“On the Cost of Sympathy: An Alternative Model for Insurance Markets”

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On the Cost of Sympathy: 
An Alternative Model for Insurance Markets*

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Abstract

The paper provides an alternative model for insurance market with three types of agents: households, providers of a service and insurance companies. Households have uncertainty about future levels of income. Providers, if hired by a household, perform a diagnoses and privately learn a signal. For each signal there is a procedure that maximizes the likelihood of the household obtaining the good state of nature. The paper assumes that providers care about their income and also about the likelihood households will obtain the good state of nature (sympathy assumption). This assumption is satisfied if, for example, they care about their reputation or if there are possible litigation costs in case they do not use the appropriate procedure. Finally, insurance companies offer contracts to both providers and households. The paper provides sufficient conditions for the existence of equilibrium and shows that the sympathy assumption leads to a loss of welfare for the households due to the need to incentive providers to choose the least expensive treatment.

*To my father, Rodolpho Paulo Rocco.
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1. Introduction

The theoretical literature on health economics usually relies on the adverse selection model to explain the apparent inefficiency observed in the provision of health insurance. In this class of model, there are several types of agents, or patients, and risk-neutral insurance companies. Patients face individual uncertainty, are strictly risk-averse and have a private information about their own risk type, which is not known by the insurance companies.

There would be obvious gains from trade to be exploited if the information about patients’ risk types were common knowledge: Insurance companies would be willing to provide insurance at actuarially fair prices, which would strictly improve each patient’s welfare. However, due to private nature of information, insurance companies cannot distinguish between patients of different risk types. One can classify the equilibria in this class of model as separating, where different types of agents accept different contracts, and pooling, when at least two distinct types of agents accept the same insurance contract.

Rothschild and Stiglitz (1976) showed that in this adverse selection model an equilibrium may not exist. Moreover, equilibria, when they exist, are necessarily separating. The low highest risk type essentially receives the same contract he would receive in the absence of asymmetry of information. The lower risk types, on the other hand, receives worse contracts, from their welfare perspective, than they would receive in the absence of asymmetry of information, in order to prevent the higher risk types to misrepresent themselves.

The adverse selection model, however, is not capable of addressing many questions which are in the core of recent debates on health economics. First, the basic feature of model is the fact that patients know more about their health than insurance companies do. However, due to pre-existing contingencies insurance companies are able to avoid this risk. Second, if some agent has a better information in the health sector is the doctor who examines a patient. However, doctors are surprisingly absent from the adverse selection models used in this analysis. This absence is even more surprising since it is precisely the change in the contracts between doctors are insurance companies that lies in the core of the recent changes in the health sector, particularly the development of HMOs. A central aspect of these changes is a risk-sharing contract between the insurance company and the primary care physician of the patient’s expected costs of treatment. The existing

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1Andrade e Lisboa (1999) review the recent evolution of the american health insurance market.
adverse selection models are unable to address the motivations for this change or to study their welfare implications.

The paper provides an alternative model for insurance market, in particular health insurance. There are three types of individuals: patients, doctors and insurance companies. Patients have uncertainty about future levels of income. Doctors, if hired by a patient, perform a diagnosis, which provides a signal about the patient's health, which is observed only by the doctor. For each signal, there is a treatment that maximizes the likelihood of the patients obtaining the good state of nature in the final period. The paper assumes that doctors care about their income and also about the likelihood that the patient will obtain the good state of nature (sympathy assumption). This assumption is satisfied if, for example, doctors care about their reputation or, in case they do not use the appropriate treatment, there is a probability of litigation (defensive medicine). There are risk neutral insurance companies that offer contracts to both the doctors and the patients. We show that the sympathy assumption leads to a loss of efficiency in the market outcome. However, given this asymmetry of information, this outcome is constrained optimal, or second best. The optimal contract in this framework presents the following feature: the doctor payment is inversely related to the cost of proposed treatment: the more expensive the treatment, the less the doctor receives.

