Seminários de Pesquisa Econômica I
(2ª parte)

"PRICE BEHAVIOR UNDER SEARCH AND INFLATION"

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DATA:  28/04/94 (quinta-feira)

HORÁRIO: 15:30h

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Price Behavior under Search and Inflation

Sheila Najberg
I. INTRODUCTION

It is a well known fact that the microeconomic background of an inflationary process is characterized by discrete jumps in individual prices. Sheshinski and Weiss' (1977) celebrated article focused economists' attention on the fact that, in an inflationary economy, firms' follow a policy whereby the nominal price is fixed over intervals, followed by discrete price adjustments between periods. The real price thus fluctuates between two fixed bounds:\((s,S)\) decreasing continuously over each interval. A rise in the inflation rate has the effect of increasing price dispersion. The new pricing policy is characterized by both a higher upper bound \(S\) and a smaller lower bound of prices \(s\).

When consumers acquire price information through costly sequential search, staggered \((s,S)\) pricing behavior has an important effect on consumers' incentive to search. A rise in the inflation rate induces consumers to search through firms more aggressively (Benabou [1988]) and to recall previously visited firms (Bonomo & Najberg [1993]) since there is a possibility of finding lower real prices in the market. Under the assumption of homogeneity of buyers and sellers, these models predict that inflation is entirely to consumers' advantage and to the detriment of firms.

In contrast, this paper demonstrates that, with heterogeneity of buyers, this result does not hold true.\(^1\) We show that an increase in the inflation rate, reduces both the welfare of non-searchers and firms' profits. For the searching consumers the negative effect is to make search a more time-consuming activity.

Under some very broad assumptions, firms increase the frequency of their price adjustments as inflation rises (Leidermen [1994]). Given the assumption that the

\(^1\) The hypothesis of heterogeneity of firms together heterogeneity of consumers is investigated in Najberg (1994). The results are very similar to those presented in this paper.
process of adjusting prices is costly, we will show that an increase in the inflation rate will induce firms to charge higher real prices and therefore sell to consumers with higher willingness to pay.

In order not to complicate the model, we make the simple assumption of two classes of consumers that differ in the levels of their search cost: consumers with high willingness to pay - "non-searchers" - and consumers with low willingness to pay - "searchers". At low inflation rates, a unique pooling equilibrium arises, with all firms setting the same \((s,S)\) path of prices and selling to all consumers. Under a **one-band pricing distribution**, inflation benefits consumer and reduces firms' profit, as in Benabou (1988). However, as inflation increases, this equilibrium is disrupted and a partial separating equilibrium characterized by a **two-band pricing distribution** emerges. In this equilibrium, consumers with low search cost purchase from firms that are selling in the lower band, and non-searchers purchase from firms they randomly happen to visit. The higher the inflation rate, the less likely it is to find firms choosing the lower-band of prices.

Although this model is not a direct extension of Salop and Stiglitz's "Bargain and Ripoffs" (1977) model, we obtain some similar results. While in their model both single-price equilibria and two-price equilibria are possible, we show that with inflation, single-band price equilibria and two-band price equilibria exists.

In contrast to Tommasi (1992), our result show that, on average, at higher inflation rates, consumers with low search cost will still purchase at lower prices, although at the expense of increasing search expenditures. In his model, higher inflation induces searching consumers to search less.

Further, a literature based on the microfoundations of the welfare impact of inflation has emerged in recent years. Most related to our model is Benabou (1992) and Ball and Romer (1992). In the first one, under the assumption of free entry, heterogeneity among buyers and homogeneity of firms, all sellers follow the same \((s,S)\) pricing rule. The model predicts that an increase in the inflation rate reduces both
upper and lower bound of prices and that profit is restored through the exit of firms. Increase in search by low search cost buyers and exit of firms limits equilibrium price dispersion and total resource spent on search. In our model, firms are able to change price strategy and still survive in the market. Higher inflation rate leads to fewer firms following the low-price strategy, which in turn increases the amount spent on search.

Using a discrete time model, Ball and Romer assume that firms adjust their prices every two periods, and the fact that consumers have equal chances of meeting the firm in either period brings benefit for the buyers. Their result is a simple case of the microeconomic principle that indirect utility functions are quasi-convex in prices (Varian (1984)). Consumers gain with inflation since they benefit from relative price variability. Although their model assumes heterogeneity on both demand and supply side of the market, they do not investigate the possibility that some firms, with an increase in the inflation rate, may prefer to increase their prices at a cost of losing a proportion of their consumers.

The remainder of the paper is organized as follows. Section II presents the model. The existence of equilibrium with both types of firms following a unique (s,S) rule ("full overlap equilibrium") is analyzed in section III. Section IV analyzes equilibria with two price bands, in which searchers purchase from firms that choose the low band ("non-overlap equilibrium"). Simulations results are presented in section V, and section VI is left for concluding remarks. Formal proof of all propositions in the text are given in the Appendix.

II. THE MODEL

We assume that a homogenous good is produced by a continuum of identical infinitely-lived firms and that the economy is on a steady perfectly anticipated inflationary path where all aggregate prices grow at a rate of \( \pi > 0 \). Firms have a
marginal cost \( c \), expressed in real terms, a discount rate \( r \) and pay \( \beta \) to adjust nominal prices.

Following Sheshinski and Weiss (1977), we assume that, in an inflationary economy, firms' optimal price policy is to follow an \((S,s)\) rule, where they adjust their nominal prices so as to achieve a real value \( S \), every time this real value has been eroded to \( s < S \). The length of time for adjustment of prices is:

\[
T = \ln \left( \frac{S}{s} \right) / r.
\]

We will require that the cross-sectional distribution of the firms' real prices be invariant over time. Given the constant rate of inflation, this corresponds to price adjustments which are uniformly staggered (Rotemberg(1983)) and the invariant distribution is the log-uniform on \((s,S)\) (Caplin and Spulber(1987)):

\[
dF(x) = \frac{dx}{x \ln(S/s)} \quad \text{for all } x \in (s,S).
\]

The demand side consists of a continuous flow into the market of \( \lambda \) consumers per firm, who enter the market to make a one-time purchase of one unit of the good. Moreover, proportion \((1-w)\) of the consumers are assumed to have such a high search cost that they will purchase as long the real price does not exceed their preference reservation price \( Z \). The proportion \( w \) of the population that searches has a constant real cost of searching \( \gamma \), and will purchase for offers below the minimum of their search reservation price \( x_s \) and \( Z \). Moreover, we assume consumers' search to be sequential, and the first quotation to be free.

\[\text{Footnote: 2 We assume that search is costly because it is time-consuming. The hypothesis that buyers' search cost } \gamma \text{ is constant, implicitly assumes that consumers have savings and income completely indexed and therefore are "inflation-protected". If not, the search cost should change accordingly to the indexation system.}\]
III. FULL OVERLAP PRICING POLICY

This section will investigate the existence of an equilibrium in which all firms follow an \((s,S)\) rule with \(S = x_s\), thus selling to both types of consumers, at all times.

