“An Economic Model of Gun Control”

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An Economic Model of Gun Control

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Abstract

Guns stolen from law-abiding households provide the principal source of guns for criminals. The lethality of crime instruments increases with the availability of guns, so the gun market is subject to externalities that generate excessive ownership and inadequate spending on protective measures to deter gun theft. One motive for gun ownership is self defense, and the gun market is subject to coordination failure: the more guns purchased lawfully, the more will be stolen by criminals, so the greater the incentive for lawful consumers to purchase guns for self defense. As a result, there may be multiple equilibria in the gun market and more than one equilibrium crime rate. We show that a simple refundable deposit for guns will internalize the externalities in the gun market and may cause large downward jumps in gun ownership, the lethality of crime instruments, and the social costs of crime.
1. INTRODUCTION

It is widely recognized that current gun-control policies fail to prevent guns from falling into the hands of criminals (Wright [1988], Wright and Rossi [1986]). Most states have waiting periods for gun purchases and require gun buyers to register with local authorities. The states without waiting periods are subject to the waiting period specified by the Brady Bill. In addition, gun sellers are prohibited from selling guns to convicted felons. The problem with the limited-access approach is that most criminals get their guns through the informal sector. Each year, about 567,000 guns are stolen (about 90% of which are taken from households) and it appears that the majority of the transactions that supply guns to proscribed people involve stolen guns (Cook, Moolconi, and Cole [1996], Moore [1981, 1983], Wright and Rossi [1986]).

As far as we know, there are no formal models of gun ownership. In our model, lawful citizens purchase guns for use in gun sports and self defense. The value of a gun held for self defense depends on the crime rate, the lethality of crime instruments (guns versus knives and clubs), and the losses experienced by armed and unarmed victims. The lethality of crime increases with the number of guns stolen from lawful citizens, consistent with the empirical evidence that the frequency of robberies committed with guns increases with the density of gun ownership (Cook [1981]). Each citizen must decide whether to buy a gun, and each purchaser must decide how much to spend on measures to protect the gun from theft. Because the social consequences of gun theft exceeds the private consequences, gun owners under-invest in protective measures. We explore the externalities in the gun market and show why they lead to excessive gun ownership and inadequate spending on protective measures. We also show that a simple refundable deposit on guns will internalize these externalities and generate efficient levels of ownership and protection.

An important feature of the gun market is that consumers are interdependent. If the lethality of crime increases with the number of guns released into the illegal sector, an
increase in the number of guns purchased by lawful consumers will increase the lethality of crime and thus increase the incentive to purchase a gun for self defense. This interdependence causes coordination failure and the possibility of multiple equilibria. We show that a small change in public policy (e.g., a small increase in a tax or a refundable deposit) may cause a large downward jump in gun ownership, the lethality of crime, and the social costs of crime.

The remainder of the paper is organized as follows. In the next two sections, we describe a model of the gun market and compare the market equilibrium to the social optimum. Then we show that a system of refundable deposits will generate the socially efficient levels of ownership and protection measures. The fourth section discusses the coordination failure in the market for guns and explores the possibility of multiple equilibria. In the fifth section we use a simple simulation model to explore the effects of public policies on gun ownership, spending on protective measures, release rates, and gun robbery rates. We show that a small change in a refundable deposit may cause a large downward jump in the crime rate. The final section summarizes the paper and discusses extensions.

2. THE MODEL AND THE EQUILIBRIUM OUTCOME

The action in the model occurs in three stages. There are N consumers, who differ in the value they place on owning a gun: the sport value of gun ownership v is distributed \( F(v) \) on support \([0, v_{\text{max}}]\). In Stage 1, each consumer decides whether to buy a single gun for use in gun sports or self defense. In Stage 2, each purchaser chooses how much to spend on protective measures that decrease the probability that the gun will be stolen. In Stage 3, some of the guns purchased by lawful citizens are released (because of theft or loss) into the illegal environment. The probability that any gun is released is \( r(e) \), where \( e \) is the owner’s level of protective effort. We assume that \( r(e) \) is decreasing and convex.
The self-defense value of a gun depends on the crime rate, the lethality of crime, and the losses experienced by unarmed and armed citizens. The probability that any consumer is a victim of crime is fixed at $c$, consistent with the empirical evidence that crime rates are independent of the availability of guns (Cook [1979], Cook [1981]). The loss experienced by an armed citizen is

