"Hedging Options in a GARCH Environment: Testing the Term Structure of Stochastic Volatility Models"

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Hedging Options in a GARCH Environment:

Testing the term structure of stochastic volatility models

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Abstract

This paper develops a methodology for testing the term structure of volatility forecasts derived from stochastic volatility models, and implements it to analyze models of S&P500 index volatility. Using measurements of the ability of volatility models to hedge and value term structure dependent option positions, we find that hedging tests support the Black-Scholes delta and gamma hedges, but not the simple vega hedge when there is no model of the term structure of volatility. With various models, it is difficult to improve on a simple gamma hedge assuming constant volatility. Of the volatility models, the GARCH components estimate of term structure is preferred. Valuation tests indicate that all the models contain term structure information not incorporated in market prices.
I. Introduction

Estimating the term structure of volatility has, in the past, focused on option implied volatility. For instance, Stein (1989) estimated the term structure of volatility using an autoregressive volatility model based on short term S&P100 option implied standard deviations. He found the actual sensitivity of medium term to short term implieds was greater than the estimated sensitivity from the term structure, and concluded that option markets overreact to information. Diz and Finucane (1993) rejected the overreaction hypothesis using different estimation techniques.

Heynen et. al. (1994) estimated the term structure of volatility by comparing how well elasticity parameters generated by autoregressive volatility (ARV), GARCH, and EGARCH models explained the relationship between long term and short term implied variances for Philips and the EOE index. The EGARCH model was found to best model this relationship, and, thus to represent the best estimate of the term structure of volatility. Xu and Taylor (1994) used regression and Kalman filter techniques to fit a term structure model to the time series of forward implied variances for currency options.

Since we cannot observe actual market volatility, tests of the performance of the term structure implied by different volatility models necessarily take an indirect form. The previous papers use the implied volatility of different maturity options as point estimates of the term structure of average volatility. This paper takes a different approach. Since option prices are an observable feature, and their behavior is determined by the underlying asset's volatility, tests based on option prices provide another metric. We perform two types of tests using option prices.
First, volatility models are compared by their success in constructing minimum risk portfolios of medium term and short term options, and, second by their ability to obtain excess profits in options trading.

In the first case, a medium term option (call, put or straddle) is held and hedged with other assets. The hedge is initially designed to eliminate first order price risk which can be undertaken by selling or buying an appropriate amount (delta) of the underlying stock. A more effective hedge also involves selling a second option to eliminate the second order risk associated with non-linearity of the price response (gamma) or the risk associated with a volatility shock (vega). It is assumed that this hedge should be constructed with the most similar available short term option.

The optimal number of short term options to hold per long term option, the hedge ratio, depends on sensitivity of the options prices to volatilities and on the estimated term structure of volatility. If movements in a state variable affect primarily short volatilities rather than long volatilities, then it will take relatively few short options to hedge this risk. The term structure of volatility forecasts measures exactly this response. In other words, understanding the term structure of expected average volatility, which generates the term structure of sensitivities, is an essential element to the hedging process.

At the end of the paper the models are used to check for mispricing of options due to the term structure. Time spreads consist of identical options with different maturities. Hence, accurate valuation requires correct estimation of the levels of medium and short term average expected volatility. Purchasing a time spread is a bet that short term volatility is underestimated and medium term volatility is overestimated by the market, while selling a time spread is a bet on the converse.
Another term structure dependent position is a jelly roll, which consists of one call
time spread held long and one put time spread held short.

This paper is structured as follows. In Section II option pricing is discussed and in
section III, a methodology for hedging options in discrete time is developed, since
the stochastic volatility models used are in discrete time. Section IV derives and
interprets hedging ratios for the stochastic volatility models. In Section V, the
stochastic volatility models are estimated and their performance is tested by
comparing hedging effectiveness and options trading profits. Finally in Section VI,
some conclusions are drawn.

II. Pricing Options in a Stochastic Volatility Environment

Pricing options in a stochastic volatility environment is not a solved problem, at
least in practice. Theoretically, the value of a European style put option which
eliminates arbitrage possibilities can be found from

\[ P_t = E_t^\circ [\max(K - S_T, 0)] \]

where the expectation is taken with respect to the risk neutral distribution as of
time t. In the equation, K is the strike price, S is the underlying price and T is the
expiration date and the risk free rate of interest is taken to be zero for ease of
notation. This expression is only useful once the risk free conditional distribution
is specified and this paper can be thought of as seeking useful parameterizations.

When the underlying asset follows a geometric Brownian motion with
constant volatility, then it is well known that a solution to (1) is the Black-
Scholes(1973) formula which can be written as
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(2) \[ P_t = BS(\sigma, S_t, T) \]

where \( \sigma \) is the constant volatility which is also the average daily volatility over the life of the option. Hull and White (1987) point out that if volatility is independent of the stock price path, then the expectation conditional on average volatility, can be taken conditionally in (1) to get

(3) \[ P_t = E_t^o \left[ E_t^o \left[ \max(K - S_T, 0) \right] | \sigma \right] = E_t^o \left[ BS(\sigma, S_t, T) \right] \]

as long as \( S_T \) is conditionally lognormal. For at-the-money options, the Black Scholes formula is approximately linear in volatility and therefore a striking simplification of Hull and White is the Black Scholes-Plug-in or BSP given as

(4) \[ P_t = BSP(E_t^o(\sigma), S_t, T) \]

Various options pricing formulae result from various assumptions about the process of the volatility.