Figure 1 - Full Overlap Pricing Policy

III-a. CONSUMERS' STRATEGY

Given that searching consumers conjecture that firms follow the above pricing policy, their reservation price \(x_s \leq Z\) is determined by: ³

\[
V = \gamma + \int_{x_s}^{x} x \cdot dF(x) + \int_{x_s}^{S} V \cdot dF(x)
\]

\[
\int_{x_s}^{s} F(x) \, dx = \gamma.
\]

It should be noted that the higher consumers' search cost \(\gamma\), the higher is their reservation price \(x_s\). Nevertheless, consumers will not purchase for prices above their preference reservation price \(Z\), thus \(x_s \leq Z\) must be imposed.

³ For details see Sargent (1987).
Define $\gamma_p$ as consumers' search cost for which $x_s = Z$.

$$\int F(x)dx = \gamma_p.$$  \hspace{1cm} (4)

For $\gamma < \gamma_p$ it follows that $x_s < Z$, while for $\gamma \geq \gamma_p$ it follows that $x_s = Z$.

![Figure 2 - Consumers' Reservation Price for full overlap price conjectures](image)

Consumers' strategy for a full overlapping pricing policy is summarized below:
1) for search cost $\gamma \in (0, \gamma_p)$, then $x_s \in (s, Z)$ and searching consumers will purchase for offers $x \leq x_s$ and search for $x > x_s$. Consumers with high willingness to pay ("non-searchers") will buy for price offer $x \leq Z$;
2) for search cost $\gamma \geq \gamma_p$, then $x_s = Z$ and all consumers will buy for price offer $x \leq Z$.

**III-b FIRMS' STRATEGY**

As seen above, for $\gamma \geq \gamma_p$, buyers will purchase for any price offer below their preference reservation price $Z$. Under the assumption of homogeneity of buyers, it is not optimal for the firms to set prices above $Z$, since no sales will take place for prices above the maximum consumers are willing to pay.

**LEMMA 1.** No firm will, in their optimal pricing strategy $(s, S)$, set $S > Z$.

**Proof:** See the appendix.
Therefore, as long as firms' upper bound of prices is not above $Z$, each firm should expect to sell to $\lambda$ consumers per period. Thus, we define the **upper band of prices** as the $(s, S)$ rule in which $S=Z$. The lower bound $s$ is obtained by maximizing the firms' intertemporal profits $V_Z$ given by:

$$V_z = -\beta + \lambda \int_0^T (Ze^{-rt} - c)e^{-rt} dt + V_z e^{-rT}.$$  \hspace{1cm} (5)

Let $T_z^*$ be the optimal periodicity of price adjustment for firms that set $S=Z$.

**THEOREM 1**: Assume consumers do not search and purchase for offers below $Z$. For sufficiently small adjustment cost, such that $\beta \leq \lambda Z/(r+\pi)$, the optimal pricing policy of a firm that sets $S=Z$ and $z=Ze^{-\sigma r}$ is determined as the unique solution to

$$rV_z = \lambda (Ze^{-\sigma r} - c).$$ \hspace{1cm} (6)

**Proof**: See the appendix.

For $\gamma < \gamma_p$, searching buyers have a reservation price $x_s < Z$, and each firm should expect to sell to $\lambda$ consumers per period, as long as their upper bound of prices is not above $x_s$. Hence, we define the **lower band of prices** as the $(s, S)$ rule in which $S=x_s$.

Let $T^*$ be the optimal periodicity of price adjustment for firms set $S=x_s$, under the assumption that searching consumers' reservation price is $x_s < Z$.

**COROLLARY**: Assume searching consumers have a search reservation price $x_s < Z$. For sufficiently small adjustment cost, such that $\beta \leq \lambda x_s/(r+\pi)$, the optimal pricing policy of a firm that sets $S=x_s$ and $s=x_se^{-\pi r}$ is determined as the unique solution to

$$rV_{x_s} = \lambda (x_se^{-\pi r} - c).$$ \hspace{1cm} (7)
The proof is obtained by replacing $Z$ by $x_s$ in the proof of Theorem 1.

Notice that a change from the lower band $(s, x_s)$ to the upper band of prices $(z, Z)$, under the assumption that searching consumers reservation price satisfies $x_s < Z$, leads to an increase in the path of prices at the expense of a reduction in sales. Searchers will only purchase for prices $\in [z, x_s]$, while non-searchers will continue to purchase for prices $\in [z, Z]$.

Assume $x_s$ is sufficiently low so that $x_s \leq z$. It follows that a firm can choose the upper band $(z, Z)$ and sell only to non-searchers, or choose the lower band $(s, x_s)$ and sell to searchers and non-searchers. Firms that choose the upper band $(z, Z)$ will optimally obtain the lower bound $z$ by maximizing the firms intertemporal profits $V_z$ now given by:

$$V_z = -\beta + (1 - w)\lambda \int_0^T (Z e^{-m} - c) e^{-\gamma t} dt + V_z e^{-\gamma t}$$

(8)

Firms will follow this pricing strategy if it does not decrease their profits, i.e. if $V_{x_s} \leq V_z$, which implies

$$(x_s e^{-\gamma t} - c) \leq (1-w)(Z e^{-\gamma t} - c).$$

(9)

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4 It will be shown, further in the paper, that $x_s \leq z$ holds in equilibrium for an important range of parameters values. The case $x_s > z$ introduces some new complications that are briefly examined at the end of the appendix.

5 Notice that the values of the upper band of prices changes when the market is composed only of non-searchers versus when it has a proportion $w$ of searchers. A higher $w$ reduces the sales of firms that follow the upper band, thus increasing the ratio "adjustment cost/ sales". Firms will then prefer to delay price adjustment and will optimally allow for a smaller $z$. 

Using (9), let us define $x_s^H$ as the reservation price at all firms are indifferent between the lower band $(s, x_s)$ and upper band $(z, Z)$. For $x_s \geq x_s^H$ all firms will follow the same pricing policy $(s, x_s)$. Given that a reduction in $x_s$ reduces the profit of the firms that choose the lower band, it follows that for $x_s \leq x_s^H$ firms will prefer $(z, Z)$ to $(s, x_s)$.

