The forward and the equity premium puzzles: Two symptoms of the same illness?

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The Forward and the Equity Premium Puzzles: Two symptoms of the same illness?¹

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Abstract

The Forward Premium Puzzle (FPP) is how the empirical observation of a negative relation between future changes in the spot rates and the forward premium is known. Modeling this forward bias as a risk premium and under weak assumptions on the behavior of the pricing kernel, we characterize the potential bias that is present in the regressions where the FPP is observed and we identify the necessary and sufficient conditions that the pricing kernel has to satisfy to account for the predictability of exchange rate movements. Next, we estimate the pricing kernel applying two methods: i) one, due to Araújo et al. (2005), that exploits the fact that the pricing kernel is a serial correlation common feature of asset prices, and ii) a traditional principal component analysis used as a procedure to generate a statistical factor model. Then, using on the sample and out of the sample exercises, we are able to show that the same kernel that explains the Equity Premium Puzzle (EPP) accounts for the FPP in all our data sets. This suggests that the quest for an economic model that generates a pricing kernel which solves the EPP may double its prize by simultaneously accounting for the FPP.

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1 Introduction

Puzzles are how economists denote systematic empirical violations of otherwise successful models. Puzzles are important for the advance of economic theory for they create research agendas aimed at incorporating to these theoretically sound models features capable of accommodating the empirical regularities that escape from their most stylized versions, or, ultimately, overthrowing these models. Two of the best known puzzles in financial economics are the equity premium — henceforth, EPP — and the forward premium puzzles — henceforth, FPP.

The EPP, commonly associated with the works of Hansen and Singleton (1983) and Mehra and Prescott (1985), is how one calls the incapacity of consumption based asset pricing models, with reasonable parameters for the representative agent’s preferences, to explain the excess return of stock market with respect to the risk free bond in the United States. The departure point of the puzzle is an attempt to fit the Euler equation of a representative agent for the American economy, which has proven to be an elusive task.

The FPP, on the other hand, relates to the difference between the forward rate and the expected future value of the exchange rate in a world with rational expectations and risk neutrality. In a risk neutral world, forward rates should coincide with expected exchange rates which is in contrast with the commonly reported conditional bias of forward rates as predictors of future spot exchange rates.

Naturally, we do not believe that agents are risk neutral. And, without risk neutrality, rational expectations alone does not restrict the behavior of forward rates for it should always be possible to, along the lines of Fama (1984), include a risk premium with the right properties for reconciling the time series. The relevant question is whether a "reasonable" economic model for the risk premium is able to generate the conditional bias of the forward premium that characterizes the FPP. The natural candidate for a sound model is Lucas’ (1978, 1982) which laid the foundation of a framework for addressing the risk premium in a general equilibrium context. Unfortunately, if it is the very failure of Lucas’ model to explain the excess return of the US stock market that defines the EPP, why should we rely on it to account for the FPP? This path for explaining the behavior of the forward premium seems to be doomed from its start.

Moreover there is a second empirical finding which defines the FPP that appears to be harder to accommodate with the available models. The forward premium, defined as the difference between the logarithm of the forward rate and of the current exchange rate, \( e^{f_{t+1} - s_t} \), is too strongly (negatively) correlated with subsequent changes in the (log of the) exchange rate, \( e^{s_{t+1} - s_t} \). Domestic currency appreciates when domestic nominal interest rates exceed foreign interest rates. The consequence is that interest rates differentials ‘predict’ differential returns, a feature that is not present in the case of the EPP.

In other words, even though the two puzzles are similar with regards to the incapacity of

\(^1\)See the comprehensive surveys by Hodrick (1987) and Engel (1996).
traditional models to account for the risk premiums involved in these different markets, there is a
non-shared characteristic: the predictability of returns based on interest rate differentials. It is
possible that it was this specificity that lead researchers to adopt distinct agendas for investigating
the two puzzles. Engel (1996), for example, argues that, since this strong power of forward
premium for forecasting \( s_{t+1} - s_t \) has no counterpart in the literature on equity returns, general
equilibrium models are not likely to replicate this finding, there being no grounds to believe that
the proposed solutions to puzzles in domestic financial markets can shed light on the FPP.

It is our goal in this paper to investigate whether the two puzzles deserve two distinct agendas,
or whether, despite this specificity of foreign exchange markets, researchers in these two areas would
do better by joining their forces in searching for a fully specified model capable of accounting for
the two phenomena. Before such a model is written a definite answer cannot be given. Our, intent
is simply to offer some evidence that the two puzzles may be but two symptoms of the same illness:
the incapacity of our consumption based models to generate the implied volatility of the SDF.
We emphasize the word the because our quest is for a single model capable of accommodating
price behavior in both markets, thus ruling out explanations based on particular characteristics
of exchange rate markets. In this sense, our approach borrows from Brandt, Santa-Clara and
Cochrane (2004) the idea of thinking about the behavior of the SDF as the implied behavior of
intertemporal marginal rates of substitution of a model yet to be written.

The fact that we lack a model for the stochastic discount factor — SDF — capable of explain-
ing either puzzle leads us to work directly with the Asset Pricing Equation. By log-linearizing this
equation, it is possible to derive an equation that describes the movement of currency depreciation
and that has the conventional regression used in most empirical studies related to FPP as a par-
ticular case. This allows us: i) to identify the potential bias in the conventional equation due to
a problem of omitted variable that may be responsible for the counterintuitive findings reported
in the literature, and; ii) to characterize necessary and sufficient conditions that the SDF must
display if it is to account for the FPP. We do not claim originality in this characterization. Using a
similar characterization, Backus et al. (1995), for example, have shown that a theory that accounts
for the FPP requires a stochastic process for the SDF and the rate of depreciation that exhibit
considerable variation in their conditional variance.

