THE IMPACT ON THE STATIONARY STATE INCOME OF A PAY-AS-YOU-GO SYSTEM OF SOCIAL SECURITY

SAMUEL DE ABREU PESSÔA

(USP)

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Samuel de Abreu Pessôa
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University of São Paulo - USP
Business Administration and Economics Faculty
Department of Economics

Abstract

A model of overlapping generations in continuous time is composed. Individuals pass through two distinct time periods during their life times. During the first period, they work, save and have a death probability equal to zero. During the second, from the periods $T$ after birth, their probability of death changes to $p$ and then they retire. Capital stock and the stationary state income are calculated for two situations: in the first, people live from their accumulated capital after retirement; in the second, they live from a state transfer payment through income tax. To simplify matters, in this preliminary version, it is supposed that there is no population growth and that the instantaneous elasticity substitution of consumption is unitary.

1. Introduction

The aim of this paper is to construct a model of an economy with overlapping generations, a finite horizon and continuous time. The starting point is the model of perpetual youth (Blanchard and Fischer, 1989, chapter 3, and Blanchard, 1985). In this model, individuals, independent of their age, are faced with a constant probability of death equal to $p$. Hence, the name of this model. It is as if people are eternally young. In this way, the model “captures the aspect of a finite life horizon, but does not alter behaviour during life, the aspect of ‘life cycle’” (Blanchard, 1985).

To incorporate the life cycle aspect into the model, it is supposed that individuals live in two periods. In the first, between the time of birth (at $s$) and $s + T$, where $T$ is known

\begin{footnote}{Agradeço o suporte financeiro da FIPE em seu programa de apoio à Pesquisa Acadêmica.}

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\end{footnote}
and identified for everyone, people work and have a probability of death equal to zero. In the second, from \( s + T \), people retire and face a probability of death equal to \( p \).

During the first period, there is an economy with overlapping generations in continuous time with a finite and determinist horizon (Cass and Yaari, 1967). During the second, there is an economy like that described by Blanchard. This paper intends to synthesise both economies. The difficulty, comparing with Blanchard’s work is that there is no closed solution for the behaviour of aggregate consumption, once the marginal propensity to consume is no longer constant during the course of life.

However, it is possible to find an equation for stationary state capital. This is used to answer the following question: what is the differential of income per capita between two economies with the same technological capacities and preferences, the only difference between the two economies being the type of retirement implemented.

In the founded system, people finance their retirement by their own savings, accumulated during their working life. In the unfounded system, the retired individuals live from state transfer payments, financed by ad valorem taxes on labour and capital income. Due to the lack of incentive to accumulation implicit in the unfounded system, it follows that the stationary state income will be smaller. This result is well established for overlapping generation models in discrete time and in two periods (see Blanchard and Fischer, 1985, chapter 3). However, using continuous time allows for a more flexible structure, ideal to quantify the effect that is already qualitatively well established.

This paper is organized in the following way: in the next section, the individual consumption paths are defined; in the following section, the paths are aggregated for all generations; in the fourth section, the necessary conditions for the stationary state equilibrium are defined; and, finally, the income differential is calculated.

2. Individual Choice

People live in two different periods. They are born in \( s \). Between \( s \) and \( s + T \), probability of death is zero, and from the period \( s + T \), it is constant and equal to \( p \). To
simplify matters, it is supposed that the individual will retire\(^2\) at the same moment as he/she starts to have a positive probability of death.

In the first period, the person’s problem is:

\[
\max_{s} \int_{t}^{T+s} e^{-\rho(t-s)} \ln c(s,t) dt
\]

subject to:

\[
\frac{\partial v(s,t)}{\partial t} = m(t)v(s,t) + y(s,t) - c(s,t)
\]

\[
v(s,s) = 0 \text{ and } v(s,T+s) = E.
\]

Where:

- \(c(s,t)\) - consumption in \(t\) of generation \(s\);
- \(v(s,t)\) - non-human wealth in \(t\) of generation \(s\);
- \(y(s,t)\) - net income of labour in \(t\) of generation \(s\);
- \(m(t)\) - net income of capital in \(t\);
- \(E\) - savings for the second period\(^3\).