Finally, let us now investigate how a change in the inflation rate affects firms behavior.

**Lemma 2.** An increase in the rate of inflation decreases the profit of firms that follow the strategy of setting $S = x_s$. The optimal lower bound of prices is reduced with a rise in the inflation rate. The effect on consumers is an increase in their welfare.

**Proof:** See the appendix.

**III-c. Equilibria**

An equilibrium is a price band $(s, S)$ with $s \leq S \leq Z$, which satisfies:

a) For the case $S = x_s = Z$:
   a1) optimality of price adjustment: $s = Z e^{-\pi r'}$, $e^{-\pi r'}$ determined by equation (a5);
   a2) best response for consumers: search cost $\gamma > \gamma_p$.

b) For the case $S = x_s < Z$:
   b1) $s = x_s e^{-\pi r'}$, with $e^{-\pi r'}$ determined by equation (a5), replacing $Z$ with $x_s$;
   b2) search cost $\gamma < \gamma_p$;
   b3) incentive compatibility for firms' choice of price band: $x_s \geq x_s^H$ determined by equation (9).

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6 A reduction in $x_s$ increases $T^*$, which in turn reduces even more the profit of firms that choose the lower band of prices. See proof of theorem 1, in the appendix.

7 Equations starting with the letter "a" can be found in the Appendix.
In an one-band pricing equilibrium all buyers accept the first price offered, and search is only a credible threat constraining S. With an increase in the level of inflation, if the pooling equilibrium can be sustained, we will observe a reduction of the whole \((s, S)\) price path. As in Benabou's model (1988), ours also predicts that for firms that follow the strategy of selling to all types of consumers, an increase in the inflation rate reduces their profit. With no resources being spent on search, to purchase at the new lower equilibrium prices, we also observe an increase in consumers welfare.

Simulations results presented in section V provide a sense of the magnitude of parameters needed for the pooling equilibrium.

**IV- NON-OVERLAPPING PRICING POLICIES**

This section will investigate the existence of an equilibrium in which some firms follow a higher pricing band \((s_H, S_H)\) while others follow a lower pricing band \((s_L, S_L)\), with the additional assumption that prices do not overlap, i.e. \(S_L < S_H\).  

\[ \begin{array}{c}
\text{s}_H \\
\text{s}_H \\
\text{s}_L \\
\text{s}_L \\
\end{array} \]

Figure 3 - Non-overlapping pricing policies

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8The case of partial overlapping prices is not considered in this paper.
IV-a. CONSUMERS' STRATEGY

Given that consumers conjecture that firms follow the above pricing policies, those with a low cost of searching will buy from firms that follow the lower band. Under the assumption of non-overlapping pricing policies, the searchers that flow in the market will be evenly divided among the firms that sell at lower prices, while the non-searchers will be distributed among all firms.

The higher consumers' search cost, the higher is searching consumers' willingness to pay and the higher the proportion of firms that will be interested in selling to searchers and non-searchers. In the limit, for sufficiently high search costs, we will show that all the firms in the market will be interested in selling to both types of consumers.\(^9\)

Thus, for low enough search cost so that, at most, only a proportion \(\alpha\) of the firms will be choosing the lower band, and continuing to require that \(x_s \leq Z\), we will have for searching consumers:

\[
V = y + \alpha \int_{x_s}^{x_f} x \cdot dF_L(x) + \int_{x_s}^{s_L} V \cdot dF_L(x) + (1 - \alpha) \int_{s_H}^{s_H} V \cdot dF_H(x)
\]

\[
\alpha \int_{x_s}^{x_f} F_L(x) \, dx = y
\]

For example, under the conjecture that \(\alpha=1/2\), \(\text{i.e.} 50\%\) of the firms follow the lower pricing strategy, searching consumers' cost \(y_{0.5}\) satisfies:

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\(^9\) It should be noted that we are assuming two-band pricing. The first band corresponds to an uniform distribution \(F_L\) at lower levels of prices, with offers \(e \in (s_L, s_L)\). The second, \(F_H\), corresponds to an uniform distribution at higher levels of prices, with offers \(e \in (s_H, s_H)\). Both distributions follow equation (2).
For high enough search cost all firms will choose to sell to all consumers. Since searchers will not purchase for prices above $x_s$, we obtain that for $\alpha=1$

$$\int_{S_L}^{S_L} F_L(x)dx = 2\gamma_{0.5} \label{eq:10}$$

At most, for $\gamma \geq \gamma_{all} \geq \gamma_p$, we obtain $x_s=Z. \label{eq:11}$

Consumers' strategy, given the non-overlapping pricing policy, is summarized below:

1) for search cost $\gamma \in (0, \gamma_{all})$, $x_s \leq S_L$ searchers will purchase from the proportion $\alpha$ of firms that choose the lower band of prices, while non-searchers will purchase from any firm, for offers $x \leq Z$.

3) for search cost $\gamma \geq \gamma_p$, then $x_s=Z$ and all consumers will buy for price offer $x \leq Z$.

\footnote{For the precise definition of $\gamma_p$ see equation (4).}
IV-b. FIRMS' STRATEGY

Suppose a proportion \( \alpha \) of the firms set \( S_L \leq x_s \), while \( (1-\alpha) \) of them set their upper bound of price at \( S_H \), with \( S_H > S_L \). Then searching consumers will end up evenly divided among the \( \alpha \) firms. The flow of consumers per firm \( \lambda \) corresponds to the total number of consumers that flow in the market per period \( (#C) \) divided by the number of firms in the market \( (#F) \). Since a proportion \( (1-w) \) of these consumers are non-searchers, all firms in the market are expected to sell at least \( (1-w)\lambda \) units. The searchers in the market \( (w#C) \) will be evenly distributed among the firms that set their upper bound of prices below or at \( x_s \). If only a proportion \( \alpha \) of the total number of \( (#F) \) follow this pricing strategy, then their sales to non-searchers and searchers are expected to be \( (1-w)\lambda + \left[ w#C \right] / \left[ (#F)\alpha \right] = (1- w + w/\alpha)\lambda \).