In each case however, it is natural to suppose that the most recent stock price \( S_t \), which is known would have potential value in forecasting average volatility over the life of an option. Thus the optimal forecast will be expressed as a function the stock price.

(5) \[ P_t = BSP(\tilde{\sigma}(S_t), S_t, T) \]

Hedging parameters are then derived by differentiating BSP with respect to the volatility model’s random variables as well as the familiar variables in BS.

An approach which does not rely on the plug-in formulation can be implemented directly from (1) using simulation methods and particular specifications of the risk neutral distribution.
stochastic volatility process and will be combinations of the BS delta, gamma and vega. For example, the stochastic volatility (SV) delta is given by

\[
\Delta_{SV} = \Delta_{BS} + \Lambda_{BS} \frac{1}{2\hat{\sigma}} \frac{\partial^2 \hat{\sigma}^2}{\partial S^2} \bigg|_{S_{t+1} = S_t}
\]

For all the volatility processes used here, the first derivative of variance with respect to price is zero since both positive and negative price movements increase volatility. However, models such as the EGARCH and AGARCH will have non-zero first derivatives and will therefore affect the delta's.

More interestingly, the gammas are potentially very different in a stochastic volatility environment from those in a Black Scholes environment.

\[
\Gamma_{SV} = \Gamma_{BS} + \Lambda_{BS} \frac{1}{2\hat{\sigma}} \frac{\partial^2 \hat{\sigma}^2}{\partial S^2} \bigg|_{S_{t+1} = S_t}
\]

The gamma effectively incorporates both a volatility hedge and a change in delta hedge. No longer is volatility hedging different from simply responding to the magnitude of possible changes in \(S\). For each proposed volatility model there is a different gamma which will be labeled for the volatility process. For example, if the underlying process is assumed to be a GARCH process, then the hedge parameter will be called a GARCH gamma.

The optimal hedge for an option position, will form a portfolio which reduces the exposure to various risk factors. Thus it forms portfolios which have zero derivatives with respect to underlying state variables. The easiest way to form a delta neutral hedge is to short \(1/\Delta\), of the underlying asset. Since a put has a negative delta, the hedge involves a long position in the underlying asset. A
straddle will typically have a delta close to zero already and therefore only small amounts of the underlying asset need be added to the portfolio.

To form a vega hedge, a second option contract is required. As both assets will have vega's the optimal portfolio is short $\Lambda_1/\Lambda_2$ contracts of the second option. Similarly, gamma hedge ratios will be given by the ratio of the gammas, $\Gamma_1/\Gamma_2$. In each case, the hedge ratio is the number of contracts of the second option which are used to hedge the first.

In order to compare the effectiveness of various hedging strategies, a series of hedged portfolios will be constructed based upon a particular theory of how the hedging should be accomplished. Each position will be held for one day. The returns on this series of portfolios will then be examined for effectiveness in hedging risks. The simplest measure of success is simply the variance of the portfolio, however measures which are insensitive to big shocks such as the inter-quartile range are also constructed. In addition, the portfolio returns are regressed on the underlying return and return squared to see whether these risk factors are effectively eliminated. Finally the portfolio returns are checked for serial correlation and ARCH. Any finding of serial correlation suggests a mispricing of the assets but there is no reason to suspect these portfolios would have constant variances so the ARCH test is merely descriptive.

III. Stochastic Volatility Models

Four models of the volatility process are examined in this paper: the constant volatility model (CV), the autoregressive volatility model (ARV) which infers the term structure from implied volatilities, the GARCH(1,1) model, and the GARCH components model. Details about each of the models are in Appendix A. These
models differ in structure and in the type of fundamental shocks they allow to drive the volatility process. In the CV model, no shock affects volatility, while in the ARV model, shocks independent of the underlying price process drive the volatility process. In the GARCH models, squared price shocks drive the volatility process. The types of shocks that must be hedged in the option portfolio depend on the volatility model.

Each of these models implies a different term structure of volatility. The CV model leads to a flat term structure while the others generally allow it to have either an upward or downward shape.

Begin by specifying the ARV model as this is an approximation of what sophisticated traders use on Wall Street and has been described by Stein (1989) and Heynen, Kemna and Vorst (1994). Let $\sigma_{1t}$ be the implied volatility of the first option contract on day $t$ and $\sigma_{2t}$ the implied volatility of the second contract which is being used for hedging. These can be related by a regression:

$$
\sigma_{1t}^2 = \lambda \sigma_{2t}^2 + \mu + \eta_t
$$

in which $\lambda$ could be estimated by least squares and is interpreted as the partial derivative of the first variance with respect to the second variance. Because of the substantial serial correlation in $\eta_t$, this can perhaps be better estimated in differenced form:

$$
\Delta \sigma_{1t}^2 = \lambda \Delta \sigma_{2t}^2 + \eta_t
$$
This $\lambda$ can be used to construct optimal vega hedge ratios. Since the long volatility is estimated to move only a fraction $\lambda$ as much as the short, the vega hedge ratio becomes $\Lambda_1 \lambda \sigma_2 / \Lambda_2 \sigma_1$.