In the second period, the hypotheses of Blanchard’s perpetual youth model are taken into consideration. In particular, every individual is faced with an effective interest rate equal to \(m(t) + p\). The second interest instalment is paid to those living in \(t\) of generation \(s\) with the resources in \(t\) from those who were born in \(s\). These payments are carried out by an insurance company which works with zero operational costs. As \(p\) is the probability of death, the insurance company’s expenditures are balanced by the revenue. It follows:

\[
\max_{T+s} \int_{T+s}^{\infty} e^{-(\rho+p)(t-(T+s))} \ln c(s,t) dt
\]

subject to:

\[
\frac{\partial v(s,t)}{\partial t} = (m(t) + p)v(s,t) + x(s,t) - c(s,t)
\]

\(^2\)Given that there is no choice between consumption and leisure, this moment should be exogenously fixed.

\(^3\)The optimal value for \(E\) will be determined later on.
\[ v(s, T + s) = E, \]

and \( x(s, t) \) is state transfer in \( t \) for a person born in \( s \).

2.1 Choice of Consumption Path

Solving (2.1), it follows:

\[ c(s, t) = c(s, s)R(t, s)e^{-\rho(t-s)} \]

and

\[ c(s, s) = \frac{\rho}{1 - e^{-\rho T}}[h(s, s) - ER(s, T + s)], \]

where:

\[ h(s, s) = \int_s^{T+s} R(t, s)g(s, t)dt \]

which is the human wealth in \( s \) of those born in \( s \), and:

\[ R(t, t') = e^{-\int_t^{t'} m(u)du} \]

which is the discount rate between \( t \) and \( t' \).

For (2.2), it follows:

\[ c(s, t) = c(s, T + s)R(t, T + s)e^{-\rho(T + s - t)} \]

and

\[ c(s, T + s) = (\rho + p)[E + g(s, T + s)] \]

where:

\[ g(s, T + s) = \int_{T+s}^{\infty} R(t, T + s)e^{-\rho(t-(T+s))}x(s, t)dt \]

is the present value in \( T + s \) of the future state transfer payments to those born in \( s \).

2.2 Choice of Wealth Left in the Second Period

\(^4\) (2.4) and (2.8) are found after integrating (2.1) and (2.2) and substituting (2.3) and (2.7).

\(^5\) (2.4) and (2.8) are found after integrating (2.1) and (2.2) and substituting (2.3) and (2.7).
From (2.3) and (2.7), the indirect utility follows:

\[ V = \int_s^{T+s} e^{-\rho(t-s)} \ln c(s, s) dt + e^{-\rho T} \int_{T+s}^{\infty} e^{-(\rho+p)(t-(T+s))} \ln c(s, T + s) dt \]

+ terms which are independent of \( E \).

Substituting (2.4) and (2.8), derived from \( E \), it follows:

\[ \frac{e^{-\rho T}}{c(s, T + S)} - \frac{R(s, T + s)}{c(s, s)} = 0 \quad (2.10) \]

Rearranging:

\[ ER(s, T + s) = \frac{1}{T} [\tilde{T}_2 h(s, s) - \tilde{T}_1 R(s, T + s) g(s, T + s)] \quad (2.11) \]

Where:

\[ \tilde{T} \equiv \tilde{T}_1 + \tilde{T}_2 \quad (2.12) \]

\[ \tilde{T}_1 \equiv \frac{1 - e^{-\rho T}}{\rho} \]

and

\[ \tilde{T}_2 \equiv \frac{e^{-\rho T}}{\rho + p} . \]

Considering:

\[ w(s, s) \equiv h(s, s) + R(s, T + s) g(s, T + s) \quad (2.13) \]

the total wealth in \( s \) of the individual born in \( s \), it follows:

\[ ER(s, T + s) = \frac{1}{T} \frac{e^{-\rho T}}{\rho + p} w(s, s) - R(s, T + s) g(s, T + s) . \quad (2.14) \]

The individual lives in two periods. The first one elapses in \( T \) years, and the second in \( p^{-1} \) years on average. As the rate of time preference is positive, the subjective assessment of these time intervals is respectively \( \tilde{T}_1 \) and \( \tilde{T}_2 \). \( \tilde{T} \) is the subjective time of life expectancy. Obviously

\[ \lim_{\rho \to 0} \tilde{T} = T + p^{-1} . \]
If there are no state transfer payments, it follows:

\[ ER(s, T + s) = \frac{T_2}{T_1} h(s, s). \]  \hspace{1cm} (2.15)

The present value at birth, of the wealth left for the second period of life is a fraction of the total wealth given by the life expectancy fraction, from the subjective point of view, which will be lived in the second period.