**LEMMA 3.** No firm will, in its optimal pricing strategy \((S_L, S_L)\), set \( S_L < x_s \).

**Proof:** See the appendix.

Analogous to the case of full overlapping pricing policy, using theorem 1, we obtain that for sufficiently small adjustment cost, such that \( \beta \leq (1-w + w/\alpha)\lambda x_g / (r+\pi) \), the optimal pricing policy of the proportion \( \alpha \) of the firms that sets \( S_L = x_s \) and sets \( s_L = x_s e^{-\pi T*} \) is determined as the unique solution to

\[
 rV_{x_s} = (1-w + w/\alpha)\lambda (x_s e^{-\pi T*} - c). \tag{12}
\]

The optimal periodicity of price adjustments given by:

\[
e^{-\pi T*} \cdot \frac{\pi}{\pi + r} e^{-(\pi + r)T*} = r \left( \frac{1}{\pi + r} - \frac{\beta}{(1-w + w/\alpha)\lambda x_s} \right) \tag{13}
\]

Notice that, in (13), \( \partial \text{RHS}/\partial x_s > 0 \), since \( \partial \text{LHS}/\partial T* < 0 \), we conclude the lower is \( x_s \) the less frequent are the price adjustment of firms that set \( S_L = x_s \).
Similar to the full overlapping case, an increase in the inflation rate reduces the profit of those firms that follow the strategy of selling to all types of consumers. Besides the increase in $\pi$ that per se reduces the profit -see equation (12)-, it decreases $x_s$ and delays price adjustment (increase in $T^*$). The final result is that, at a rise in the level of inflation, firms that are selling to searchers and non-searchers will reduce both their upper and lower bounds of prices.

We now further characterize low-cost firms' optimal pricing strategy.

**Lemma 4.** No firm will, in their optimal pricing strategy $(s_L, s_H)$, set $s_L > Z$.

*Proof:* See the appendix.

Since we have shown that $s_L < x_s$ and $s_L > Z$ are not optimal, we finally need to consider policies $s_L \in [x_s, Z]$ with $s_L = S_L e^{-\pi T^*}$. The purpose of the next lemmas is to show, that under not very restricted conditions, firms will maximize their profit either by setting very high prices and selling exclusively to non-searchers or by offering low enough prices to attract all consumers. In other words, we will demonstrate, that under fairly broad conditions, firms will have two local maximum consisting of the bands $(s_L, x_s)$ and $(s_H, Z)$.

**Lemma 5.** If the search reservation price is so low that firms prefer to sell only to non-searchers, the optimal strategy is to set their upper level of prices at the maximum value that non-searchers are willing to pay.

*Proof:* See the appendix.

Again, in this case, the optimal pricing policy can be established with the use of Theorem 1. For sufficiently small adjustment cost, such that $\beta \leq (1-w)\lambda Z/(r+\pi)$, the optimal pricing policy of a firm that sets $s_L = Z$ and obtains $s_L = Z e^{-\pi T^*} \geq x_s$, is determined as the unique solution to $r V_z = (1-w)\lambda (Z e^{-\pi T^*} - c)$.  

(13)
By analogy, \( e^{-nT^*} - \pi/\pi(\pi+r) e^{-n(\pi+r)} = r[1/(\pi+r) - \beta/(1-w)\lambda Z] \).  

(14)  

**Lemma 6.** For sufficiently high search reservation price so that \( x_S \geq s_L \), for any choice of upper bound of prices, and if \( \frac{\partial V_S}{\partial S_L} |_{s_L=x_S} < 0 \) holds, then firms will either be following the price strategy \((Z e^{-nT^*}, Z)\) or \((x_S e^{-nT^*}, x_S)\).

**Proof:** See the appendix.

The above lemmas show us that if consumers have small search cost, a proportion of the firms will prefer to sell only to non-searchers thus will follow the high band \((s_H, Z)\).

If consumers have higher search cost, these firms will optimally decide between selling to searchers and non-searchers at a lower price band \((s_L, x_S)\), or to follow the higher price band \((s_H, Z)\). The next lemma precisely defines the size of the search cost that makes low costs firms prefer one strategy to the other.

**Lemma 7.** A proportion \( \alpha \) of the firms will set \( S_L = x_S \) with \( s_L = x_S e^{-\pi T^*} \)

\[
\begin{align*}
\text{if } & \quad \frac{\partial V_S}{\partial S_L} |_{s_L=x_S} < 0 \quad \text{and if} \\
& \quad s_L \geq \frac{(1-w)}{(1-w + \frac{w/\alpha}{\alpha})} (s_H - c) + c \\
& \quad (15)
\end{align*}
\]

**Proof:** See the appendix.

Assume \( s_L \geq [(1-w)/(1-w + w/\alpha)](s_H - c) + c \), for all \( \alpha \in (0, 1] \). Moreover, assume for \( \alpha = 1 \), that \( s_L = (1-w)(s_H - c) + c \). For firms that set \( S_L = x_S \), a rise in the
inflation rate decreases $x_s$ and leads to a less frequent price adjustment (equation (13)). This in turn, decreases $s_L$. It is easy to see that a decrease in the entire path of prices can only be sustained with an increase in sales. Thus, at the new equilibrium, fewer firms will be selling at low-prices to the searching consumers and a higher proportion of firms will switching to a high-prices strategy.

Had we initially assumed $s_L>(1-w)(s_H - c) + c$, and if with an increase in the inflation rate the above inequality continued to hold, none of the low-cost firms would change their pricing policy.

We can conclude that increase in the inflation rate above the level at which all firms are indifferent between the pricing policy $(s_H, Z)$ and $(s_L, x_s)$, will have the effect of inducing a proportion of these firms to prefer to offer higher prices.

IV-c. EQUILIBRIA

An equilibrium is a quadruple $E = (x_s, s_L, s_H, \alpha)$, with $s < x_s \leq Z$, which solves:

1) optimality of price adjustment: $s_L = x_s e^{-\pi T^*}$, with $T^*$ determined by (12) and $s_H = Ze^{-\pi T^*_Z}$, with $T^*_Z$ determined by (14).

2) best response for searching consumers: $y = y(s, s_L, x_s)$

3) non-overlapping $(s, S)$ rules: $s_H \geq x_S$

4) conditions for $(s_L, x_s)$ and $(s_H, Z)$ be the only local maximizers: $\frac{\partial V_s}{\partial s_L} \bigg| _{s=s_s} < 0$

5) incentive compatibility partial separating: $s_L \geq (1-w)/(1-w^+ w/\alpha)(s_H-c) + c$

and for $\alpha' > \alpha$ $s_L \leq (1-w)/(1-w^+ w/\alpha')(s_H-c) + c$
Our model predicts that inflation increases production efficiency, because there is a level of inflation at which only high-cost firms will split off to the upper band. Since, at this level, low-cost firms will have an increase their sales, we conclude that increase in the inflation rate increases production efficiency.

