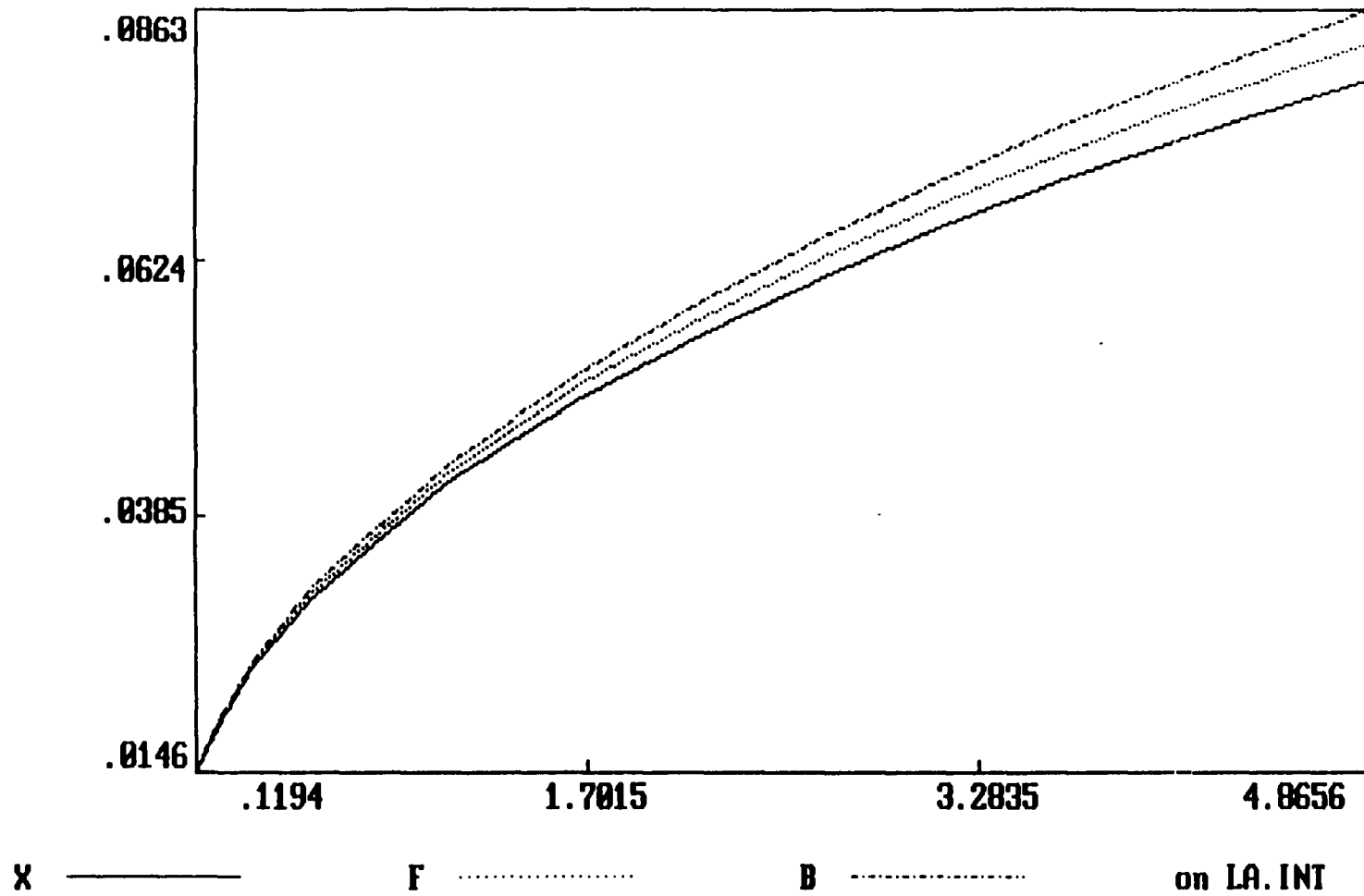


FIGURE 3
Welfare Cost of Inflation: Bailey's (B) x Our Formula (F)



Obs.: X represents the lower bound and F the upper bound to the correct value of the welfare cost of inflation $s(r)$. Bailey's expression, on the other hand, provides an approximate solution to $s(r)$ worse than the one given by the closed form F. As proved in the text, Bailey's solution lies over F, the upper bound to $s(r)$ which we present.