This lambda however refers only to one pair of maturities so that it is not very useful. Because each of these volatilities is the average of the volatilities for the remaining life of an option, it is natural to parameterize these in terms of the one day volatility parameter.

\begin{equation}
\sigma_t^2 = \rho (\sigma_{t-1}^2 - \omega) + \omega + \epsilon_t
\end{equation}

where $\omega$ is the long run constant variance, so that

\begin{equation}
\sigma_{it}^2 = \omega + \frac{\left(1 - \rho_{T_i}^t\right)}{(1 - \rho)} \frac{1}{T_i} (\sigma_t^2 - \omega)
\end{equation}

so that the lambda for contracts with maturity $T_1$ and $T_2$ is:

\begin{equation}
\lambda_{12t} = \frac{T_2}{T_1} \left[ \frac{(1 - \rho_{T_1}^t)}{(1 - \rho_{T_2}^t)} \right]
\end{equation}

This allows time variation in the hedge ratio and sensitivity to the maturity of the contracts. To estimate $\rho$, requires simply backing it out from (13) where a particular $\lambda$ is estimated from the data on implieds. Notice that there is no historical data on the underlying asset used in this procedure.
The GARCH model does use historical data from the underlying asset to estimate the process of volatility; these parameters are then used to forecast the term structure of volatility. The GARCH model can be expressed as

\[ \sigma_t^2 = \omega + \alpha (\varepsilon_{t-1}^2 - \omega) + \beta (\sigma_{t-1}^2 - \omega) \]  

(14)

where \( \varepsilon_t = \log(S_t/S_{t-1}) \), \( \sigma^2 \) is the one day volatility, and \( \omega \) is the long run volatility. For this model, multistep forecasts are easily computed from the one step conditional variances. The average variance from \( t \) until \( T \) is given by

\[ \sigma_{tt}^2 = \omega + \frac{1}{T_{t}} \left[ \frac{1 - (\alpha + \beta)^T_{t}}{1 - \alpha - \beta} \right] (\sigma_{t+1}^2 - \omega) \]  

(15)

This model, like the ARV model, implies a monotonic upward or downwards sloping term structure which mean-reverts at a rate \( \alpha + \beta \).

The GARCH components model was proposed by Engle and Lee(1993) and allows more complex lag distributions. It models volatility as mean reverting to a long run component of volatility, but this long run component itself mean reverts to a constant level. The process for volatility can be written as:

\[ \sigma_t^2 = q_t + \alpha (\varepsilon_{t-1}^2 - q_t) + \beta (\sigma_{t-1}^2 - q_t) \]

(16)

\[ q_t = \omega + \rho (q_{t-1} - \omega) + \phi (\varepsilon_{t-1}^2 - \sigma_{t-1}^2) \]

where \( q \) is the long run component which mean reverts to \( \omega \) with a rate \( \rho \), while instantaneous volatility \( \sigma \) mean reverts to \( q \) at a rate \( \alpha + \beta \). This term structure is not necessarily monotonic since the day ahead, trend, and long term forecast all
influence the n-step ahead forecasts. The forecast of the average volatility from \( t \) to \( T_1 \) is now given by:

\[
\sigma_{t+1}^2 = \omega + \frac{1}{T_1} \left[ \frac{1-(\alpha+\beta)^{T_1}}{1-\alpha-\beta} \right] (\sigma_{t+1}^2 - q_{t+1}) + \frac{1}{T_1} \left[ \frac{1-(\rho)^{T_1}}{1-\rho} \right] (q_{t+1} - \omega)
\]

To compute GARCH gammas, the derivatives of (15) and (17) are simply substituted into (8). Since these derivatives are taken with respect to \( S_{t+1} \), these formulae are derived by first advancing time one day further and then recognizing that there are only \( T_1-1 \) days remaining in the contract.

To get a sense of the differences between these hedging parameters, consider first the Black Scholes hedge ratios for a standard maturity where the long option has 30 days to maturity while the option used to hedge has only 10 days to maturity. In this case the familiar vega hedge shorts 1.73 of the near contracts for each far contract. The hedge ratio exceeds one which is counterintuitive but is a reflection of the increase in vega with maturity. After all, this corresponds to an experiment where the volatility parameter is changed once and for all and therefore has a bigger impact on the longer lived options.

In contrast, the BS gamma hedge ratio is only .57 indicating just over half a near contract be shorted for each far contract. Again, this is intuitive since the second derivative of the current stock price has dramatically reduced impact over longer horizons. For at-the-money options, the term structure of gamma is upward sloping indicating that nonlinear impact of a price shock is greater for short term options than longer term options. This implies that the gamma hedge ratio is less than one, which is consistent with intuition. In contrast, away-from-the-money, the term structure of gamma is downward sloping, since close to maturity in-the-
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money options behave like the underlying asset, and out-of-the-money options are insensitive to underlying price moves. Thus, the out-of-the-money hedge ratio will be greater than one. See charts 3 and 4.

The hedge parameters from the stochastic volatility models are linear combinations of these two BS parameters. The ARV model operates through vega so that the mean reversion in volatility counteracts the rise in vega. For this standard case and the estimated model, the hedge ratio is .99 requiring a one-for-one hedge strategy.