$$c A y(R)$$

(1)

where $A$ is the "base" loss for an armed victim, $y(R)$ is an index of the lethality of crime instruments, and $R$ is the number of guns released into the illegal sector. The more lethal the crime instrument, the greater the loss experienced by a victim, consistent with the empirical evidence that crimes committed with guns (as opposed to knives or clubs) have higher success rates, larger coercive transfer payments, and higher death rates for victims (Cook [1981], Cook and Leitzel [1996]). The loss experienced by an unarmed citizen is

$$c U y(R)$$

(2)

where $U > A$, reflecting the empirical evidence that unarmed victims experience higher victim costs (Kleck [1991], Hinderlang [1976], Kleck and DeLone [1993], Kleck [1988]).

The lethality of crime instruments depends on the number of guns released into the illegal sector. The expected number of guns released is

$$R = \int_{v^*}^{v_{max}} r(e) dF(v)$$

(3)

where $v^*$ is the type of the marginal purchaser. We assume that $y(R)$ is increasing and convex, consistent with the empirical evidence that the frequency of criminal gun use increases with the general prevalence of guns. For example, Cook [1979] estimates and elasticity of gun robbery with respect to the density of gun ownership of 0.50: a 10% increase in the ownership rate increases the gun robbery rate by 5%.

Our model explains the connection between gun ownership and the gun robbery rate. The more guns purchased lawfully, the more guns released into the illegal sector where they are purchased by proscribed individuals. The street prices of guns (prices paid
by proscribed individuals) is higher in cities with a relatively small supply of lawful guns, with street prices up to five times the retail prices (Cook, Molliconi, and Cole [1995]). Implicit in our model is the notion that the street price is decreasing in the number of guns released, and criminals substitute guns for other instruments as the street price drops.

In stage 2, each gun owner chooses e to maximize her expected payoff

\[ \pi_G(v, e) = v(1 - r(e)) - cy(R)A(1 - r(e)) - cy(R)Ur(e) - P - e, \]  

where P is the retail price of a gun. The first term in (4) is the expected personal value of ownership, equal to the personal value times the probability of retaining the gun. The second and third terms show the expected costs of being victimized. The last two terms incorporate the costs of purchasing and protecting the gun.

In stage 1, each consumer decides whether to purchase a gun. The expected payoff to a consumer who does not purchase is

\[ \pi_N = -cy(R)U \]  

Each consumer purchases a gun if it is in her interest to do so, that is, if \( \pi_G(v, e) - \pi_N \geq 0 \). We assume that individuals ignore the effects of their own decisions on expected releases and the lethality of crime.

2.1 The Protection Decision

Consider first a purchaser's protection decision. Differentiating (4), the first-order condition for a payoff-maximizing choice of e is

\[ -r'(e)[v + cy(R)(U - A)] = 1 \]  

The expression in square brackets is the value of possessing a gun, equal to the sum of the personal and self-defense values of ownership. The self-defense value equals the expected savings in victimization costs from possessing a gun. The left side of (6) is the private
marginal benefit of protective measures (the change in probability of release times the value of possession) and the right side is the marginal cost of effort.

Equation (6) implicitly defines an optimum effort level \( e^* \) for each purchaser. The optimum effort is increasing in the personal value of gun ownership,

\[
\frac{\partial e^*}{\partial v} = \frac{r'(e)}{\Delta} > 0
\]  

(7)

where

\[
\Delta = -(v + cy(R)(U - A))r''(e) < 0
\]  

(8)

This is sensible because a larger \( v \) implies a higher cost associated with losing a gun and thus a greater incentive to protect it. The optimum effort is also increasing in the expected number of releases,

\[
\frac{\partial e^*}{\partial R} = \frac{cy'(R)(U - A)r'(e)}{\Delta} > 0.
\]

(9)