In the perfectly separating equilibrium, low costs firms are offering lower prices than high-costs firms. Given our assumption of non-overlapping price distributions, the whole path of prices reveals the firm's type. It is the most productive efficient equilibrium since searchers purchase only from low-costs firms.

Tommasi (1992), in a repeated purchase model, obtains the opposite result. In his model, consumers will optimally search less as inflation rises, since prices do not reveal costs. Firms are then able to increase their prices without loosing consumers. Sales are redistributed from the more efficient to the less efficient firms and in our model, the above result could be obtained, if with an increase in the inflation rate all low-cost firm choose the lower band of prices. Only in this case, our models predicts market failure.

It is interesting to evaluate searching consumers expected total search expenditures (SE).

**PROPOSITION**: Increase in the inflation rate increases expected total search expenditure.

**Proof**: Each buyer with search cost $\gamma$ searches on average $1/F(x_s(\gamma, \alpha))$ times. Since the first one is free, it should be expected that $SE = \gamma \cdot [1/F(x_s(\gamma, \alpha)) - 1]$. Increase in the inflation rate reduces the number of firms that are selling for prices equal or below $x_s$, thus increases $SE$.

In Benabou (1992), under the assumptions of free entry and homogeneity of firms, inflation increases search by low search-cost buyers and the exit of firms. Both
these effects impose a limit for price dispersion and total search. In our model, the existence of a two band price distribution changes this result. We show that the higher the inflation rate the less likely it is to find firms that choose the lower band of prices, thus an increase in the search expenditure should be expected.

Finally, let us see how a reduction in the adjustment cost affects firms' pricing strategy. Clearly, firms will prefer to adjust more often, thus there will be a reduction in the price dispersion of the market. This will increase consumers' search reservation price, which in turn may increase the number of firms offering prices attractive to all types of consumers. In the equilibrium, a higher proportion of firms will be offering attractive prices to all types of consumers. However, given the increase in consumers' search reservation price, the new \((s_l, x_g)\) prices will be higher then before.

Numerical simulations presented in section V provide a sense of the different equilibria that may spring and indicates their magnitude.

V- SIMULATIONS

The purpose of this section is not to match actual data, but simply to explore the various channels through which inflation operates in equilibrium, and provide a sense of the magnitudes involved. Since this section is a reproduction of Najberg (1994), it is assumed that the market is evenly composed by high and low costs firms.

The reference set of parameters is: inflation rates ranges from zero to 40% per period, total number of firms in the market = 10, proportion of low costs firms = 1/2, flow of consumers per period = 1000, proportion of searchers \(w=1/2\), consumers preference reservation price \(Z=100\), price adjustment cost \(\beta = 1000\), production cost of low-cost firms \(c_L = 40\), production cost of high-cost firms \(c_H = 50\) and firms discount rate \(r=0.05\).

Figure 5 and Table 1 show the possible full overlap equilibria.
Figure 5 - Full Overlap Equilibrium (with $\beta \leq \lambda x$ for $\pi + r$)

Table 1 - Consumers' reservation price for full overlap equilibria

<table>
<thead>
<tr>
<th>Inflation (%)</th>
<th>10 firms $S = x &lt; Z$</th>
<th>10 firms $S = x = Z$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00001</td>
<td>0.50</td>
<td>0.50</td>
</tr>
<tr>
<td>1</td>
<td>4.07</td>
<td>4.65</td>
</tr>
<tr>
<td>5</td>
<td>8.94</td>
<td>10.18</td>
</tr>
<tr>
<td>10</td>
<td>12.64</td>
<td>14.32</td>
</tr>
<tr>
<td>15</td>
<td>16.04</td>
<td>17.50</td>
</tr>
<tr>
<td>20</td>
<td>18.47</td>
<td>20.19</td>
</tr>
<tr>
<td>25</td>
<td>21.23</td>
<td>22.56</td>
</tr>
<tr>
<td>30</td>
<td>23.61</td>
<td>24.69</td>
</tr>
<tr>
<td>34</td>
<td>25.74</td>
<td>26.29</td>
</tr>
<tr>
<td>40</td>
<td>27.92</td>
<td>28.50</td>
</tr>
</tbody>
</table>
Result 1. In region I, all the firms in the market follow the same pricing strategy, with the upper bound of prices equal to consumers' preference reservation price \( (S = Z) \). As the rate of inflation rises, the lower bound of prices decreases while the upper bound of prices continues to be equal to \( Z \). Thus, in this region, an increase in the inflation rate reduces real prices and increases consumers' welfare.

Result 2. In region II, all the firms in the market follow the same pricing strategy, with the upper bound of prices equal to consumers' search reservation price \( (S = x_s) \). As the rate of inflation raises, both the lower bound and the upper bound of prices decrease. Thus, in this region, an increase in the inflation rate reduces real price and increases consumers' welfare.

Result 3. In region III, the full overlap equilibrium does not exist. For \((\pi, \gamma)\) close to region II, a proportion of the high-cost firms will prefer to set their upper bound of prices at \( Z \). For an increase in the level of inflation and/or decrease in search cost, a higher number of firms, including the low-cost ones, will prefer to set \( S = Z \).

Figure 6 and Table 2 show the possible non-overlapping equilibria. Under the assumption of heterogeneity of firms' costs, it is intuitive that the first firms to split off to the upper band of prices are those with higher costs. At increasing levels of inflation, also low-cost firms will switch to the upper-band of prices. In the limit, for very high inflation rates, only a proportion of the low-cost firms will be interested in selling at low-band prices.

\[ ^{11} \text{The upper bound of region III (Fig. 7) is strictly above the upper bound of region IV-a (Fig. 8). That is to say, the full overlap equilibria region lays above non-overlapping equilibria region. As the number of firms approaches infinity, the upper bound of region III comes closer to the upper bound of region IV-a.} \]
Figure 6 - Non-Overlap Equilibrium

Table 2 - Consumers' reservation price for non-overlap equilibria
Result 4. Region IV-a is characterized by having a proportion $\alpha_H$ of the high-cost firms only selling to non-searchers ($S=Z$) and all low-cost firms together with the proportion $(1-\alpha_H)$ selling both types of consumers ($S=x_S$). If inflation increases and/or search cost is reduced, the smaller is the proportion of high-cost firms that are willing to sell searchers.