The GARCH gamma using these maturities, estimated parameters and typical values is .90 while the GARCH components model is .78. Since these are linear combinations of the BS gamma and vega, they will lie between these extremes. The shape of the term structure and the persistence of shocks determines how these are weighted. In general, if only near variances are sensitive to volatility shocks, then the weights will give more emphasis to the BS gamma. If the process is IGARCH, then more weight will be given to BS vega.

V. Results

We compare the implied term structures of the four volatility models by constructing hedges for near-the-money medium term calls, puts, and straddles using short term calls, puts, and straddles. The best model should construct the lowest risk portfolios. In addition, we compare the term structures by measuring their profits in trading time spreads and jelly rolls.

This paper uses daily data for the nearest-to-the-money Standard and Poor's 500 Index put and call option with closest and next-closest maturities from October
1985 through February 1992. The data was gathered by Chicago Board Options Exchange. Only the 654 data points for which option prices are available for the medium and short term calls and puts are included in the analysis. Options that are further than five percent from the money are excluded. See Table 1 for details.

**Estimating the Volatility Models**

The first step in the testing procedure was estimation of the stochastic volatility models. See Table 2 for details. The ARV model was estimated using an OLS regression of differenced implied medium term variance on the differenced implied short term variance. The beta from this regression, .54, implies a rho of .93, assuming that the long term option has 37 days to maturity and the short term option 13. Diz and Finucane use Maximum Likelihood Estimation to arrive at beta parameters ranging from .43 to .55 for S&P100 implieds. The rho implied by these betas range from .92 to .94.

From the implieds, there is strong evidence for mean reversion in volatility. The change in medium term implied volatility is a fraction of the change in the short term implied. Also, short term implied variances and their first differences are more volatile than for medium term. That is, the volatility of volatility is declining with maturity. This is consistent with raw data for option prices which indicates that short term option portfolios are substantially more volatile than medium term portfolios.

The GARCH(1,1) and GARCH components models are estimated using maximum likelihood estimation. The GARCH(1,1) model assumes underlying normal density, and the components model assumes that the underlying density is Student's-t. The GARCH decay parameter is .97 which implies mean reversion in
volatility and a declining volatility of volatility. The components model also exhibits this pattern. The CV model uses the one step ahead GARCH(1,1) forecast as the estimate of 'constant' volatility.

Using these models, it is straightforward to estimate hedge ratios. For an at-the-money call position, all hedge ratios are less than one except for the CV and ARV vega hedges. This is due to the strong upward slope of the vega term structure. In addition, the slope of the term structure of hedge ratios flattens for all models except the CV and ARV. The CV gamma hedge and the GARCH components hedge have the lowest hedge parameters and are also closest to the empirical best hedge parameters. Details are in Table 3.

Hedging tests

The first test of the term structure implied by the volatility models is the hedging test. It is implemented as follows. Each trading day, three one hundred dollar portfolios are purchased. One contains medium term calls, one medium term puts, and one medium term 1:1 straddles. Then, hedge ratios are calculated based on the appropriate formula from each stochastic volatility model. In each portfolio, delta shares of the underlying are sold short, generating a zero exposure to first order price shocks.

For the ARV model, each portfolio is vega hedged by selling the appropriate number of short term options. For the GARCH(1,1) and components model, each portfolio is gamma hedged by selling short term options. The short contract is designed to match the long contract so that when long calls are being hedged, a short call is sold with the same strike. The same is true for puts and straddles. Finally, the entire portfolio is delta hedged with the composite delta. Two
The second best model, the GARCH components model, reduced risk over the delta hedge by 1-10%. Its success may be due its flexible parameterization of the term structure of volatility, which generates faster time decay, and lower hedge ratios.

The GARCH(1,1) model was unable to improve on the delta hedge for calls, and reduced risk by 2-5% for puts and straddles. The portfolios with hedge ratios greater than one on average, the CV and ARV vega hedge, are the poorest performers. They are unable to improve on the delta hedge for any option position.

A perfectly hedged portfolio would have no structure in its price changes. None of the hedge portfolios are able to attain this goal. However, the price changes for hedge portfolios constructed using the CV gamma and GARCH components models are uncorrelated with the index price change and square. No model consistently generates hedge price changes free of autocorrelation and heteroscedasticity.

**Interpretation of the Results**

It is rather surprising that the best hedge is the BS gamma hedge. Essentially this says that the best assumption about volatility is that it is constant. This is clearly totally inconsistent with the movements in both implied and historical volatilities.

Is it possible that we have simply considered the wrong stochastic volatility models? While this is of course true, it will not resolve the paradox. Any stochastic volatility process will add some vega to the Black Scholes gamma and will generally give too large a hedge ratio. Only if movements in short term
volatility are systematically reversed after the short term option expires can the hedge ratio be reduced; this seems highly implausible as a continuing process.

Another potential explanation for this puzzle is that the Black Scholes Plug-in approach to option pricing is systematically incorrect and produces too large hedge ratios. To examine this possibility, we revert to the basic option pricing model in equation (1) and simulate the risk neutral terminal distribution under several assumptions. In particular, this distribution is simulated under the GARCH(1,1) and component GARCH processes estimated in this paper. In each case, the standardized shocks are drawn from a student-t distribution with 6 degrees of freedom to represent the leptokurtosis in the standardized residuals from any volatility model. These are simulated for 40 days with 10,000 replications and option prices are calculated as the expected payoff from this risk neutral distribution. The simulation is then restarted with the same random numbers and $S_{t+1}$ increased and decreased by .5%. Then the three option prices all with the same strike and maturity are used to estimate the second derivative. This is a numerical approach to correctly calculating the GARCH gamma without using the Black Scholes formula at all.