When the pay-as-you-go system works fully, the State gauges the current value of the transfers in such a way that \( E = 0 \). From (2.11), it follows that:

\[ R(s, T + s)g(s, T + s) = \frac{T_1}{T_1} w(s, s). \]  \hspace{1cm} (2.16)

2.3 Determination of the Consumption Path

From (2.3) and (2.4), it follows that:

\[ c(s, t) = \frac{\rho}{1 - e^{-\rho T}} [h(s, s) - ER(s, T + s)] R(t, s) e^{-\rho(t-s)}, \; s \leq t \leq T + s. \]  \hspace{1cm} (2.17)

From (2.10) and (2.7), it follows that:

\[ c(s, t) = \frac{\rho}{1 - e^{-\rho T}} [h(s, s) - ER(s, T + s)] R(t, s) e^{-\rho(t-s)}, \; t > T + s. \]  \hspace{1cm} (2.18)

There are obviously no breaks in the consumption path.

From (2.11), it follows:

\[ h(s, s) - ER(s, T + s) = \frac{T_1}{T} w(s, s). \]

Therefore:

\[ c(s, t) = \frac{1}{T} w(s, s) R(t, s) e^{-\rho(t-s)}, \forall t. \]  \hspace{1cm} (2.19)

Integrating the budget restriction between \( s \) and \( t \) for \( s \leq t \leq T + s \), and remembering that

\[ [R(s, t)]^{-1} = R(t, s) \]
It follows:

\[ v(s, t) = \int_s^t y(s, t') R(t, t') dt' - \frac{1}{T} w(s, s) \frac{1 - e^{-\rho(t-s)}}{\rho} R(t, s), \quad s \leq t \leq T + s. \quad (2.20) \]

Analogically, for \( t \geq T + s \) it follows:

\[ v(s, t) = \frac{1}{T} e^{-\rho(t-s)} R(t, s) w(s, s) - \int_t^\infty x(s, t') R(t, t') e^{-\rho(t'-t)} dt', \quad t > T + s. \quad (2.21) \]

3. Aggregation

3.1 Population

At every moment, \( n \) people are being born. The number of individuals living in \( t \), born in \( s \) will be:

\[ n \quad \text{if } t - T \leq s \leq t \]
\[ ne^{-\rho(t-(T+s))} \quad \text{if } s < t - T \]

Therefore, the population will be:

\[ N = \int_{t-T}^t nds + \int_{-\infty}^{t-T} ne^{-\rho(t-(T+s))} ds = n(T + p^{-1}) \quad (3.1) \]

To normalise, consider \( n = \frac{1}{T+p} \).

3.2 Aggregated Consumption and Wealth

It follows:

\[ c(t) = \int_{-\infty}^{t-T} \frac{e^{-\rho(t-(T+s))}}{T + p^{-1}} c(s, t) ds + \int_{T-T}^t \frac{c(s, t)}{T + p^{-1}} ds \]
\[ = \frac{1}{T(T + p^{-1})} \left\{ \int_{-\infty}^{t-T} e^{-\rho(t-(T+s))} w(s, s) e^{\int_s^t m(t') dt'} e^{-\rho(t-s)} ds \right. \]
\[ + \int_{t-T}^t w(s, s) e^{\int_s^t m(t') dt'} e^{-\rho(t-s)} ds \left\} \quad (3.2) \]
Analogically, for wealth it follows:

\[ v(t) = \frac{1}{T+p^{-1}} \int_{-\infty}^{t-T} e^{-\rho(t-(T+s))} \left[ \frac{1}{T} \rho + p \right] e^{\int_{t}^{s} m(t')dt'} w(s, s) \]

\[ - \int_{t}^{\infty} e^{-\int_{i}^{t'} (m(t')+p)dt''} x(s, t')dt' ds \]

\[ + \frac{1}{T+p^{-1}} \int_{t-T}^{t} \left[ \int_{s}^{t} e^{-\int_{s}^{t'} m(t'')dt''} y(s, t')dt' - \frac{1}{T} \left( 1 - e^{-\rho(t-s)} \right) e^{\int_{s}^{t} m(t')dt'} \right] ds \]  

(3.3)

3.3 The Equilibrium Condition

To have equilibrium, it is necessary that:

\[ v(t) = k(t) \]  

(3.4)

and

\[ \dot{k}(t) = f(k(t)) - \delta k(t) - r(t) \]  

(3.5)

where:

\( f(\cdot) \) - production function;

\( \delta \) - depreciation rate.

