"PRICING THE OPTION ADJUSTED SPREAD OF BRAZILIAN EUROBONDS"

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Coordenação: Prof. Pedro Cavalcanti Ferreira
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“Pricing the Option Adjusted Spread of Brazilian Eurobonds”

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Abstract

This paper presents results of a pricing system to compute the option adjusted spread (“OAS”) of Eurobonds issued by Brazilian firms. The system computes the “OAS” over US treasury rates taking into account the embedded options present on these bonds. These options can be calls (“callable bond”), puts (“putable bond”) or combinations (“callable and putable bond”). The pricing model takes into account the evolution of the term structure along time, is compatible with the observable market term structure and is able to compute risk measures such as duration and convexity, and pricing and hedging of options on these bonds. Examples show the effects of the embedded options on the spread and risk measures as well as the effects on the spread due to variations on the volatility parameters of the short rate.

Keywords: Derivatives pricing, risk management, term structure dynamics, intertemporal models, dynamic programming, object oriented programming.

JEL Codes: C61, G12.

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Introduction

The market for bonds issued by Brazilian firms, denominated in US dollars, the Eurobonds, has become very active worldwide in the recent past. The attraction of these securities to international investors are the high spreads over the US treasury rates, and the embedded options that guarantee an eventual minimum payment before maturity.

These embedded options can be call options ("callable bond"), put options ("putable bond") or combinations ("putable and callable bond"). The call option belongs to the issuer and the put option to the buyer. Therefore the embedded call option limits the gains of the buyer and the embedded put option limits its losses. The opposite occurs to the issuer.

Thus, the fact that embedded options are present in Brazilian Eurobonds, it is necessary to take them into account when determining their risk spread over the US yield curve, that is, how much are investors really paying for the likelihood of default of the issuing firm\(^2\). In the international market this "risk spread", is called "option adjusted spread", "OAS", since in general, but not always, the "OAS" take into account these embedded options\(^3\).

In this paper, results of a computational pricing system for the "OAS" of Brazilian Eurobonds are presented. A comparison between the spreads taking into account the embedded options, the "dynamic spread" ("DOAS") and not considering them, the "static spread" ("SOAS") is presented. It is also discussed the classical measures of risk: duration and convexity, that will also be affected by these options as well as by the direction of the movement of the term structure considered in their computations. It will be shown that the higher the intrinsic value of the embedded option the larger are the differences between the "dynamic" and "static" spreads. The sensitivity of the spread to changes in the volatility parameters of the short rate are also investigated.

The Pricing System

Pricing models for options on assets that depend on the term structure of interest rates require the modeling of the evolution of the term structure as whole along time. That is,

\(^2\) The default may be caused by the financial condition of the firm ("idiosyncratic or credit risk") and/or due to Brazilian macroeconomic conditions ("Brazil risk"). Currently, the spread to be determined does not distinguishes between these two sources of default risk.

\(^3\) In order to take into account the embedded options in the determination of the spread, it is necessary a fixed-income derivative pricing model as described below. Due to its complexity, these models are not always available in the trading desks of security firms.
for each future date and state of nature, it is necessary to generate a new term structure\(^4\). It is also desirable that the model be compatible with the observable term structure in the market at the analysis date.

The starting point for most fixed-income derivative pricing models is given by the dynamics of the instantaneous rate. This paper's pricing system implements the Gaussian model, developed originally by Vasicek [1977] and extended by Hull and White [1990] to make it compatible with the Heath, Jarrow, Morton [1992] paradigm\(^5\). For this model, the dynamics for the instantaneous interest rate, in continuous-time, is given by:

\[
dr = (\theta(t) - \alpha r)dt + \sigma dZ
\]  

(1)

Where \(\theta(t)\) is the long term interest rate, \(\alpha\) the speed of mean reversion, \(\sigma\) the volatility and \(dZ\) is a Brownian motion.