Result 5. Region IV-b is characterized by having all the high-cost firms ($\alpha_H=1$) selling only to non-searchers ($S=Z$) and all low-cost firms selling to both searchers and non-searchers ($S=x_S$). Despite the appearance in Fig. 8, this is a nontrivial region. As we have shown in Lemma 8, there is a range of possible values for the inflation rate that will induce low-cost firms to choose the lower band of prices while high-cost firms choose the upper band.

Result 6. Region IV-c is characterized by having all high-cost firms together with a proportion $\alpha_L$ of the low-cost firms selling only to non-searchers ($S=Z$) and a proportion $(1-\alpha_L)$ of low-cost firms selling to non-searchers and searchers that flow in the market ($S=x_S$). If inflation increases and/or search cost is reduced, the smaller is the proportion of low firms that are willing to sell searchers.

Result 7. In region IV-a and IV-c, the higher the inflation the smaller is the proportion of firms that set their upper bound of prices at $S=x_S$. Thus, an increase in the inflation rate makes it less likely for searching consumers to buy immediately.

Result 8. In region IV-a, an increase in the inflation rate increases production efficiency. A higher inflation rate will induce high-costs firms to sell at higher prices, and searchers will be only purchasing from low-cost firms (region IV-b and IV-c).
Result 9. Region IV-b is the most informative of the regions concerning firm's type. In this region, prices reveals type and searchers will buy only from the low-cost firms.

Result 10. In region IV, increase in the inflation rate will increase cross-sectional variances of prices. At a higher inflation rate, firms that set their upper bound of prices at $Z$ will select a lower $z$, while firms that are selling to both types of consumers will decrease both their upper and lower bounds of prices.

Inflation operates through several channels in equilibrium. The next results derives exclusively from our numerical simulation.

Result 11. In region IV, increase in the inflation rate decreases non-searchers' welfare. According to our simulations, the possible gains, due to the higher price dispersion in the market, are offset by the increase in the quantity of firms that choose the upper band of prices.

Result 12. In region IV, increase in the inflation rate increases searchers' welfare. At a higher inflation, since less firms choose the lower band of prices, it is expected that searching consumers will have to search more and therefore search expenditure will increase. Yet, this negative effect is offset by the possibility of finding lower prices in the market.

Below are two numerical examples to illustrate the above results.
Example 1: Assume a high enough search cost so all low-cost together with a proportion of the high-costs firms choose the lower band of price. Moreover, assume search cost is $\gamma = 7.76$.

For a level of inflation = 5%, we will obtain in equilibrium,
90% firms choose lower band (67.04, 75.5) & 10% firms choose upper band (85.86, 100),
Average price for non-searchers $73.50,
Average price for searchers $71.198 + search expenditures $0.862 \Rightarrow 72.06$

-For a level of inflation = 10%, we will obtain in equilibrium,
69.5% firms choose lower band (63.12, 74) & 30.5% firms choose upper band (80.34, 100),
Average price for non-searchers $75.75,
Average price for searchers $68.418 + search expenditures $3.405 \Rightarrow 71.82$

-For a level of inflation = 15%, we will obtain in equilibrium,
60% firms choose lower band (60.44, 73) & 40% firms choose upper band (76.19, 100),
Average price for non-searchers $76.36,
Average price for searchers $66.52 + search expenditures $5.17 \Rightarrow 71.69
Figure 7 - The effect of inflation on the equilibrium with high-search cost consumers

**Example 2:** Assume a low enough search cost so only low-cost firms choose the lower band of price. Moreover, assume search cost is $\gamma = 1.63$.

For a level of inflation=5%, we will obtain in equilibrium, 50% firms choose lower band (55.19, 61.6) & 50% firms choose upper band (85.86, 100),

Average price for non-searchers $\$ 76.52$,
Average price for searchers $\$ 58.34 + search expenditures $\$ 1.63 \Rightarrow \$ 59.97$. 

For a level of inflation = 10 %, we will obtain in equilibrium,
25% firms choose lower band (48.01, 54.5) & 75% firms choose upper band (80.34, 100),
Average price for non-searchers $ 83.40,
Average price for searchers $ 51.185 + search expenditures $ 4.996 \Rightarrow $ 56.18

For a level of inflation = 15 %, we will obtain in equilibrium,
21.5% firms choose lower band (46.56, 54) & 78.5% firms choose upper band (76.19, 100)
Average price for non-searchers $ 83.79,
Average price for searchers $ 50.12 + search expenditures $ 5.98 \Rightarrow $ 56.10.

For a level of inflation = 20 %, we will obtain in equilibrium,
19.5% firms lower band (45.32, 53.5) & 80.5% firms upper band (72.75, 100),
Average price for non-searchers $ 83.84,
Average price for searchers $ 49.30 + search expenditures $ 6.772 \Rightarrow $ 56.07.
Result 13. A decrease in the cost of adjustment ($\beta$) reduces price dispersion and increases consumers' reservation price. At the new equilibrium, a higher proportion of firms offering lower prices should be found, reducing expected search expenditure (SE) at the expense of purchasing for higher prices. Moreover, a decrease in $\beta$ may reduces production efficiency.

Below are the two-band pricing equilibrium for a market in which $\gamma = 0.67$, $\pi = 25\%$ and

1) $\beta = 1000 \Rightarrow$ 10\% of the firms offer prices $\in (42.91, 49.5)$

90\% of the firms offer prices $\in (69.76, 100)$.

2) $\beta = 100, \Rightarrow$ 22\% of the firms offer prices $\in (50.42, 53.5)$

78\% of the firms offer prices $\in (100, 94.27)$.

A decrease in $\beta$ increases the proportion of firms that choose the lower band of prices. If a decrease in $\beta$ induces also high-costs firms to choose the upper band of prices, production efficiency is reduced.

Below are the average price found by each type of consumers in a market in which $\gamma = 0.67$, $\pi = 25\%$ and

1) $\beta = 1000 \Rightarrow$ non-searchers = $88.48$

searchers = $52.16 (46.13 + 6.03)$.

2) $\beta = 100 \Rightarrow$ non-searchers = $83.86$

searchers = $52.23 (49.85 + 2.37)$.

VI- CONCLUSION

This paper has examined the functioning of a market with heterogeneity of consumers under inflation. For low levels of inflation, a pooling equilibrium exists with all firms selling to all types of consumers. An increase in the inflation reduces firms' profits and increases consumers' welfare. However, above a certain level of inflation this type of equilibrium is disrupted. A degree of separation begins, with a proportion of firms choosing to set higher prices at the expense of not selling to searchers. Our numerical simulations suggest that non-searchers' welfare decreases with increase in the inflation rate.