The results are rather surprising. The simulated values are very close to the BSP values and much higher than the BS gammas. Even the GARCH component model with leverage which is simulated with just the same persistence as the component model, has only slightly lower hedge ratio. There is no clear definition of the BSP for this case, as the function is non-differentiable.
HEDGE RATIOS 30days/10days

<table>
<thead>
<tr>
<th>Model</th>
<th>Simulated</th>
<th>BSP</th>
<th>BS</th>
</tr>
</thead>
<tbody>
<tr>
<td>GARCH</td>
<td>.83</td>
<td>.90</td>
<td>.57</td>
</tr>
<tr>
<td>GARCH component</td>
<td>.78</td>
<td>.78</td>
<td>.57</td>
</tr>
<tr>
<td>GARCH component with leverage</td>
<td>.77</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

It appears that the problem with the hedge ratios is not a problem with the option pricing formulation, at least for these at-the-money options.

The remaining solution is that the options are mispriced. We consider the possibility that the market misprices these options although it is also possible that the CBOE who collected the data, did the mispricing. In either case, the theoretical and empirical hedge ratios could be different. In fact, using a portion of the same data set, Noh, Engle and Kane(1994) found substantial evidence that GARCH models could profitably suggest when to be short or long at-the-money straddles. Furthermore, Stein(1989) claims to have found that options prices overreact to news. A systematic overreaction of short maturity options would make them excessively variable and would make a low hedge ratio optimal. This possibility is explored in the next section.

Option Pricing tests

If the options market is not efficient in that it does not incorporate all available information into current prices, then it may be possible to earn excess profits by trading options. The options pricing tests we implement focus on whether there are better volatility forecasts than those impounded into market prices. We design a trading strategy that uses volatility forecasts from each stochastic volatility
model to price term structure dependent option positions. By calculating excess profits from each strategy, we can compare the accuracy of the model forecasts compared to the market forecasts.

The trading strategy we use is to calculate the market price of one option position. We then estimate the price of this position using the Black-Scholes formula, evaluated at forecast average volatility using a stochastic volatility model. The difference between the model price and the market price is the trading signal.

The trading decision is binary: only one position is purchased or sold, no matter what the magnitude of the trading signal is. If the model price is greater than the market price, we assume that the market has underpriced the option position, and purchase the position. If the model price is less than the market price, we assume the market has overpriced the position, and we sell it. At the end of each day, the position is established, and it is sold at the end of the following day. If the market prices the next day move in the direction of the model prices, the model earns a profit. Otherwise, it loses money. If the model volatility forecasts are superior to the market forecasts, then the model prices are more accurate than the market prices, and the trading strategy should earn excess profit over time. The methodology is similar to that of Noh, Engle, and Kane (1993).

In order to test the forecasts of the term structure implied by the models, we select three term structure sensitive positions for our analysis. These are a 1:1 call time spreads, a 1:1 put time spread, and a jelly roll. The time spreads consist of a medium term at-the-money option held long and a short term at-the-money option held short. The jelly roll consists of a 1:1 call time spread held long and a 1:1 put time spread held short. As a comparison to Noh et. al., we also test two volatility
sensitive positions: short term at-the-money straddles and medium term at-the-money straddles.

The four volatility models compared in the tests are the GARCH(1,1) model, the ARCH components model, the CV model, and implicitly, the ARV model. For the constant volatility model, we assume that the best forecast of future constant volatility is given by the one step ahead forecast from a GARCH(1,1) model. This would be more appropriate if the model estimated were an IGARCH model, which actually has a flat term structure of volatility, like the CV model.

The ARV model is not explicitly tested, since it implies that the prices derived from implied volatility are the correct prices. Of course, this will result in no trades, since model prices will be equal to market prices by definition. The ARV model is taken to imply a strategy of always buying one position. Of course, it is just as sensible to consider it as an always sell strategy, or probably even more reasonable is to use forecasts derived from an implied volatility regression as in Day and Lewis (1988).

The first part of Table 5 shows the average profits and standard deviation of profits for each of the four models. It is noteworthy that the 'Always buy' position earns trading profits that are not statistically different from zero at the .01 percent level for the term structure sensitive portfolios, but the trading profits are significantly negative for the volatility sensitive portfolios. This indicates that consistently selling straddles was profitable over the period, implying that the market volatility forecasts were too high on average, while the market term structure of volatilities was not biased in this way.
The GARCH models and CV model earn excess profits for all the term structure sensitive positions, while the ARV model, which uses the market forecasts earns zero profits. This indicates that there is an inefficiency in the market's processing of volatility information to forecast the term structure. For the call time spreads, the models earn an average profit of about 15 cents per position with a standard deviation of about 1 dollar. The profits are slightly higher and the volatility of price changes is lower for the put time spreads. The average prices of the call time spreads are about $4.30 compared to $3.40 for the puts.

All of the models are predominantly option sellers as shown on the Summary of Trading Signals. Each of the models, other than the ARV model, chooses to sell positions at least 80% of the time. This indicates that overall, the models find time spreads, jelly rolls, and straddles to be overpriced. In fact, all of the models, except the ARV model, are able to earn profits significantly greater than zero.