The basic structure of the model seems to apply to any provision of any specialized service in which the proposed service depends upon a diagnosis performed by the provider himself. Households may benefit from buying an insurance against the necessity of having to buy such a service. The provider of the insurance, on the other hand, has to design a contract with the provider taking into account that he has a privileged information about the service that may be needed. Moreover, due to gains due to reputation, the provider of the service may have incentives, in the presence of a contract that pays for service performed, to always choose the service that minimizes the probability that a new problem may happen again, independently of the cost of such service. Despite these possible applications of the basic model, in the remainder of the paper we describe the model using the health insurance market as a benchmark case.
2. The basic model

2.1. Uncertainty, patients and doctors

Consider a partial equilibrium model with a single commodity, three periods - *ex-ante, interim, ex-post* - and three types of individuals: *patients, doctors and insurance companies*. A patient faces uncertainty about her initial endowment in the *ex-post* period: there are two individual states of nature and her endowment in the second period is strictly larger than her endowment in the first period. We refer to the patient endowment in state \( s \) by \( e^s \). Her preferences over consumption bundles is represented by a state independent utility function \( u : \mathbb{R}_+ \rightarrow \mathbb{R} \), strictly increasing and concave in the differentiable sense:

\[
(H1) \; u \in C^2 \text{ and satisfies: i) } du > 0 \; \text{; ii) } d^2 u < 0. \; \text{Moreover,} \; 0 < e^1 < e^2
\]

Under these assumptions of random endowments and strict risk-aversion, the patient is willing to buy an insurance contract, provided that the price of the contract is not much higher than the actuarially fair price. The standard insurance literature assumes the existence of a finite collection of risk neutrals insurance companies, who offer insurance contracts simultaneously and independently. Under these assumptions, the outcome of the model with at least two companies is that they offer actuarially fair contracts, which are accepted by the patients.

The model proposed here departs from the standard literature by assuming the existence of a third type of individual, whom we refer to as *doctor*. A doctor examines a patient and chooses an action, or treatment, in the interim period that affects the patient's probability of the good state of nature. This action is supposed to be perfectly observable, however its effectiveness depends on a signal privately observed by the doctor.

To make the argument precise, suppose that in the interim period the doctor observes a private signal, \( s \in [0,1] \). The probability of a signal \( s \) is described by the cumulative probability function \( F(s) \in C^1, dF(s) > 0 \) for every \( s \in (0,1) \). There are 2 types of actions available for the doctor:

\[
A := \{a^0, a^1\}
\]

Action \( i \) costs \( a_i \) to be implemented in addition to the doctors' payments, \( 0 = a^0 < a^1 \). Action \( a^0 \) should be interpreted as the doctor choosing *no action*. Given
this signal, the probability of the good state of nature for a patient depend upon the action chosen and it is given by $\pi(a, s)$ for every $a$. We assume:

\[(H2) \text{ For each } a \text{ the function } \pi(a, \cdot) \in C^2, \text{ for every } s \in (0, 1)\]

\[d\pi(a^1, s) > d\pi(a^0, s)\]

and

\[0 < \pi(a^1, 0) < \pi(a^0, 0) < 1\]

\[0 < \pi(a^0, 1) < \pi(a^1, 1) < 1\]

Therefore, there is a signal $s^* \in (0, 1)$ such that both actions generate the same probability of the good state of nature

\[\pi(a^1, s^*) = \pi(a^0, s^*)\]

Moreover, for every signal $s < s^*$ action $a^0$ generates the higher probability of the good state of nature in the second period, while for every signal $s > s^*$ the reverse happens. We refer to the action that maximizes the probability of the good state of nature as the \textit{best action}.

We assume that doctors care about their income and about choosing the best action for the patients, given the signal they observe. This second assumption, which we refer to as \textit{sympathy assumption}, is the distinctive assumption of our model and it implies the following result. Suppose the doctor income does not depend upon the action chosen. Then he always choose the best action. \textit{Notice that the term best here has no economic meaning: the best treatment can actually be very costly and patients may prefer an alternative treatment, which is less effective but cheaper}. This result is trivially consistent with the stylized facts from health insurance markets, in particular the overuse, from the economic perspective, of exams and treatments the system if patients are provided a full insurance system.