FIGURE 4

Welfare Costs of Inflation: $X(r) \times s(r) \times F(r) \times B(r)$

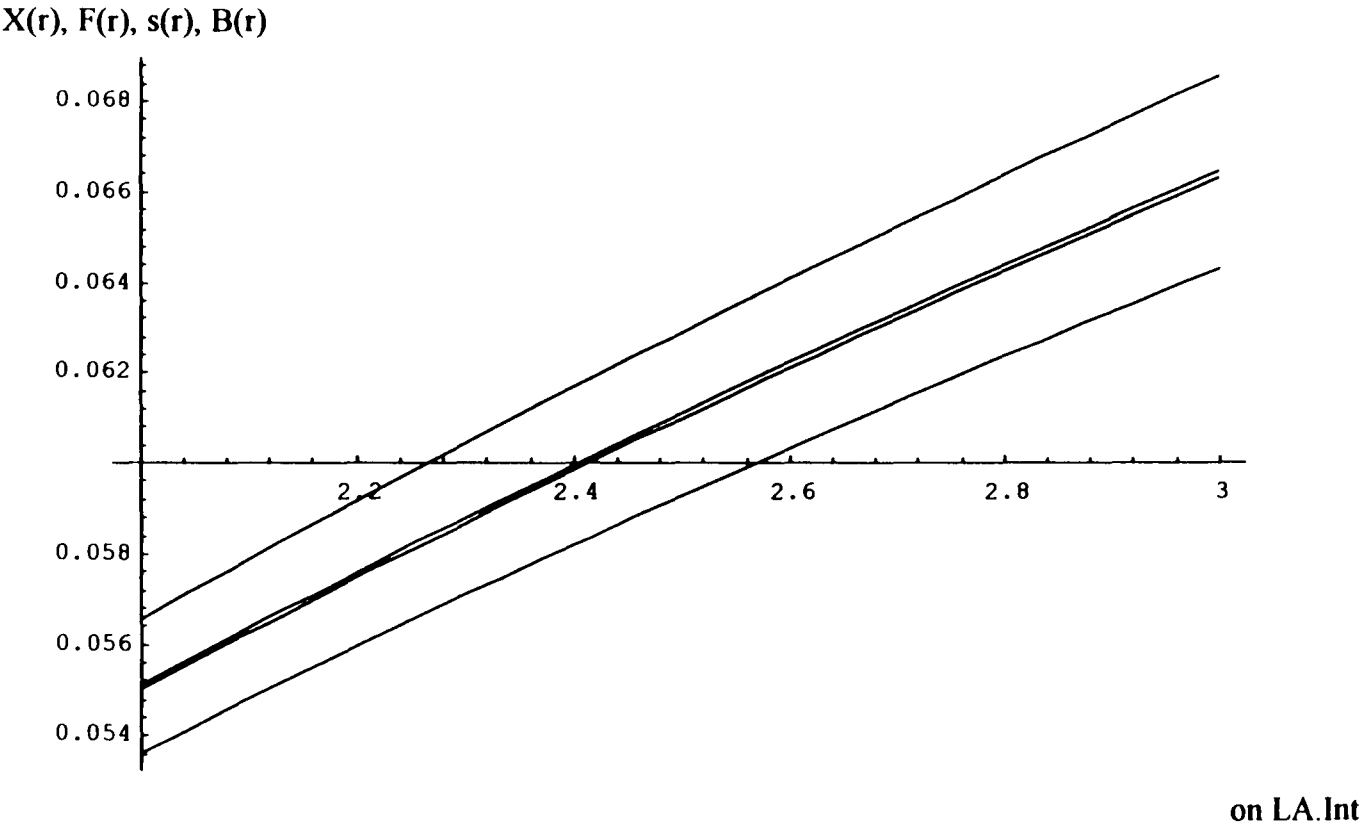


FIGURE 5

Welfare Costs of Inflation: $F(r) - s(r)$ X $B(r) - s(r)$