These tests indicate that the GARCH models are superior in pricing volatility sensitive portfolios, and that all of the models are able to generate excess profits for term structure dependent positions. However, the tests are not conclusive about the superiority of any model in estimating the term structure of volatility.

V. Conclusions

This paper provides a methodology for testing the term structure of volatility implied by stochastic volatility models, and implements it to analyze the term structure of S&P500 index volatility. Hedging tests select the Constant Volatility model using a gamma hedge, followed by the GARCH components model as best at forecasting the term structure. It is argued that the surprising success of the constant volatility model is likely to be a result of option mispricing. The
valuation tests concur, indicating the superiority of the GARCH models in pricing volatility sensitive portfolios, although all the models excel in pricing term structure dependent portfolios.
APPENDIX A.

Stochastic volatility model

ARV:
\[ \sigma^2_{t+1} - \sigma^2_t = \rho(\sigma^2_t - \sigma^2) + e_t \]

GARCH (1,1):
\[ \sigma^2_{t+1} = \omega(1-\alpha-\beta) + \alpha e^2_t + \beta \sigma^2_t \]

GARCHcomp:
\[ \sigma^2_{t+1} = q_{t+1} + \alpha(e^2_t - q_t) + \beta(\sigma^2_t - q_t) \]
\[ q_{t+1} = \omega(1-\rho) + \rho q_t + \phi(e^2_t - \sigma^2_t) \]

n-step ahead volatility

ARV:
\[ \sigma^2_{t+k|t} = \sigma^2 + \rho^k-1[\sigma^2_t - \sigma^2] \]

GARCH (1,1):
\[ \sigma^2_{t+k|t} = \omega + (\alpha + \beta)^{k-1}[\sigma^2_{t+1} - \omega] \]

GARCHcomp:
\[ \sigma^2_{t+k|t} = \omega + (\alpha + \beta)^{k-1}(\sigma^2_{t+1} - q_{t+1}) + \rho^{k-1}(q_{t+1} - \omega) \]

Average volatility over T periods

for GARCH, \( S_{t+1} = S_t \)

ARV:
\[ \sigma^2_{t+1}(T) = \sigma^2 + \frac{1}{T} \left[ \frac{1-\rho^T}{1-\rho} \right] (\sigma^2_{t+1} - \sigma^2) \]

GARCH (1,1):
\[ \sigma^2_{t+2}(T-1) = \omega + \frac{1}{T-1} \left[ \frac{1-(\alpha + \beta)^{T-1}}{1-(\alpha + \beta)} \right] (\sigma^2_{t+2} - \omega) \]

GARCHcomp:
\[ \sigma^2_{t+2}(T-1) = \omega + \frac{1}{T-1} \left[ \frac{1-(\alpha + \beta)^{T-1}}{1-(\alpha + \beta)} \right] (\sigma^2_{t+2} - q_{t+2}) + \left[ \frac{1-\rho^T}{1-\rho} \right] (q_{t+2} - \omega) \]

Sensitivity of average standard deviation to a shock

ARV:
\[ \frac{d\sigma^2_{t+1}(T)}{de_t} = \frac{1}{2\sigma^2_{t+1}(T)T} \left[ \frac{1-\rho^T}{1-\rho} \right] \]

GARCH:
\[ \frac{d\sigma^2_{t+2}(T-1)}{dS^2_{t+1}} = \frac{1}{2\sigma^2_{t+2}(T-1)} \left[ \frac{1-(\alpha + \beta)^{T-1}}{1-(\alpha + \beta)} \right] \frac{d^2\sigma^2_{t+2}}{dS^2_{t+1}} \]

GARCHcomp:
\[ \frac{d\sigma^2_{t+2}(T-1)}{dS^2_{t+1}} = \frac{1}{2\sigma^2_{t+2}(T-1)} \left[ \frac{1-(\alpha + \beta)^{T-1}}{1-(\alpha + \beta)} \right] \frac{d^2\sigma^2_{t+2}}{dS^2_{t+1}} + \left[ \frac{1-\rho^T}{1-\rho} \right] \frac{d^2q_{t+2}}{dS^2_{t+1}} \]
Elasticity of medium term average standard deviation with respect to short term average standard deviation

\[
ARV : \lambda = \frac{T_s}{T_m} \left[ 1 - \frac{\rho_{T_s}}{1 - \rho_{T_s}^r} \right] \frac{\sigma_{t+1}(T_s)}{\sigma_{t+1}(T_m)}
\]

\[
GARCH (1, 1) : \lambda = \frac{T_s - 1}{T_m - 1} \left[ 1 - \frac{(\alpha + \beta) T_m^{-1}}{1 - (\alpha + \beta) T_s^{-1}} \right] \frac{\sigma_{t+2}(T_s - 1)}{\sigma_{t+2}(T_m - 1)}
\]

\[
GARCH_{comp} : \lambda = \frac{T_s}{T_m} \frac{\sigma_{t+2}(T_s - 1)}{\sigma_{t+2}(T_m - 1)} \left[ 1 - \frac{(\alpha + \beta) T_s^{-1}}{1 - (\alpha + \beta) T_m^{-1}} \right] \frac{d^2 \sigma_{t+1}}{dS_{t+1}^2} + \frac{1 - \rho T_m^{-1}}{1 - \rho} \frac{d^2 S_{t+1}}{d^2 S_{t+1}^2}
\]