The lethality of crime increases with \( R \), so the benefit from possession (lower victimization costs) increases with \( R \), encouraging more protection. The optimum effort is increasing in the gap between the victimization costs of armed and unarmed citizens, meaning that the derivatives with respect to \( U \) and \( A \) are positive and negative, respectively:

\[
\frac{\partial e^*}{\partial U} = \frac{cy(R)r'(e)}{\Delta} > 0, \quad \frac{\partial e^*}{\partial A} \frac{-cy(R)r'(e)}{\Delta} < 0.
\]

(10)

2.2 The Purchase Decision

Consider next the consumer's decision about whether to purchase a gun. The consumer will purchase if

\[
\Delta \pi = \pi_G(v, e^*) - \pi_N = [v + cy(R)(U - A)](1 - r(e^*)) - P - e^* \geq 0.
\]

(11)

By the envelope theorem, the slope of \( \Delta \pi \) reduces to
\[
\frac{d\Delta \pi}{dv} = (1 - r(e^*)) > 0
\]  

Thus, provided \( \pi_G(0, e^*) - \pi_N < 0 \) and \( \pi_G(v^{\max}, e^*) - \pi_N > 0 \), there is a unique \( v^* \in (0, v^{\max}) \) such that \( \Delta \pi = 0 \).

The equal-payoff condition \( \Delta \pi = 0 \) implicitly defines the type of the marginal gun purchaser \( v^* \). The number of purchasers, \( N - F(v^*) \), is decreasing in \( v^* \). Differentiating \( \Delta \pi = 0 \) using (11) and (6) implies that fewer consumers purchase as the price of a gun rises:

\[
\frac{\partial v^*}{\partial P} = \frac{1}{1 - r} > 0
\]  

The self-defense value of ownership increases with \( R \), so more consumers purchase as \( R \) increases:

\[
\frac{\partial v^*}{\partial R} = -c y'(R)(U - A) < 0.
\]  

The larger the difference in victim costs, the larger the benefit from ownership, and so the greater the number of gun purchasers. This means that the derivatives of \( v^* \) with respect to \( U \) and \( A \) are negative and positive, respectively:

\[
\frac{\partial v^*}{\partial U} = -c y(R) < 0; \quad \frac{\partial v^*}{\partial A} = c(R) > 0.
\]  

2.3 Equilibrium

The payoff functions underlying consumers' purchase and protection decisions depend on the lethality of crime, which depends on how many guns are released into the environment. The number of guns released depends in turn on how many guns are purchased and how much consumers spend on protective measures. In equilibrium, consumers' purchase decisions must be consistent with their expectations about the number of releases. Formally, \( \Delta \pi = 0 \) implies
\[ v^* = \frac{P + e^*}{1 - r(e^*)} - cy(R)(U - A) \]  
(16)

where

\[ R = \int_{v_*}^{\text{max}} r(e^*) dF(v) \]  
(17)

The equilibrium values of \( v^* \) and \( R \) must solve both (16) and (17). We return to this requirement later in the paper.

3. **Efficiency and Public Policy**

The efficient allocation maximizes expected social welfare, equal to the sum of the expected payoffs to non-buyers \( v < v^* \) and buyers \( v \geq v^* \),

\[
W = \int_0^{v_*} -cy(R)U dF(v) + \int_{v_*}^{\text{max}} ((v - cy(R)A)(1 - r(e)) - cy(R)Ur(e) - P - e) dF(v),
\]  
(18)

subject to the definition of expected total releases from (3). The planner has two choice variables. The first is the threshold value \( v^* \), which determines how many consumers buy guns. The second is the level of protection for each gun buyer. Although each consumer ignores the effect of her purchase and protection decisions on expected releases and the crime rate, the social planner does not. Formally, the planner's problem is to maximize the Lagrangian

\[
L = \int_0^{v_*} -cy(R)U dF(v) + \int_{v_*}^{\text{max}} ((v - cy(R)A)(1 - r(e)) - cy(R)Ur(e) - P - e) dF(v) - \lambda \left( \int_{v_*}^{\text{max}} r(e) dF(v) \right)
\]  
(19)

3.1 **Efficient Participation and Protection**
Consider first the participation decision. Differentiating the Lagrangian with respect to \( v^* \), the first-order condition for efficient participation is

\[
\frac{\partial L}{\partial v^*} = -cy(R)Uf(v^*) - ((v^* - cy(R)A)(1 - r(e)) - cy(R)Ur(e) - e - P)f(v^*)
\]

\[
- \lambda r(e)f(v^*) = 0
\]