Note that (3.4) results in (3.5), but the contrary is not necessarily true. A constant is lost when going from (3.4) to (3.5), which is not necessarily recovered when going back to (3.4).

4. The Stationary State Solution

4.1 Equilibrium Condition

It is possible to greatly simplify equations (3.2) and (3.4) in the stationary state. Given that the capital/labour relation is fixed in the stationary state, from (3.2) it follows that⁶:

\[ c^* = \frac{\omega^*}{(T + p^{-1}) T} \left\{ \frac{1 - e^{-(\rho - m^*)T}}{\rho - m^*} + \frac{e^{-(\rho - m^*)T}}{\rho + p - m^*} \right\} . \]  

(4.1)

⁶\( a^* \) indicates the value of the variable \( a \) in the stationary state.
it is a necessary condition for the stationary state. To calculate (4.1), the following condition is used:

\[ \rho + p - m^* > 0. \] (4.2)

So that the consumption is bounded.

From (3.3), it follows:

\[
v^* = \frac{e^{m^* T}}{T + p^{-1}} \left\{ \int_{0}^{t-T} e^{-e^{t-(T+s)}} \left[ \frac{1}{T} \frac{e^{-p(t-s)}}{\rho + p} e^{-m^* w^*} \right. \right.
\]
\[- \left. e^{p t} x^* \int_{1}^{\infty} e^{-(m^* + p) t'} dt' \right] ds + \int_{0}^{t} \left[ \int_{s}^{t} y^* e^{-m^* t'} dt' \right. \right.
\[- \left. \frac{1}{T} \frac{1 - e^{-p(t-s)}}{\rho} e^{-m^* w^*} \right] ds \}.
\]

In order to know whether the solution for (4.1) agrees with (3.4), the previous expression is derived related to time. Then, after substituted (4.1), it follows:

\[ 0 = m^* v^* + \frac{p^{-1}}{T + p^{-1}} x^* + \frac{T}{T + p^{-1}} y^* - c^*. \] (4.3)

But

\[ c^* = f(k^*) - \delta k^* \]
\[ y^* \frac{T}{T + p^{-1}} = (1 - \tau)[f(k^*) - k^* f'(k^*)] \]
\[ x^* \frac{p^{-1}}{T + p^{-1}} = \tau[f(k^*) - \delta k^*] \]
\[ m^* = (1 - \tau) r^* = (1 - \tau)[f'(k^*) - \rho]. \] (4.4)

Where:

\( \tau \) - rate of income tax;
\[ \frac{T}{T + p^{-1}} \] - working population;
\[ \frac{p^{-1}}{T + p^{-1}} \] - retired population.
Substituting (4.4) into (4.3), it follows:

\[(1 - \tau)[f'(k^*) - \rho](v^* - k^*) = 0 \Rightarrow v^* = k^*, \text{ if } m > 0.\]

From (2.13), it follows:

\[w^* = y^* \frac{1 - e^{-m^*T}}{m^*} + x^* \frac{e^{-m^*T}}{m^* + p}. \tag{4.5}\]

In addition to (4.1) and (4.5), if the system takes the form of a pay-as-you-go system, from (2.16), it follows that:

\[x^* \frac{e^{-m^*T}}{m^* + p} = \frac{w^*}{T} \frac{e^{-pT}}{\rho + p}. \tag{4.6}\]

This condition ensures that the state transfer payment value is exactly sufficient to make sure that individuals do not leave any resources for the second period.

Note that in (4.1), the marginal propensity to consume of wealth does not depend on how the social security is established. However, when income is taken into consideration, the same does not occur. From (4.6) and (4.5), it follows that:

\[\text{If } x^* = 0 \Rightarrow \frac{w^*}{T} = \frac{y^*}{T} \frac{1 - e^{-m^*T}}{m^*}. \tag{4.7}\]

\[\text{If } x^* \neq 0 \Rightarrow \frac{w^*}{T} = \frac{y^*}{T} \frac{1 - e^{-m^*T}}{m^*}. \tag{4.8}\]

As \(\overline{T}_1 < \overline{T}\), it follows that the marginal propensity to consume of labour income will be greater in the pay-as-you-go system.