Hedge Ratios

\[
CV: \text{Gamma:} \quad \Gamma = \frac{\Gamma_m(\sigma)}{\Gamma_s(\sigma)}
\]

\[
CV: \text{Vega:} \quad \Lambda = \frac{\Lambda_m(\sigma)}{\Lambda_s(\sigma)}
\]

\[
ARV : -\frac{\lambda_m(\sigma_{t+1}(T_m)) \frac{d\sigma_{t+1}(T_m)}{d\epsilon}}{\lambda_s(\sigma_{t+1}(T_s)) \frac{d\sigma_{t+1}(T_s)}{d\epsilon}}
\]

\[
\Gamma_m(\sigma_{t+2}(T_m - 1)) + \lambda_m(\sigma_{t+2}(T_m - 1)) \frac{d^2 \sigma_{t+2}(T_m - 1)}{dS_{t+1}^2}
\]

\[
GARCH (1, 1) : -\frac{\Gamma_s(\sigma_{t+2}(T_s - 1)) + \lambda_s(\sigma_{t+2}(T_s - 1)) \frac{d^2 \sigma_{t+2}(T_s - 1)}{dS_{t+1}^2}}{\Gamma_s(\sigma_{t+2}(T_s - 1)) + \lambda_s(\sigma_{t+2}(T_s - 1)) \frac{d^2 \sigma_{t+2}(T_s - 1)}{dS_{t+1}^2}}
\]

\[
GARCH_{comp} : \text{same as GARCH}(1, 1) \text{ but with different 2d derivative}
\]
Bibliography


Table 1 - Summary of Options Data

Data gathered by the Chicago Board Options Exchange, Daily, Oct. 1987-Feb. 1992
Nearest-to-the-money options for which current and next day's price are available are used in the study.

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<th></th>
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<td>180</td>
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</table>

*Only 5 points are used from Oct. 1987, these are before Oct. 14

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<th></th>
<th>Medium Term Call Options</th>
<th>Short Term Call Options</th>
<th>Medium Term Put Options</th>
<th>Short Term Put Options</th>
<th>Medium Term Straddles</th>
<th>Short Term Straddles</th>
<th>Call 1:1 Time Spreads</th>
<th>Put 1:1 Time Spreads</th>
<th>Jelly rolls</th>
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<td>Number</td>
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<td>654</td>
<td>654</td>
<td>654</td>
<td>654</td>
<td>654</td>
<td>654</td>
<td>654</td>
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<td>3.131</td>
<td>2.330</td>
<td>2.559</td>
<td>2.071</td>
<td>2.559</td>
<td>4.733</td>
<td>0.966</td>
<td>1.548</td>
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<td>13.5</td>
<td>37.2</td>
<td>13.5</td>
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<td>13.7</td>
<td>6.5</td>
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Table 2 - Estimation of Stochastic Volatility Models

Autoregressive Volatility Model
Regression of medium term differenced implied variance on short term
Daily implied variance used is average of put and call implied variances
678 observations based on data availability from Oct. 1985-Feb. 1992

<table>
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<tr>
<th></th>
<th>Coefficient</th>
<th>Std Error</th>
<th>t-stat</th>
<th>Prob &gt;</th>
<th>Adj-R²</th>
<th>DW</th>
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<tr>
<td>In t</td>
<td>0.0000</td>
<td>0.0001</td>
<td>-0.27</td>
<td>0.7873</td>
<td>0.7879</td>
<td>2.672</td>
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<td>Beta</td>
<td>0.5418</td>
<td>0.0108</td>
<td>50.19</td>
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<tr>
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<td>0.9325</td>
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GARCH(1,1)
Daily returns (price appreciation) for S&P500 index from CRSP
4551 observations (1975-1992)
Maximum Likelihood estimation with Normal as the underlying density

<table>
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<tr>
<th></th>
<th>Coefficient</th>
<th>Std Error</th>
<th>t-stat</th>
<th>Prob &gt;</th>
<th>Robust t-stat</th>
<th>Ljung-Box(15)</th>
<th>Jarque-Bera(1980) normality test</th>
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</thead>
<tbody>
<tr>
<td>Omega (ω)</td>
<td>0.0000</td>
<td>0.0000</td>
<td>7.95</td>
<td>0.0000</td>
<td>2.6404</td>
<td>10.26</td>
<td>5808.86</td>
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<td>Alpha (α)</td>
<td>0.0671</td>
<td>0.0018</td>
<td>37.14</td>
<td>0.0399</td>
<td>1.6829</td>
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<tr>
<td>Beta (β)</td>
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<td>0.0049</td>
<td>18.796</td>
<td>0.0398</td>
<td>22.9188</td>
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GARCH Components Model
Daily returns (price appreciation) for S&P500 index from CRSP
4551 observations (1975-1992)
Maximum Likelihood estimation with Students-t as the underlying density