From the above stochastic partial differential equation and an arbitrage condition between bonds with different times to maturity, an analytical term structure is derived. This analytical term structure between dates \(t\) and \(T\) \((t \leq T)\), is given by:

\[
P(t, T) = A(t, T)e^{B(t, T)t}
\]  

(2)

Where:

\[
B(t, T) = \frac{1-e^{-\alpha(T-t)}}{\alpha}
\]  

(3)

\[
\log A(t, T) = \log \frac{P(0, T)}{P(0, t)} - B(t, T) \frac{\partial \log P(0, t)}{\partial t} - \frac{1}{4\alpha^2} \sigma^2(e^{\alpha T} - e^{-\alpha t})^2(e^{2\alpha t} - 1)
\]  

(4)

Note that \(P(0, t)\) is the observable term structure in the market\(^6\) at the analysis date (date 0), and it is assumed that \(P(T, T) = 1\).

\(^4\) Notice that we are dealing with the dynamics of a curve rather than a point.

\(^5\) To be compatible with the Heath, Jarrow, Morton [1992] paradigm means, among other things, that the model is compatible with the observed term structure and that model parameters are stable.

\(^6\) The observable term structure represents the risk-free spot rates. For the case of Eurobonds, these spot rates are the rates implied in the US treasury bonds.
The embedded options are characterized by the following optimal exercise conditions:

For the case of a call option ("callable bond") at the option exercise date $t$ and state of nature $i$, the value of the Eurobond is given by:

$$ P(t, T, i) = \min \{ P(t, T, i) ; P_c \} $$  \hspace{1cm} (5)

For the case of a put option ("putable bond") at the option exercise date $t$ and state of nature $i$, the value of the Eurobond is given by:

$$ P(t, T, i) = \max \{ P(t, T, i) ; P_p \} $$  \hspace{1cm} (6)

For the more general case of a combination of call and put options ("callable and putable bond") at the option exercise date $t$ and state of nature $i$, the value of the Eurobond is given by:

$$ P(t, T, i) = \max \{ P_p ; \min \{ P(t, T, i) ; P_c \} \} $$  \hspace{1cm} (7)

Where $P_c$ and $P_p$ are the strike prices for the embedded call and put options respectively.

The above model was implemented in a dynamic programming algorithm based on arbitrage free recombining trees\textsuperscript{7}. The system consider early exercise provisions, so that it is possible to price Eurobonds with more than one option date and also exercise periods\textsuperscript{8}. The system is also capable to price American and European style options on these bonds. The system was written in C++, exploiting the object orientation paradigm\textsuperscript{9}, in order to reinforce the relations between the underlying bonds, derivatives and the term structure\textsuperscript{10}.

\textsuperscript{7} The fact that the Gaussian model generate recombining trees, is perhaps its main advantage vis-à-vis other models in the area. When the pricing trees do not recombine, the computational effort grows exponentially.

\textsuperscript{8} This is equivalent to the Bermuda style option. See Gonçalves and Souza [1996].

\textsuperscript{9} The system was developed in the Unix operating system on a Silicon Graphics workstation. For more details on the C++ language and the object oriented programming technology, see Stroustrup [1991].

\textsuperscript{10} Exploiting the fact that, in this case, underlying bonds and derivatives are dependent on the term structure for pricing and risk. Object orientation will also facilitate the development of interface software for real-time trading.
Figure 1 below presents a schematic description of the observed term structure, the pricing tree and the term structure for a state of nature at a future date.

![Diagram of Pricing Tree]

\[ \text{Figure 1: Description of the Pricing Tree} \]

The pricing tree is composed of two stages: forward induction ("present-to-future") and backward induction ("future-to-present"). In the forward induction stage the future term structures are generated for the states of nature considered and the respective risk adjusted probabilities ("martingale measures") determined. In the backward induction stage, the present value of expected value of the cash-flows (principal and coupons), are computed taking into account the optimal early exercise conditions given by equations (5), (6) and/or (7) above, via a dynamic programming algorithm.

The system also computes the classical risk measures, duration (the sensitivity of the Eurobond price to changes in interest rates) and convexity (the sensitivity of the duration to changes in interest rates), taking into account the embedded options, as well as price and hedge of American and European options on these bonds\(^\text{11}\). Note that due to the non-linear behavior of options, the risk measures, duration and convexity, will be different if we consider an increase or decrease in interest rates. That is, the direction of a movement in the term structure will affect the risk measures as will be shown in the next section.