The sympathy assumption can be derived in a dynamic game where the historical rate of patients achieving the good state of nature is used by the doctor as a signal either to differentiate himself from different types of doctors or to show his commitment in choosing the appropriate action for a certain class of signals. Alternatively, one can generate the sympathy assumption by allowing the possibility of doctors being sued in case they do not use the best treatment (defensive medicine). In order to keep the model simple, we focus on the static game where the sympathy is simply assumed.
More formally, if the doctor has income $r$ in the ex-post period, observes signal $s$ and chooses action $a$, his utility is given by

$$v \left( \pi (a, s) - \max_{a'} \pi (a', s), r \right)$$

where $v : [-1, 0] \times \mathbb{R}_+ \rightarrow \mathbb{R}$ is strictly increasing and strictly concave in the differentiable sense:

(H3) $v \in C^2$ and satisfies: i) $dv \gg 0$; ii) $d^2v$ is negative definite; and iii)

$$\lim_{y \rightarrow 0} v \left( \pi (a, s) - \max_{a'} \pi (a', s), y \right) = -\infty$$

A doctor has a reservation utility denoted by $\bar{v}$ and there is $r$ such that $v (0, r) = \bar{v}$. Let $\Delta \pi (a, s) := \pi (a, s) - \max_{a'} \pi (a', s)$.

There are $J > 1$ principals, or insurance companies, that provide insurance for the patients and intermediate their relation with the doctors. We suppose that these principals are risk-neutral. In the ex-ante period they hire doctors and offer contracts for the doctors and patients. The doctor's contract specify how much the principal will pay for the doctor in the interim period. As we will see later, in the optimal contract may be optimal to make the doctors' payments contingent upon the action chosen.

The patient's contract specify how much she pays for the insurance company in each individual state of nature and the type of contract the insurance has established with the doctor. Indeed, in choosing the contract that maximizes her expected utility, the patient takes into account that the doctor's choice of action does depend on his contract with the insurance company.

Suppose a patient has accepted a particular principal offer in the first period. In the second period the patient meets the doctor who is the only one who observes the private signal $s$. Given this signal and the contract the doctor has with the principal he chooses an action. Both the principal and the patients correctly anticipates how the doctor behaves for each possible signal for each given contract established with the principal.\(^2\)

The timing of the model is as follows. The principals simultaneously offer contracts to the patients and doctors. Household and doctor choose to accept the

\(^2\)There is a natural extension of the model in which the insurance companies can verify the signal, but there is a cost in doing so.
contracts that maximizes their respective expected utility. The interim period come and the doctor observes the private signal and choose the action. Household uncertainty is resolved and contracts are enforced.

2.2. The optimal contract

We start the analysis of the model investigating the insurance companies’ offers to the doctor. Let \( r^i \) be the payment the principal makes to the doctor if he chooses action \( a^i \). In what follows we restrict the analysis to the case where \( r^1 \leq r^0 \).

Suppose the doctor has accepted the offer and has observed signal \( s \). He, then, solves the problem

\[
\max_{i=0,1} v \left( \Delta \pi (a_i, s), r^i \right)
\]

This problem has at least one solution. Moreover, from the monotonicity of the probability and utility function, there is a critical signal \( s(r) > 0 \) such that for all \( s < s(r) \) the doctor chooses \( a_0 \) and for all \( s > s(r) \) the doctor chooses \( a_1 \). At \( s = s(r) \) the doctor is indifferent between the two actions, provided that \( s(r) < 1 \).

Therefore, the doctor problem reduces to choosing the critical signal \( s(r) \). Let

\[
V (r^0, r^1, \tilde{s}) := \int_0^{s} v \left( \Delta \pi (a_0, s), r^0 \right) \frac{dF}{ds}(s) + \int_{s}^{1} v \left( 0, r^1 \right) \frac{dF}{ds}(s)
\]

More formally, for each given contract \( r \) the doctor chooses the critical signal \( s(r) \) to solve the problem

\[
\max_{\tilde{s}} V (r^0, r^1, \tilde{s})
\]

The set \( R \) of contracts for the doctor that satisfy participation constraint can be described as follows

\[
R := \left\{ (r^0, r^1) \geq 0 / \max_{\tilde{s}} V (r^0, r^1, \tilde{s}) \geq \bar{v} \right\}
\]

where \( \bar{v} \) is the doctor reservation utility.