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<tr>
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<th>Coefficient</th>
<th>Std Error</th>
<th>t-stat</th>
<th>Robust t-stat</th>
<th>Ljung-Box(15)</th>
<th>Jarque-Bera(1980) normality test</th>
</tr>
</thead>
<tbody>
<tr>
<td>Omega (ω)</td>
<td>0.0000</td>
<td>0.0000</td>
<td>1.27</td>
<td>0.0000</td>
<td>0.7238</td>
<td>25.08</td>
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<td>Alpha (α)</td>
<td>0.0139</td>
<td>0.0405</td>
<td>0.34</td>
<td>0.0480</td>
<td>0.2897</td>
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<td>Beta (β)</td>
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<td>0.0377</td>
<td>25.37</td>
<td>0.1310</td>
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<td>Phi (φ)</td>
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<td>0.0422</td>
<td>0.58</td>
<td>0.0885</td>
<td>0.2767</td>
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<td>rho</td>
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<td>0.0087</td>
<td>113.96</td>
<td>0.0122</td>
<td>80.9810</td>
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</table>
Table 3 - Hedge Ratios

Number of short term options to hold per long term option
S=200, K=200, h(t+1)=sqrt(q(t+1))=i(medium)=i(short)=sig=.01, rf=.0002

<table>
<thead>
<tr>
<th>Model</th>
<th>Medium Mat=20 / Short=5</th>
<th>Medium Mat=30 / Short=10</th>
<th>Medium Mat=40 / Short=20</th>
<th>Average Hedge Ratio over sample</th>
</tr>
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<tbody>
<tr>
<td>B-S Gamma</td>
<td>-0.50</td>
<td>-0.57</td>
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<tr>
<td>B-S Vega</td>
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<td>GARCH components</td>
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<tr>
<td>Hedge, Straddles</td>
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<td>Options</td>
<td>-0.55</td>
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</table>
Table 4 - Option Hedging Results

Buy 100$ portfolio of medium term positions. Delta and gamma or vega hedge. Rebalance daily.
Data: CBOE S&P500 Index Options (Oct. 1, 1985 - Feb. 28, 1992)
Ljung-Box and ARCH tests are on first six lags of portfolio return
Regression is portfolio price change on contemporaneous S&P500 Index change and square
Delta hedge uses GARCH(1,1) volatility forecast

<table>
<thead>
<tr>
<th>Hedging Medium Term Calls</th>
<th>Average Daily Portfolio Price Change</th>
<th>Standard Deviation of Daily Portfolio Price Changes</th>
<th>Raw Data Ljung-Box prob. (&lt; .01)</th>
<th>Raw Data Engle ARCH prob. (&lt; .01)</th>
<th>Regression F prob. (&lt; .01)</th>
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<tr>
<td>100$ Medium Term Calls</td>
<td>0.69</td>
<td>18.23</td>
<td>*</td>
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<tr>
<td>100$ Medium Term Calls</td>
<td>0.69</td>
<td>18.23</td>
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<tr>
<td>Delta Hedged</td>
<td>-0.43</td>
<td>9.27</td>
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<tr>
<td>100$ S&amp;P500 Index</td>
<td>0.04</td>
<td>0.96</td>
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<td>Black-Scholes Gamma Hedge</td>
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<tr>
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<td>Average Daily Portfolio Price Change</td>
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<td>Raw Data Ljung-Box prob. (&lt; .01)</td>
<td>Raw Data Engle ARCH prob. (&lt; .01)</td>
<td>Regres sion F prob. (&lt; .01)</td>
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**Hedging medium term straddles**

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<th>Hedge</th>
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<th>Standard Deviation of Unexpected Daily Portfolio Price Changes</th>
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<th>Raw Data Engle ARCH prob. (&lt;.01)</th>
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Table 5 - Option Trading Results

Value portfolio of 1:1 time spreads (calls, puts), jelly roll, medium and short term straddle.
Sell if market price is greater than model price, otherwise buy. Rebalance daily.

* Average not different from zero at .01 level.

<table>
<thead>
<tr>
<th>Model</th>
<th>Always Buy - ARV model</th>
<th>CV Model</th>
<th>GARCH(1,1) avg</th>
<th>Garch Components</th>
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<td>1 position</td>
<td>1:1 Call Time Spread</td>
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<td>1:1 Put Time Spread</td>
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<td></td>
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Summary of Trading Signals

<table>
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<tr>
<th>Number of Buy signals</th>
<th>Average Profit on Buy/Sell different from 0 at .01 level</th>
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<td>ARV model</td>
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<td>1 position</td>
<td>Jelly Roll</td>
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<td>Straddle</td>
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<tr>
<td>100$ Short Term</td>
<td>Straddle</td>
</tr>
</tbody>
</table>

| Position              | ARV Model | CV Model | GARCH(1,1) avg vol | Garch Components |
| 1 position            | 1:1 Call Time Spread | 0 / 0 | ./+ + | ./+ + | ./+ + |
| 1 position            | 1:1 Put Time Spread  | .0 / 0 | ./+ + | ./+ + | ./+ + |
| 1 position            | Jelly Roll     | .0 / 0 | ./+ + | ./+ + | ./+ + |
| 100$ Medium Term      | Straddle      | .0 / 0 | .0 / + | .0 / + | .0 / + |
| 100$ Short Term       | Straddle      | .0 / 0 | .0 / + | .0 / + | .0 / + |
Figure 1- Vega Term Structure

Term Structure of Vega
(S=100, K=100, sig=.01, rf=.0002)
Figure 2: Gamma Term Structure

Term Structure of Gamma
(S=100, K=100, sig=.01, rf=.0002)
Hedging options in a GARCH environment: testing