As discussed and estimated in Gonçalves and Issler [1996], the values of the parameters \(a\) and \(\sigma\) are crucial in the implementation and use of the pricing system. Next section will present a sensitivity analysis of these parameters on the value of the spread.

\(^{11}\) For options pricing with the Eurobonds as underlying asset, an additional pricing tree is generated on top of the bond tree described above.
Figure 2 below presents an schematic description of the pricing and risk system as a whole.

![Diagram of pricing and risk system]

**Figure 2: Description of the Pricing and Risk System**

**Sensitivity Analysis**

In this section a comparison between the Eurobonds' static and dynamic spreads, their respective risk measures, duration and convexity, as well as the spread sensitivity to short rate parameters, are presented.

The securities data and analysis date refers to November 27, 1996. The US term structure observed in the market at that date is given at table 1 below.

**Table 1: US Observed Term Structure**

<table>
<thead>
<tr>
<th>Analysis Date</th>
<th>3 Months</th>
<th>6 Months</th>
<th>1 Year</th>
<th>3 Years</th>
<th>5 Years</th>
<th>30 Years</th>
</tr>
</thead>
<tbody>
<tr>
<td>11/27/96</td>
<td>5.14</td>
<td>5.25</td>
<td>5.38</td>
<td>5.76</td>
<td>5.91</td>
<td>6.44</td>
</tr>
</tbody>
</table>

12 This information as well as the Eurobonds terms and conditions, was given at the Bloomberg Financial Markets.
Table 2 below presents the eight Brazilian Eurobonds considered in the analysis, assuming a face value of USD 100.00.

<table>
<thead>
<tr>
<th>Firm</th>
<th>Mar. Price</th>
<th>Maturity</th>
<th>Coupon</th>
<th>Call Date</th>
<th>Strike</th>
<th>Put Date</th>
<th>Strike</th>
</tr>
</thead>
<tbody>
<tr>
<td>Acesita</td>
<td>100.00</td>
<td>10/15/2004</td>
<td>11.125</td>
<td>-</td>
<td>-</td>
<td>10/15/2001</td>
<td>99.25</td>
</tr>
<tr>
<td>Bahia Sul</td>
<td>102.625</td>
<td>07/10/2004</td>
<td>10.625</td>
<td>07/10/2001</td>
<td>100.00</td>
<td>07/10/2001</td>
<td>99.375</td>
</tr>
<tr>
<td>Copene</td>
<td>100.00</td>
<td>10/19/2001</td>
<td>9.50</td>
<td>-</td>
<td>-</td>
<td>10/19/1998</td>
<td>98.781</td>
</tr>
<tr>
<td>CVRD</td>
<td>103.750</td>
<td>04/02/2004</td>
<td>10.00</td>
<td>04/02/2001</td>
<td>100.50</td>
<td>04/02/2001</td>
<td>99.80</td>
</tr>
<tr>
<td>Eletrobrás</td>
<td>102.375</td>
<td>07/06/2004</td>
<td>10.00</td>
<td>-</td>
<td>-</td>
<td>07/06/2001</td>
<td>98.125</td>
</tr>
<tr>
<td>Klabin</td>
<td>102.875</td>
<td>08/12/2004</td>
<td>11.00</td>
<td>08/12/2000</td>
<td>99.20</td>
<td>08/12/2000</td>
<td>98.50</td>
</tr>
<tr>
<td>Net Sat</td>
<td>107.00</td>
<td>08/05/2004</td>
<td>12.750</td>
<td>from 8/5/2001 to 9/3/2001</td>
<td>107.00</td>
<td>08/15/2001</td>
<td>101.00</td>
</tr>
<tr>
<td>Opp Petr.</td>
<td>102.250</td>
<td>02/23/2004</td>
<td>11.50</td>
<td>-</td>
<td>-</td>
<td>02/23/1999</td>
<td>99.00</td>
</tr>
</tbody>
</table>

Table 3 below presents the results of the pricing system for the dynamic and static spreads expressed in basis points (bps). The parameters of the instantaneous rate used in the computations are the ones given in Gonçalves and Issler [1996], that is, $\alpha = 0.0014$ and $\sigma = 0.0146$.