Substituting (4.7) into (4.1) and (4.8) into (4.1) and (4.6), it follows:

Founded system:

\[c^* = \frac{y^*}{(T + p^{-1})\overline{T}} \frac{1 - e^{-m^*T}}{m^*} \left\{ \frac{1 - e^{-(\rho-m^*)T}}{\rho - m^*} + \frac{e^{-(\rho-m^*)T}}{\rho + p - m^*} \right\}. \tag{4.9}\]

Unfounded system:

\[c^* = \frac{y^*}{(T + p^{-1})\overline{T}_1} \frac{1 - e^{-m^*T}}{m^*} \left\{ \frac{1 - e^{-(\rho-m^*)T}}{\rho - m^*} + \frac{e^{-(\rho-m^*)T}}{\rho + p - m^*} \right\}. \tag{4.10}\]
and
\[ x^* e^{-m^* T} \frac{m^* + p}{T^2} \frac{1 - e^{-m^* T}}{m^*} = y^* \frac{T}{T^2} \frac{1 - e^{-m^* T}}{m^*}. \] (4.11)

Supposing \( f(k) = k^\alpha \), from (4.4), it follows that:
\[ \frac{c^*}{y^*} = \frac{f(k^*) - \delta k^*}{f(k^*) - k^* f'(k^*)} \left[ \frac{T + p^{-1}}{T} \right] \frac{1}{1 - \tau} = \frac{T + p^{-1}}{T} \frac{1}{1 - \tau} \frac{1}{1 - \alpha} (1 - \frac{\delta \alpha}{1 - \tau} + \delta). \] (4.12)

and
\[ \frac{x^*}{y^*} = \frac{T \tau}{p^{-1} (1 - \tau)} \frac{f(k^*) - \delta k^*}{f(k^*) - k^* f'(k^*)} = \frac{T \tau}{p^{-1}} \frac{1}{1 - \tau} \frac{1}{1 - \alpha} (1 - \frac{\delta \alpha}{1 - \tau} + \delta). \] (4.13)

Since
\[ m^* = (1 - \tau) \epsilon^* = (1 - \tau) [f'(k^*) - \delta]. \]

Note that in (4.11), if \( m^* = \rho \), it follows that the state transfer payment has to have the exact same salary value so to make people leave nothing for the second period of life.

5. Calculation of the Stationary State Income

Considering the following configuration:
\[ \delta = 0.05 \]
\[ \rho = 0.04 \]
\[ p = 0.05 (p^{-1} = 20 \text{ years}) \]
\[ T = 35 \text{ years} \]
\[ \alpha = 0.6 \]
\[ m^* = r^* = 0.05103 \text{ is obtained} \] and, hence is 14.736 for the income in the founded, and \( m^* = 0.06561, \tau^* = 0.3975 \) and 7.422 for the income in the pay-as-you-go system. In this configuration, there is a 100% gain in the stationary state income related to the changes in the pay-as-you-go system.

\[ \text{The value of } m^* \text{ in the founded system is calculated } r^* \text{ from (4.9) and (4.12). To calculate and } m^* \text{ in the pay-as-you-go system, simply substitute (4.13) into (4.11) and (4.12) into (4.10). The resulting non-linear system is solved for } r^* \text{ and } m^*. \]
<table>
<thead>
<tr>
<th>( p )</th>
<th>( \tau )</th>
<th>( m )</th>
<th>( y^u )</th>
<th>( \tau )</th>
<th>( y^F )</th>
<th>( y^u/y^F )</th>
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<td>0.51</td>
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</table>

Note that the relative income behaviour \( (y^u/y^F) \) is not monotonic for changes in \( p \) (table 1), and in \( \alpha \) (table 4). Given that the pay-as-you-go system does not represent an efficient Pareto allocation, comparative statics may produce counter intuitive results as is common in second best situations.

The relative income sensitivity is low for parameter \( p \) of reasonable variation band (table 1). This sensitivity is very low regarding variations in the time preference rate (table 4).

The fraction of capital income is the critical parameter (tables 2 and 3). For both cases studied (\( p = 0.05 \) [Table 2] and \( p = 0.1 \) [Table 1] ), when the income fraction of the factors of production, which have been accumulated, rises, the impact on relative income is greater. This effect is quantitively quite strong. Considering capital in a broad sense (i.e. an aggregate of human and physical capital) it is reasonable to use \( \alpha > 0.5 \). If \( 4/5 \) of the income are factor rewards which have been accumulated due to rational economic decisions, the change in the social security system produces a 300% income gain (if \( p = 0.05 \)) or even up to a 450.