Inflation increases price dispersion in the market. The possibility of finding better prices induces search, but the chances of finding firms that follow the low-band pricing strategy is reduced as inflation increases. For the searching consumers, this
represents a more time-consuming activity, which can be interpreted as an increase in the resources spent on search.

Had we extended to a repeated purchase model, we believe to have a model, with microeconomic foundations, that explains why consumers are frequently surprised with the prices they expect to observe in previously visited firms, even under increasing expected levels of inflation. There is no need to assume unexpected inflation, to justify an increase in prices above past period's level of inflation rate. The increase in the frequency of price adjustment- with its reflection on the firms' costs- that results from an increase in inflation rate, is sufficient to explain why firms will adjust their prices beyond past period's inflation rate.

Our model predicts that, increase in the inflation rate, will require searching consumers to allocate more time to search for a better price. In a model with a greater variety of types, both of consumers and firms, it should be expected that, each type of consumers will need to seek more intensively to find lower price/lower cost firm. Intuition leads to believe that consumers may prefer to purchase product of inferior quality, in order to reduce time/resources spent on search and to meet their budget constraint. Inflation alters firms equilibrium price distribution and the distribution of buyers across firms.

This result holds true, under the assumption that buyers are fully inflation protected. Had we assumed consumers are only partially protected, the above results would be strengthen.
APPENDIX

Proof of Lemma 1. Suppose a firm sets $S > Z$, so that $S = Z e^{\tau T'}$. After an adjustment to $S$, sales are zero during a length of time $t > 0$. Thus, the firm real intertemporal profits is given by:

$$V_s = -\beta + \lambda \int_0^T (Ze^{-\eta (1-t')} - c)e^{-\eta t} + e^{-rT'}V_s.$$  \hspace{1cm} (a1)

Now, imagine the firm in the next adjustment, sets $S = Z$, then resuming the same $(s, S)$ policy. Therefore:

$$W_Z = -\beta + \lambda \int_0^{T-t'} (Ze^{-\eta t} - c)e^{-\eta dt} + e^{-r(T-t')}V_s.$$ \hspace{1cm} (a2)

It is immediate to see that $W_Z > V_S$, thus $S > Z$ cannot be optimal.

Proof of Theorem 1: The profit stream of a firm that sets $S = Z$ can be rewritten as below:

$$V_Z = \frac{-\beta + \lambda \int_0^T (Ze^{-\eta t} - c)e^{-\eta dt}}{(1 - e^{-rT'})}$$

$$V_Z = \frac{-\beta + \lambda \int_0^T Ze^{-\eta t}e^{-\eta dt}}{(1 - e^{-rT'})} - \frac{\lambda(c)}{r}.$$ \hspace{1cm} (a3)
First order condition \( \frac{\partial V_z}{\partial T_z} = 0 \Rightarrow V_z = \frac{\lambda (Ze^{-\pi T_z} - c)}{r} \) (a4)

with \( e^{-\pi T_z} - \frac{\pi}{\pi + r} e^{-(\pi + r)T_z} = r(\frac{1}{\pi + r} - \frac{\beta}{\lambda Z}) \). (a5)

Since \( \frac{\partial V_z}{\partial T_z} \bigg|_{T_z=0} > 0 \), to ensure the existence of an optimal policy we need to impose that

\( \frac{\partial V_z}{\partial T_z} \bigg|_{T_z=\infty} < 0 \), thus it follows that \( \beta < \frac{\lambda Z}{\pi + r} \) (a6)

**Proof of Lemma 2.** Initially assume search cost is so high that \( x_s = Z \).
Define \( \sigma = s/Z \) so that (a3) can be written as:

\[ \frac{\partial V_z^*}{\partial T} \sigma = (r + \pi) \sigma - \pi \sigma^1 + r/\pi - r[1 - (r + \pi)\beta/\lambda Z]. \]

We know that

\[ \frac{\partial (\frac{\partial V_z^*}{\partial T})}{\partial \sigma} = \sigma \left[ 1 - \sigma \frac{r}{\pi} (1 - r/\pi \ln(\sigma)) \right] + r\beta/\lambda Z > 0, \]

since \( \ln(1/\sigma r/\pi) < 1/\sigma r/\pi - 1 \).

Moreover \( \frac{\partial (\frac{\partial V_z^*}{\partial T})}{\partial \sigma} = (r + \pi)(1 - \sigma r/\pi) > 0 \).

Thus \( \frac{\partial \sigma}{\partial \pi} = -\left[ \frac{\partial (\frac{\partial V_z^*}{\partial T})}{\partial \sigma} / \partial \pi \right] / \left[ \frac{\partial (\frac{\partial V_z^*}{\partial T})}{\partial \sigma} \right] < 0. \) (a7)

We can conclude that an increase in the inflation rate increases price dispersion. Under the assumption of no search in the market, firms will continue to set their upper bound of prices at \( Z \) but will optimally allow for a decrease in the lower bound of prices \( s \).

For firms that are setting \( S = x_s < Z \), due to the existence of search in the market \( (\gamma < \gamma_p) \), the reduction on profits caused by a higher inflation rate is even stronger. The possibility of finding lower prices causes a reduction in consumers reservation price (see equation 3, in the main text).

The discounted profit stream of firms that set \( S = x_s \), can be obtained straightforward from theorem 1:

\[ V_{x_s}^* = (x_s e^{-\pi r} - c). \] (a8)

From ((a3), we have that a decrease in \( Z \) has the effect of delaying price adjustment (increasing \( T^* \)). By analogy, a decrease in \( x_s \) increases \( T^* \). The final result is a reduction in firms' profit due to a decrease in both the upper and lower bounds of prices.
Proof of Lemma 3. Suppose a firm sets $s_L \leq x_s$ and any length of time adjustment $T = \ln(S_L/s_L)/\pi$. The firm real intertemporal profits is given by:

$$V_s = -\beta + (1- w + w/\alpha)\int_0^T (S_L e^{-m} - c)e^{-\pi t} dt + e^{-\pi T} V_s$$  \hspace{1cm} (a9)$$

Assume in (a9), $S_L < x_s$. Imagine now that when a price change is expected to occur, the firm deviates once, sets $S_L = x_s$, and then resume after a time $T$ the same $(s_L, S_L)$ rule. Clearly, for a given $T$, in the class of policies $S_L \leq x_s$, $S_L = x_s$ is optimal.

Proof of Lemma 4. The proof is obtained by replacing $S$ by $S_L$ and by the use of using the coefficient $(1- w)\lambda$ instead $\lambda$ in the proof of Lemma 1.