<table>
<thead>
<tr>
<th>Firm</th>
<th>Static Spread</th>
<th>Dynamic Spread</th>
<th>Difference Static-Dyna</th>
</tr>
</thead>
<tbody>
<tr>
<td>Acesita</td>
<td>510.44</td>
<td>529</td>
<td>-18.56</td>
</tr>
<tr>
<td>Bahia Sul</td>
<td>473.21</td>
<td>379</td>
<td>44.21</td>
</tr>
<tr>
<td>Copene</td>
<td>364.74</td>
<td>394</td>
<td>-29.26</td>
</tr>
<tr>
<td>CVRD</td>
<td>333.34</td>
<td>291</td>
<td>42.34</td>
</tr>
<tr>
<td>Eletrobrás</td>
<td>413.02</td>
<td>374</td>
<td>39.02</td>
</tr>
<tr>
<td>Klabin</td>
<td>486.10</td>
<td>378</td>
<td>98.10</td>
</tr>
<tr>
<td>Net Sat</td>
<td>652.65</td>
<td>515</td>
<td>137.65</td>
</tr>
<tr>
<td>Opp Petroquímica</td>
<td>545.23</td>
<td>550</td>
<td>-4.77</td>
</tr>
</tbody>
</table>

First, note that Acesita, Copene and Opp Petroquímica bonds have the difference between the static and dynamic spreads smaller than 30 bps, indicating a low option sensitivity for the implied risk spread, given their market prices. The Net Sat bond had the most expressive difference, 134 bps, due to the somewhat low strike price of its embedded call.

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13 Market prices given by Reuters.
14 Prices computed at ex-coupon dates, excluding accrued interest.
15 100 bps = 1 %.
option 107, vis-à-vis a market price of 107 and coupons of 12.50 and the fact that this bond can be called any time during one month and it is putable during only one day, making this embedded call option very valuable and thus reducing the potential gains of this bond. The Klabin bond has also a large difference between the static and dynamic spreads, 98 bps. Only three Eurobonds had their dynamic spreads higher than the static: Acesita, Copene and OPP Petroquímica. These are the “cheap” bonds and the remaining are the “expensive” ones.

Table 4 below presents the risk measures, duration and convexity, for the static, dynamic and static up to the first option date duration and convexity. These measures are given in USD assuming a face value of USD 100.00 for the bond. The table shows both positive duration (PD) and convexity (PC) as the risk measures due to an increase in the observed term structure of 100 bps, and negative duration (ND) and convexity (NC) as the measures of risk due to a decrease of 100 bps in the observed term structure.

<table>
<thead>
<tr>
<th>Firm</th>
<th>Dynamic</th>
<th>Static</th>
<th>1st Option Static</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>PD</td>
<td>ND</td>
<td>PC</td>
</tr>
<tr>
<td>Acesita</td>
<td>4.49</td>
<td>4.96</td>
<td>0.34</td>
</tr>
<tr>
<td>Bahia Sul</td>
<td>3.68</td>
<td>3.85</td>
<td>0.16</td>
</tr>
<tr>
<td>Copene</td>
<td>2.67</td>
<td>3.21</td>
<td>0.31</td>
</tr>
<tr>
<td>CVRD</td>
<td>3.69</td>
<td>3.84</td>
<td>0.15</td>
</tr>
<tr>
<td>Eletrobr.</td>
<td>4.44</td>
<td>4.91</td>
<td>0.34</td>
</tr>
<tr>
<td>Klabin</td>
<td>3.12</td>
<td>3.23</td>
<td>0.12</td>
</tr>
<tr>
<td>Net Sat</td>
<td>3.62</td>
<td>3.78</td>
<td>0.15</td>
</tr>
<tr>
<td>Opp Petr.</td>
<td>3.43</td>
<td>4.11</td>
<td>0.45</td>
</tr>
</tbody>
</table>

Note that as for the risk measures, duration and convexity, the difference between the static and dynamic cases, increases as the intrinsic value of the embedded options increase. Copene was the Eurobond most affected in terms of risk, indicating that its embedded option is “at-the-money”. Note also that the embedded options tends to reduce the duration, while for the convexity, the options may either reduce or decrease its value.