Note the slight variation of the rate of the income tax in both cases (\( p = 0.05 \) and \( p = 0.1 \)). The difference between the greatest value (\( \tau = 0.42 \)) and the smallest (\( \tau = 0.32 \)), taking the average of the two extremes into consideration, it is 2.

### 6. Conclusion

It has been shown that a change, such as was established in the retirement system, may produce significant income gains in the stationary state. Two economies, which
Given that the model is very simple, various simulations can be performed. Table 1 illustrates the results when $p$ changes, all other parameters remaining constant. In table 2 $\alpha$ changes and, in table 3 $\alpha$ changes, considering $p = 0.1$ (i.e. $p^{-1} = 10$). Obviously, the rate column refers to the following result in the unfunded system, and $y^u$ corresponds to income in this system.

**TABLE 1**

<table>
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<th>$p$</th>
<th>$\tau$</th>
<th>$m$</th>
<th>$y^u$</th>
<th>$r$</th>
<th>$y^F$</th>
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<td>0.1101</td>
<td>6.80</td>
<td>0.1053</td>
<td>7.69</td>
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<td>2.00</td>
<td>0.04</td>
<td>0.1097</td>
<td>7.07</td>
<td>0.1071</td>
<td>7.55</td>
<td>0.93</td>
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</table>

**TABLE 2**

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>$\tau$</th>
<th>$m$</th>
<th>$y^u$</th>
<th>$r$</th>
<th>$y^F$</th>
<th>$y^u/y^F$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.3</td>
<td>0.42</td>
<td>0.0540</td>
<td>1.38</td>
<td>0.0271</td>
<td>1.80</td>
<td>0.77</td>
</tr>
<tr>
<td>0.4</td>
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<td>0.0579</td>
<td>1.93</td>
<td>0.0351</td>
<td>2.83</td>
<td>0.68</td>
</tr>
<tr>
<td>0.5</td>
<td>0.41</td>
<td>0.0618</td>
<td>3.25</td>
<td>0.0430</td>
<td>5.45</td>
<td>0.60</td>
</tr>
<tr>
<td>0.6</td>
<td>0.40</td>
<td>0.0656</td>
<td>7.41</td>
<td>0.0510</td>
<td>14.74</td>
<td>0.50</td>
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<td>0.37</td>
<td>0.0697</td>
<td>31.67</td>
<td>0.0593</td>
<td>78.12</td>
<td>0.41</td>
</tr>
<tr>
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<td>0.32</td>
<td>0.0742</td>
<td>656.75</td>
<td>0.0682</td>
<td>2188.44</td>
<td>0.30</td>
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**TABLE 3** $p = 0.1$

<table>
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<th>$m$</th>
<th>$y^u$</th>
<th>$r$</th>
<th>$y^F$</th>
<th>$y^u/y^F$</th>
</tr>
</thead>
<tbody>
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<td>0.33</td>
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<td>1.35</td>
<td>0.0410</td>
<td>1.68</td>
<td>0.81</td>
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<tr>
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<td>0.0742</td>
<td>1.82</td>
<td>0.0503</td>
<td>2.53</td>
<td>0.72</td>
</tr>
<tr>
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<td>0.37</td>
<td>0.0817</td>
<td>2.81</td>
<td>0.0600</td>
<td>4.60</td>
<td>0.61</td>
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<tr>
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<td>0.0700</td>
<td>11.35</td>
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<td>0.0812</td>
<td>50.81</td>
<td>0.36</td>
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<tr>
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<td>214.46</td>
<td>0.0943</td>
<td>976.42</td>
<td>0.22</td>
</tr>
</tbody>
</table>

**TABLE 4**
are identical from the preferences, technology and initial endowment points of view, will present quantitively different stationary state incomes.

Three limitations in the current study must be solved:

i.) The introduction of population growth;

ii). The study of sensitivity in relation to the instantaneous elasticity substitution of consumption;

iii) The study of sensitivity in relation to the elasticity of substitution capital/labour.

It is believed that these generalisations produce factual value for income tax rate values.

References


Autor: Pessoa, Samuel de Abreu.
Título: The impact on the stationary state income of a