\[3\]Suppose a principal offer \( r^0 < r^1 \). In this case, the doctor, if have accepted the principal’s offer, chooses action \( a^0 \) for signals \( s < s^* \). That is to say, the principal is given incentives for the doctor to choose the most expensive action even when the cheapest action is strictly better. One can easily verify that this offer is a strictly dominated strategy for the insurance company.
Since $r^0 \geq r^1$, the doctor chooses $s(r) \geq s^* > 0$. The optimal solution of the doctor problem must satisfy the following first order equations

$$
v \left( \Delta \pi \left( \bar{s}, a^0 \right), r^0 \right) \frac{dF(\bar{s})}{ds}(\bar{s}) - v \left( 0, r^1 \right) \frac{dF(\bar{s})}{ds}(\bar{s}) - \mu_1 = 0
$$

$$
\min \{\mu_1, 1 - \bar{s}\} = 0
$$

To simplify the notation, and prevent the doctor from choosing $s(r) = 1$, we impose the following simplifying assumption:

$$(H3) \lim_{r \to \infty} v \left( \pi \left( 1, a^0 \right), r \right) < \bar{s}.$$ 

Therefore, the solution of the doctor problem for any contract in the set $R$ is completely characterized by the following equation

$$v \left( \Delta \pi \left( \bar{s}, a^0 \right), r^0 \right) - v \left( 0, r^1 \right) = 0$$

since $dF(\bar{s}) > 0$ for every $\bar{s}$ by assumption. Moreover, an easy application of the implicit function theorem shows that this equation has a unique solution $\bar{s}(r) \in C^1$. Let

$$G(\bar{s}(r)) := \int_0^{\bar{s}} \pi \left( s, a^0 \right) dF(s) + \int_{s}^{1} \pi \left( s, a^1 \right) dF(s)$$

be the probability of the good state of nature given the contract offered which results in the choice of $\bar{s}$.

Suppose insurance company $j$ offers a contract $r_j$ for the doctor, $r_j \in R$, and that the patient pays $d^2_j$ for the insurance company in the patient's good state of nature and that the insurance company pays $d^1_j$ to the patient in the bad state of nature. The company $j$'s payoff is then given by

$$\Pi(r_j, d_j) := G(\bar{s}(r_j)) d^2_j - (1 - G(\bar{s}(r_j))) d^1_j - F(\bar{s}(r_j)) r^0_j - (1 - F(\bar{s}(r_j))) (r^1_j + a_1)$$

If the patient accepts this offer her utility is then given by

$$U(r_j, d_j) := (1 - G(r_j)) u(e^1 + d^1_j) + G(r_j) u(e^2 - d^2_j)$$

Suppose the remaining companies had offered contracts $(r_{j'}, d_{j'})_{j' \neq j}$. The patient would only accept company $j$'s offer if

$$U(r_j, d_j) \geq U(r_{j'}, d_{j'}) \text{ for all } j' \neq j$$
and
\[ U(r_j, d_j) \geq U(0, 0) \]
since the patient may always reject all offers. Let \((r_0, d_0) := (0, 0)\).

**Definition 2.1.** An equilibrium is a collection of strategies \(\{(r_j, d_j)\}_{j}\) where for each \(j\) the contract \((r_j, d_j)\) solves the problem

\[
\max \Pi_j(r, d) \quad \text{subject to } U(r, d) \geq \max_{j=0, \ldots, J} U(r', d') \\
\quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad
Notice that in our model, as in the standard moral hazard model, the critical signal chosen by the doctor can be inferred from the contract offered. This suggests an alternative approach to analyzing the doctor problem, which is summarized in the next lemma.

**Lemma 2.3.** There is an open and non-empty set $S \subset [s^*, 1]$ such that for each $s \in S$ there is a minimum expected cost, $C(s)$, that the insurance company has to incur in order to induce the doctor to choose $s$ as the critical signal. Moreover, $C \in C^1$ for every $s \in S$ and

$$C(s) \to \infty \text{ as } s \to 1$$

In the appendix we prove the lemma and provide a characterization of the function $C(s)$.