Proof of Lemma 5. Consider policies $S_L \in [x_s, Z]$ with $s_L = S_L e^{-\pi T*}$. Define $x_s = S_L e^{-\pi t'}$, so that $t = \ln(S_L/x_s)/\pi$. If a firm is going to sell only to non-searches this implies that $s_L \geq x_s$, thus $T^* \leq t'$. In this case, the firms intertemporal profits is given by:

$$V_Z = -\beta + (1- w)\lambda \int_0^{T*} (S_L e^{-m} - c)e^{-\pi t} dt + V_Z e^{-\pi T*}$$  \hspace{1cm} (a10)$$

Clearly, to set $S_L = Z$ is optimal in this class of policies.

Proof of Lemma 6: The case $s_L \geq x_s$, i.e. $T^* \leq t'$, was already investigated in lemma 5, we are left to analyze firms behavior for which $s_L < x_s$, i.e. $T^* > t'$.

In this case, firms that follows the pricing strategy $(S_L, S_L)$ will sell only to non-searchers for offers $\in [S_L, x_s)$ and for all consumers for offers $\in [x_s, S_L]$. Its profit stream will be given by:

$$V_s = -\beta + (1- w)\lambda \int_0^{T*} (S_L e^{-m} - c)e^{-\pi t} dt + (1- w + w/\alpha)\lambda \int_{t'}^{T*} (S_L e^{-m} - c)e^{-\pi t} dt + V_s e^{-\pi T^*}$$  \hspace{1cm} (a11)$$
\[
\frac{\partial V_s}{\partial S_L} = \frac{\lambda}{(1-e^{-\gamma T})} \left\{ (1-w) \int_0^t e^{-(\pi+r)\gamma t} \, dt + (1-w+w/\alpha) \int_t^{T^*} e^{-(\pi+r)\gamma t} \, dt \right. \\
- \left[ \frac{w}{\alpha} (x_s - c)e^{\gamma T} \right] \frac{\partial \gamma}{\partial S_L} \right\}
\]

\[
= \frac{\lambda}{(1-e^{-\gamma T})} \left( \frac{1-w}{\pi+r} \right) + \frac{1}{(1-w+w/\alpha)(\pi+r)} \left( e^{-(\pi+r)\gamma T} - e^{-(\pi+r)\gamma t} \right) - \left[ \frac{w}{\alpha} (x_s - c)e^{\gamma T} \right] \frac{1}{\pi S_L} \]

\[
\frac{\partial^2 V_s}{\partial S_L^2} = -\frac{\lambda (w/\alpha)}{(1-e^{-\gamma T})\pi^2} \left( \frac{S_L}{x_s} \right)^{-2-\gamma T} \left[-\gamma x_s + (\pi+r)c \right] \tag{a13}
\]

For \( \frac{x_s}{c} \geq \frac{(\pi+r)}{r} \Rightarrow \frac{\partial^2 V_s}{\partial S_L^2} \geq 0. \tag{a14} \]

For \( \frac{x_s}{c} \leq \frac{(\pi+r)}{r} \Rightarrow \frac{\partial^2 V_s}{\partial S_L^2} \leq 0. \tag{a15} \]

Initially assume \( Z \geq x_s \geq c(\pi+r)/r \), then the intertemporal profit function \( V_s \) is convex concerning \( S_L \). Figures A1 and A2 illustrate the different \( V_s \) graphs.
Under the assumption that $a v < 0$, firms either select $(z, Z)$ or $(s, x_s)$.

For $x_s \leq c(\pi + r)/r$, the intertemporal profit function is concave concerning $S_L$. But nothing more can be said about the convexity/concavity of the profit function for values higher than $x_s$.

By analogy, under the assumption that $\frac{\partial V_s}{\partial S_l}|_{s_L = x_s} < 0$, low-cost firm either select $(z, Z)$ or $(s, x_s)$.

Thus for $x_s > s_L$ and $\frac{\partial V_s}{\partial S_l}|_{s_L = x_s} < 0$ firms will either be following one of the two pricing bands: $(z, Z)$ or $(s, x_s)$.

**Proof of Lemma 7:** With non-overlapping pricing policies, and with $\alpha$ of the firms setting $S_L = x_s$, each one will obtain a profit of $(1-w+w/\alpha)\lambda(x_se^{-\pi T^*} - c)$, while if one chooses to set $S_L = Z$ the profit would be of $(1-w)\lambda(Ze^{-\pi T^*} - c)$.

**The case $x_s > z$, under the assumption of a pooling equilibrium**

Define $x_s = Ze^{-\pi T^*}$. Then, the intertemporal profit of the firms that choose the upper band is:
\[ V_Z = \frac{-\beta + (1-w)\lambda \int_0^{T_r} (Ze^{-\pi t} - c)e^{-rt} dt + w\lambda \int_{t^*}^{T_r} (Ze^{-\pi t} - c)e^{-rt} dt}{(1 - e^{-rT_Z})} \]

First order condition: 
\[ \frac{\partial V_Z}{\partial T_Z} = 0 \Rightarrow V_Z = \frac{\lambda(Ze^{-\pi T_Z} - c)}{r} \]

Notice that now the periodicity of price adjustment depends on the firms' costs, since 
\[ Ze^{-\pi T_Z} - \frac{\pi}{\pi + r} Ze^{-(\pi + r)T_Z} = r\left(\frac{Z}{\pi + r} - \frac{\beta}{\lambda}\right) - wrZ\left(1 - e^{-(\pi + r)t^*}\right) + wc(1 - e^{-rt^*}) \]

We can see from above that decrease in \( x_s \) implies an increase in \( t^* \) and in \( T_z \). Moreover, since \( \partial \text{RHS} / \partial c > 0 \) and \( \partial \text{LHS} / \partial T_Z > 0 \), in case of heterogeneity on the supply, high-cost firms will split off first.

The case of a two price band distribution with partial overlap \( (x_s > z) \), introduces some new complication. The basic intuition is very simple. The closer is \( x_s \) to \( Z \), the higher the sales of those firms that choose the upper band. This has the effect of reducing the ratio "adjustment cost/ per sale ", inducing more frequent price adjustment (reduction in \( T_z \)) and increasing profit. A change in \( x_s \) affects the gains of the firms that set \( S = Z \) in two ways: a direct effect on sales and an indirect effect on the periodicity of price adjustment. Moreover, the profit of the firms that choose to set \( S = x_s \) is affected directly with a change on \( x_s \), besides also being affected in the above two ways.
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Autor: Najberg, Sheila.
Título: Price behavior under search and inflation.