(20)

where \( \partial L/\partial R = 0 \) implies

\[
\lambda = -\left[ \int_{0}^{v^*} cy'(R)dF(v) + \int_{v^*}^{v_{\text{max}}} ((cy'(R)A)(1 - r(e)) + cy'(R)Ur(e))f(v) \, dv \right] < 0
\]

(21)

The multiplier \( \lambda \) is the shadow price of releases, the decrease in welfare caused by a small increase in \( R \). Equation (20) can be written as

\[
[v^* + cy(R)(U - A)](1 - r(e)) - P - e = -\lambda r(e).
\]

(22)

The marginal consumer increases aggregate expected releases by \( r(e) \), so the external cost of another buyer is \(-\lambda \cdot r(e)\). Thus, at the optimum \( v^* \), the difference in payoffs between purchasing and not purchasing, \( \Delta \pi \), equals \(-\lambda \cdot r(e)\), which from (21) is greater than zero.

Comparing (11) and (22) implies that the efficient level of participation is less than the equilibrium level: too many consumers purchase guns in equilibrium.

Consider next the efficient levels of protection. From the Lagrangian, the first-order condition for the level of protection is

\[
\frac{\partial L}{\partial e} = \int_{v^*}^{v_{\text{max}}} ((v - cy(R)A)(-r'(e)) - cy(R)Ur'(e) - 1)f(v) \, dv + \lambda \left\{ \int_{v^*}^{v_{\text{max}}} r'(e)f(v) dv \right\} = 0
\]

(23)

Equation (23) can be written in terms of marginal benefits and costs:

\[
-r'(e)[v + cy(R)(U - A) - \lambda] = 1.
\]

(24)
The marginal social benefit is $r'(e)$ times the sum of the private and external benefits of protective effort (the increase in welfare per unit change in aggregate expected releases). Comparing (6) and (24) shows that the equilibrium effort level for each buyer is too low: gun buyers undertake too little protective effort because they ignore the effect of their efforts on expected releases and the crime rate.

3.2 Public Policy: Refundable Deposit

A simple policy will correct the market failures associated with the purchase and protection of guns. Suppose the government imposes a refundable deposit $D$ on gun purchases. At the end of the time period, each buyer must establish that the gun is in her possession, or $D$ will be forfeited.

With a refundable deposit, the expected payoff to a consumer who buys a gun and undertakes protective effort $e$ is

$$
\hat{\pi}_G(v,e;D) = v(1 - r(e)) - cy(R)A(1 - r(e)) - cy(R)Ur(e) - Dr(e) - P - e. \quad (25)
$$

Consider the implications of a refundable deposit on protective efforts. In Stage 2, the equilibrium choice of $e$ is characterized by

$$
\frac{\partial \hat{\pi}_G}{\partial e} = -(v - cy(R)A)r'(e) - (D + cy(R)U)r'(e) - 1 = 0. \quad (26)
$$

This implies

$$
-r'(e)[v + cy(R)(U - A) + D] = 1. \quad (27)
$$

If $D = -\lambda$, this is identical to (24), the equation for the efficient level of protection. To generate efficient protective effort, the deposit should equal the negative of the shadow price of releases.

This refundable deposit will also generate the efficient level of participation. In Stage 1, the marginal purchaser $v^*$ is now characterized by
\[
\hat{\pi}_G(v,e;D) - \pi_N = 0
\]

or

\[
[v + cy(R)(U - A)](1 - r(e)) - P - e = Dr(e). \tag{28}
\]

This is identical to (22), the equation for efficient participation if \( D = -\lambda \). Therefore, the deposit system will implement the first-best efficient allocation.

4. **COORDINATION FAILURE IN THE MARKET FOR GUNS**

One motive for purchasing a gun is to protect yourself from predation by criminals, and some criminals will be armed with guns stolen from lawful consumers. The more guns bought by lawful consumers, the more will be released to be used by criminals, so the greater the lethality of crime and the greater the incentive to buy a gun for self defense. This interdependence makes it possible to have multiple equilibria in the market for guns, one of which is more efficient than the others.