In order to characterize the optimal contract is convenient to define the patient's utility level associated with refusing the insurance companies contracts which provides doctors service for the patients and just buying an insurance contracts that smooth her consumption over the individual states of nature

$$U(1) := u(e_1 G(1) + e_2 (1 - G(1)))$$

We define

$$C(1) := 0$$

**Lemma 2.4.** The contract $(r^*, d^*)$ is the outcome of an equilibrium, with associated critical signal $s^*$, if and only if $(d^*, s^*)$ solves the following problem

$$\max_{\bar{s}, \bar{x}} u(e_1 + d_1) (1 - G(\bar{s})) + u(e_2 - d_2) G(\bar{s})$$

$$d^2 G(\bar{s}) - d^1 (1 - G(\bar{s})) - C(\bar{s}) \geq 0$$

$$e_1 + d_1 \geq 0, e_2 - d_2 \geq 0$$

By the previous lemma, at any equilibrium the patient must have perfectly smooth consumption over the individual states of nature and the budget constraint must be binding

$$e_1 + d_1 = e_2 - d_2$$

$$d^2 G(\bar{s}) - d^1 (1 - G(\bar{s})) = C(\bar{s})$$
and thus the patient's income in any state of nature is given

\[ e^1 + d^1 = e_1 (1 - G(\bar{s})) + e_2 G(\bar{s}) - C(\bar{s}) \]

But then, it follows that the patient's optimal contract must be associated with the critical signal that maximizes the above income.

**Lemma 2.5.** The critical signal \( \bar{s}^* \) associated with the outcome of an equilibrium \((d^*, r^*)\) must solve the following programing problem

\[
\max_{\bar{s} \in [0,1]} e_1 (1 - G(\bar{s})) + e_2 G(\bar{s}) - C(\bar{s})
\]

**Proof.** Suppose the claim is not true, and therefore there are \( \bar{s} \) and \( \bar{s}' \), and corresponding \((d^V, d^V')\) and \((d^I, d^I')\), satisfying

\[
\begin{align*}
(e_2 - e_1) G(\bar{s}') + C(\bar{s}') &< (e_2 - e_1) G(\bar{s}) + C(\bar{s}) \\
u (e^1 + d^1) G(\bar{s}') + u (e^2 - d^2) (1 - G(\bar{s}')) &> u (e^1 + d^1) G(\bar{s}) + u (e^2 - d^2) (1 - G(\bar{s})) \\
\text{where } d^2 (1 - G(\bar{s})) - d^I G(\bar{s}) &= C(\bar{s})
\end{align*}
\]

As before, we can restrict the analysis to contracts that leads to state independent income for the patient

\[
\begin{align*}
(e^1 + d^2) \left(1 - \frac{G(\bar{s})}{G(\bar{s}')}\right) - \frac{C(\bar{s})}{G(\bar{s})} &= (e^2 - d^2) \\
(G(\bar{s})e^1 + d^2 (1 - G(\bar{s})) - C(\bar{s})) &= G(\bar{s}) (e^2 - d^2) \\
d^2 &= (e_2 - e_1) G(\bar{s}) + C(\bar{s})
\end{align*}
\]

and thus

\[
(e_2 - e_1) G(\bar{s}') + C(\bar{s}') > (e_2 - e_1) G(\bar{s}) + C(\bar{s})
\]

which is a contradiction. \(\square\)

**Proposition 2.6.** Every equilibrium is second best.

This proposition follows immediately from lemma 2.1 and the definition of second best allocation.

**Proposition 2.7.** There is an equilibrium.
Proof. Let
\[ S := \{ \bar{s} \in [s^*, 1) / V_r(\bar{s}) = \bar{v} \} \]
As we show in the appendix, \( S \) is non-empty and bounded. Moreover, the functions \( G(\bar{s}) \) and \( C(\bar{s}) \) are continuous for \( \bar{s} \in S \). If there is no solution to the above problem it means that there is a sequence \( \bar{s}_n \rightarrow \bar{s} \notin S \), \( \{ \bar{s}_n \} \subset S \), such that
\[ (e_2 - e_1) G(\bar{s}_n) + C(\bar{s}_n) > (e_2 - e_1) G(\bar{s}_{n+1}) + C(\bar{s}_{n+1}) \]
for every \( n \). By definition of \( S \), this implies \( r_0(\bar{s}_n) \rightarrow \infty \), otherwise, taking a subsequence if necessary, \( \bar{s}_n \rightarrow \bar{s} \in [0, 1] \), \( r(\bar{s}_n) \rightarrow r \) and thus by continuity
\[ V(\bar{s}_n, r(\bar{s}_n)) \rightarrow V(\bar{s}, r) = \bar{v} \]
which implies \( \bar{s} \in S \), a contradiction. Therefore, \( r_0(\bar{s}_n) \rightarrow \infty \), and since \( r_1(r(\bar{s}_n), \bar{s}_n) \geq 0 \) for every \( n \), we obtain
\[ (e_2 - e_1) G(\bar{s}_n) + C(\bar{s}_n) \rightarrow \infty \]
which is an absurd. Therefore, the above minimization problem has a solution. \( \square \)

Proposition 2.8. At an optimal contract \( r \) associated with a critical signal \( \bar{s} < 1 \) we must have \( r_0 > r_1 \) and at any such contract, \( C(\bar{s}) \) is locally a decreasing function of \( \bar{s} \). Moreover, there is a threshold level of expected income below which households do not hire a doctor.