Next, a study of the effects on the spread due to variations in the short rate parameters is presented. Figures 3 to 6 below present the sensitivities of the dynamic spread to the parameters of the instantaneous rate, a (speed of mean reversion) and σ (volatility) for the Eurobonds of Bahia Sul, CVRD, OPP Petroquímica and Net Sat. Pictures on the left show the variation of the DOAS against volatility (varying from 0.005 to 0.1000) for two values

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16 For instance, for an increase in the observed term structure of a 100 bps, the OPP bond will have its price reduced in 3.43 USD, and for a reduction of 100 bps, its price will increase in 4.11 USD, for the dynamic duration case. For the case of convexity, a negative sign implies a reduction and an increase in duration, respectively.

17 This is because the embedded options will be exercised in some states of nature, “shortening” the life of the bond.
of \( a \) (0.0014 and 0.1000). Pictures on the right show the variation of the \( \text{DOAS} \) against the speed of mean reversion, \( a \) (varying from 0.001 to 0.150) for three values of volatility (0.005, 0.015 and 0.050).

Figure 3: Sensitivity of the spread to short rate parameters

Figure 4: Sensitivity of the spread to short rate parameters

Figure 5: Sensitivity of the spread to short rate parameters
The above figures indicate that OPP Petroquímica has the most sensitive dynamic spreads to short rate parameters. In this case, the risk spread increases in more than a 100% when the volatility changes from 0.005 to 0.1000. For the case of Net Sat, the effects of volatility parameters on spreads was also considerable. The effect of short rate volatility parameter values on the remaining bonds was small, in the order of 10 bps or less. Finally, the effect of variations in the speed of mean reversion was not substantial for most bonds, with the exception of the Opp Petroquímica bond, which experienced changes in the risk spread of up to 100 bps.

Also of interest is the shape of the spread versus parameters curves. For the case of spread versus volatility curves, for Bahia Sul and CVRD, they are convex, with the spread decreasing with an increase in volatility, fixing the speed of mean reversion. For the case of Net Sat the spread volatility curve is concave, increasing and then decreasing the spread, as volatility increases. The Opp Petroquímica spread-volatility curves are linear, with spread increasing with volatility.

For the case of spreads against mean reversion, Bahia Sul and CVRD have their spreads increasing with the speed of mean reversion, while for Opp Petroquímica, the spread is decreasing. Interestingly, for the case of Net Sat, depending on the value of the volatility parameter, the spread sensitivity curve can be upward sloping, downward sloping or flat.

The above analysis indicate that the interactions of the market dynamics, expressed by the short rate parameters, and the risk spread is quite intricate, possibly non-linear, and highly dependent on the terms and conditions of the embedded options.

**Conclusions**

Results from a pricing and risk system for Brazilian Eurobonds were presented, taking into account their embedded options. It was shown through several examples, that these embedded options may substantially affect the price and risk of these bonds.
The implemented pricing model takes into consideration the evolution of the term structure along time, is compatible with the observable term structure, computes the risk measures duration and convexity taking into account the embedded options and computes the price and hedge of options having the Eurobond as underlying asset. The system was developed using the C++ programming language and the object oriented paradigm, exploiting the relations between the term structure, bonds and options.

The presence of embedded options affect the magnitude of the spread over the US yield curve as well as the risk measures duration and convexity. It is shown that the size of these measures will depend on the intrinsic value of the embedded options as well as the direction of the movement of the term structure. The sensitivity of the dynamic risk spread to parameters of the instantaneous rate of the Gaussian model, was also investigated and it was shown that the magnitude of the spread was also affected by parameters for bonds with high embedded options values. Furthermore, it was shown that the way the market parameters and risk spreads interact is highly dependent on the embedded options terms and conditions.

In terms of future research, the inclusion of a second stochastic factor in order to account for the systemic country risk (“Brazil Risk”) is under development. It will then be possible to determine the idiosyncratic or credit risk of Brazilian Eurobonds taking into account the dynamics of “Brazil” risk in addition to the US yield curve.

References