To simplify matters, we'll assume that the probability of release (\( r \)) is exogenous, meaning that the consumer's only choice concerns whether or not to buy a gun. In this case, the equilibrium conditions for \( v^* \) and \( R \), equations (16) and (17), reduce to

\[
v^* = \frac{P}{1 - r} - cy(R)(U - A) \tag{29}
\]

and

\[
R = r(N - F(v^*)) \tag{30}
\]

Substituting for \( R \) in (29) gives

\[
v^* = \frac{P}{1 - r} - cy(r(N - F(v^*)))(U - A). \tag{31}
\]

The equilibrium value of \( v^* \) is a solution to (31). In other words, the equilibrium value of \( v^* \) is a fixed point of the mapping \( \Phi(v^*) \), where
\( \Phi(v^*) = \frac{P}{1-r} - cy(r(N - F(v^*))(U - A)). \) (32)

The purchase decisions of consumers are interdependent in this model. If more consumers purchase a gun, more guns are (expected to be) released into the environment, which increases the lethality of crime instruments and hence the desire to own a gun for self defense. In problems like this one, the existence and uniqueness of equilibrium can be problematic. Here we briefly outline restrictions that are sufficient to ensure that the equilibrium value of \( v^* \) is unique and interior to the choice set.

First, note that since \( \Phi(v^*) \) is a composition of continuous functions, it is continuous on the interior of \([0,v_{\text{max}}]\). Second, note that

\[
\Phi(0) = \frac{P}{1-r} - cy(rN)(U - A); \tag{33}
\]

\[
\Phi(v_{\text{max}}) = \frac{P}{1-r}. \tag{34}
\]

If \( \Phi(0) > 0 \) and \( \Phi(v_{\text{max}}) < v_{\text{max}} \), then by continuity, \( \Phi(v^*) \) must have at least one interior fixed point.

Under what circumstances will there be a unique equilibrium value of \( v^* \)? The slope and curvature of \( \Phi(v^*) \) are

\[
\Phi'(v^*) = rcy'(R)(U - A)f(v^*) > 0; \tag{35}
\]

\[
\Phi''(v^*) = r(U - A)(cy'(R)f'(v^*) - rcy''(R)f(v^*)^2). \tag{36}
\]

Suppose consumers are uniformly distributed on \([0,v_{\text{max}}]\). Then a sufficient condition for the existence of a unique equilibrium is

\[
\frac{rcy'(rN)(U - A)N}{v_{\text{max}}} < 1. \tag{37}
\]

In this case \( \Phi(v^*) \) is increasing and strictly concave, with \( \Phi'(0) < 1 \).
Other equilibrium outcomes are possible, although it is difficult to say much about them in the general case. If $f(v^*)$ changes sign on $(0, v_{\text{max}})$, then $\Phi''(v^*)$ may change sign as well, leading to the possibility of multiple equilibria and coordination failure, in the spirit of Cooper and John [1988]. Figure 1 illustrates the nature of the coordination failure. Here there are two stable equilibria (points L and H) and one unstable equilibrium (point U). At point L, $v^*$ is such that relatively few people purchase guns, few are released into the environment, and consequently the desire to purchase guns for self defense is small. In this case, the decisions of consumers are mutually consistent at a low level of gun ownership.

In contrast, at point H, $v^*$ is such that most people purchase guns, many are released into the environment, and so the desire to purchase guns for self protection is large. In this case, the decisions of consumers are mutually consistent at a high level of gun ownership. In the context of our model, the equilibrium shown by point L dominates the equilibrium shown by point H because the loss from victimization is lower and fewer resources are devoted to gun ownership.

5. Summary

This paper is motivated by the fact that about half the guns used by criminals have been stolen from households, gun shops, or manufacturers. The release of legally purchased guns into the hands of criminals generates externalities in gun ownership that can be internalized by a refundable deposit. The deposit decreases the number of guns released into the illegal sector by decreasing gun ownership and increasing spending on protective measures. If some consumers purchase guns for self-defense, there is a coordination failure in the gun market, making multiple equilibria possible. As a result, a change in public policy may lead to large changes in gun ownership, the lethality of crime, and the social costs of crime.
References


Figure 1: Coordination Failure and Multiple Equilibria
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