Proof: From lemma 2.5, at the optimal contract \( (r_0, r_1) \) associated with a critical signal \( \bar{s} < 1 \) we must have
\[ (d^2 + d^1) dG(\bar{s}) = dC(\bar{s}) \]
\[ \Rightarrow (e^2 - e^1) dG(\bar{s}) = dC(\bar{s}) \]
where
\[ dG(\bar{s}) = (\pi(\bar{s}, a^0) - \pi(\bar{s}, a^1)) dF(\bar{s}) \]
\[ = : \Delta \pi(\bar{s}) dF(\bar{s}) \]
Therefore
\[ dG(\bar{s}) < 0 \Rightarrow dC(\bar{s}) < 0 \]
Moreover, from Lemma 2.5 household will only decide to hire doctors if
\[ \max_{\bar{s} \in S} e_1 (1 - G(\bar{s})) + e_2 G(\bar{s}) - C(\bar{s}) \geq e_1 (1 - G(1)) + e_2 G(1) - C(1) > 0 \]
which completes the argument. \( \square \)
3. Appendix

Fix a signal $\bar{s} \in (s^*, 1)$ and an doctor’s payment in case he chooses action $a_1$, $r_1$. Consider the equation

$$v(\pi(\bar{s}, a_0) - \pi(\bar{s}), x) - v(0, r_1) = 0$$

It is simple to verify that this equation has a unique solution for $s$ close to $s^*$, denoted by $r_0(r_1, \bar{s})$. Moreover, since $d_r v(\Delta \pi, r) > 0$, by the implicit function theorem $r_0(\cdot) \in C^1$. Let

$$V(r_1, \bar{s}) := \int_0^{\bar{s}} v(\Delta \pi(s, a_0), r_0(r_1, \bar{s})) dF(s) + \int^{1}_{\bar{s}} v(0, r_1) dF(s)$$

Consider the set

$$S := \{s \in [s^*, 1)/ V(r, s) = \bar{v} \}$$

Since $V(\bar{r}, s^*) = \bar{v}$, where $v(\bar{r}) = \bar{v}$, it is simple to verify that $S$ is non-empty. Moreover, by construction for each $\bar{s} \in S$ there is a unique $r_1(\bar{s})$ satisfying

$$V(r_1(\bar{s}), \bar{s}) = \bar{v}$$

where $r_1(\bar{s}) \in C^1$ since

$$\frac{dV}{dr_1} = \int_0^{\bar{s}} v(\Delta \pi(s, a_0), r_0(r_1, \bar{s})) \frac{dr_0}{dr_1} dF(s) + \int^{1}_{\bar{s}} \frac{dv(0, r_1)}{dr_1} dF(s) > 0$$

and

$$\frac{dr_0}{dr_1} = \frac{dv(0, r_1)}{dv(\Delta \pi(\bar{s}), r_0)} > 0$$

Therefore, for every $\bar{s} \in S$ there is a neighborhood of $\bar{s}$, $V_{\bar{s}}$, such that $V_{\bar{s}} \cap [s^*, 1] \subset S$. Moreover, for each $\bar{s} \in S$ there is a unique corresponding contract that induces the doctor to choose the signal $\bar{s}$ and that provides an utility level identical to the reservation utility,

$$r(\bar{s}) := (r_0(r_1(\bar{s}), \bar{s}), r_1(\bar{s}))$$

The expected cost for the insurance company of implementing this contract is given by

$$C(\bar{s}) := r_0(r_1(\bar{s}), \bar{s}) F(\bar{s}) + (r_1(\bar{s}) + a_1)(1 - F(\bar{s}))$$

where $C \in C^1$.  

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4. References

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