Our main contribution is to extract, using two different model-free methodologies, a time series

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2Research regarding the FPP is often done in the international area like Fama and Farber (1979), Hodrick (1981)
and Lucas (1982). It is finally worth mentioning the hedging pressure models developed by Keynes (1923) and Hicks
(1939) which also generates foreign exchange risk premium.

3Certainly, this a relevant issue, since it is possible to observe that the emphasis of the modern forward premium
literature is on international affine term structure models and on a microstructure approach used to address the
FPP, besides other exchange rate puzzles, while the latter one has concentrated on models that use additional state
variables and consequently make changes in the description of stock market risk, as Campbell and Cochrane (1999)
habit persistence model and Constantinides and Duffie (1996) model with uninsured idiosyncratic risks.

4The log-linearization is only for illustrative purposes. In the empirical exercise we adopt the assumptions described
on Araújo et al. (2005) that allows for a more general specification.
for the SDF that accounts for the EPP and verify that it does account for the FPP. The two methodologies used to estimate the SDF as a function of the asset returns are: i) a new methodology due to Araújo, Fernandes and Issler (2005) — AFI, henceforth — based on the fact that the SDF is a serial correlation common feature of asset prices, and ii) a statistical factor model. Applying both methodologies to different data sets we extract the SDF and show that the SDF compatible with the Equity premium behavior is also capable of accommodating the FPP.

Our first data set provides us with an estimator of the SDF that "prices correctly" excess return of S&P500 with respect to short-term American and of uncovered over covered purchasing of foreign government bonds, we show that this SDF also accounts for the FPP. Since foreign assets have been used to estimate the SDF, one may view our first exercise as being, in some sense, "in the sample". We then proceed a series of exercises that excludes foreign assets and show that it is still possible to account for the FPP using only US domestic assets. Our results, which are similar when using these two methods, thus, seem to indicate not only that the two puzzles are intertwined but also that the American domestic assets we have used are "representative" in a sense to be made precise later on.

The remainder of the paper is organized as follows. Section 2 gives account of the issues related to these asset pricing puzzles in a consumption-based framework, while in section 3 we concentrate on showing that the predictability of exchange rates may not be seen as a specificity. In section 4, the techniques used to estimate the SDF are emphasized and three empirical applications are done. In section 5, the results are analyzed to understand the behavior of the SDF that solves FPP and EPP and then to draw parallels between them. The conclusion is in the last section.

2 Economic Theory, the EPP and the FPP

2.1 Stochastic Discount Factor and Asset returns

Following Harrison and Kreps (1979) and Hansen and Richard (1987), we write the asset pricing equations,

$$1 = \mathbb{E}_t \left( M_{t+1} R^i_{t+1} \right),$$

and

$$0 = \mathbb{E}_t \left[ M_{t+1} \left( R^i_{t+1} - R^j_{t+1} \right) \right].$$

Here, $\mathbb{E}_t(\cdot)$ denotes the conditional expectation given the information available at time $t$, $R^i_{t+1}$ and $R^j_{t+1}$ represent, respectively, the real gross return on assets $i$ and $j$ at time $t+1$ and $M_{t+1}$ is the SDF, a random variable that generates unitary prices from asset returns.

It is well known that the law of one price — the fact that two assets with the same payoff in all states of nature must have the same price — is equivalent to the existence of a SDF through Riesz representation theorem. The correlation with the SDF is the only measure of risk that matters for pricing assets. It is, therefore, clear that we need not rely on the strong assumptions to guarantee
the validity of (1) and (2). Given free portfolio formation, and under the assumption of no arbitrage, we can further guarantee that there exists a strictly positive SDF that correctly prices the returns of all assets.

Uniqueness of $M_{t+1}$, however, is harder to come about. In general, if markets are not complete there will be a continuum of SDF’s pricing all traded securities. Yet, there will still exist a unique SDF, $M^*_t$, in the span of traded assets, labeled mimicking portfolio. This SDF is the unique element of the payoff space that prices all traded securities. Any SDF, $M_{t+1}$, may thus be written $M_{t+1} = M^*_t + \nu_{t+1}$ for some $\nu_{t+1}$ with $E_t [\nu_{t+1} R_{t+1}^i] = 0 \forall i$.

This approach allows us to conveniently specify the assumptions about the economy that are implicit in the choice of the function that determines the SDF with the intent to obtain further results.

2.2 Consumption-Based Model and Asset Pricing Puzzles

The Consumption Capital Asset Pricing - CCAPM - is an economic model that specifies from an equilibrium relation $M_{t+1}$ as the intertemporal marginal rate of substitution for the representative agent.

Assume that the economy has an infinitely lived representative consumer for the reference country, whose preferences are representable by a von Neumann-Morgenstern utility function. Then, the first order conditions for his choice of an optimal portfolio yield,

$$1 = \beta E_t \left[ \frac{u'(c_{t+1})}{u'(c_t)} R_{t+1}^i \right] \forall i \quad (3)$$

and, consequently,

$$0 = E_t \left[ \frac{u'(c_{t+1})}{u'(c_t)} \left( R_{t+1}^j - R_{t+1}^i \right) \right] \forall i, j \quad (4)$$

where $\beta \in (0, 1)$ is the discount factor in the representative agent’s utility function and $c_t$ represents aggregate consumption at time $t$. It is apparent that equations (3) and (4) are consistent with equations (1) and (2) if we define the SDF at time $t+1$, as

$$M_{t+1} = \beta \frac{u'(c_{t+1})}{u'(c_t)} \quad (5)$$

This representation for asset prices lies in the Arrow Debreu model of general equilibrium and although it has been extensively used in finance literature due to its empirically, analytically and intuitive convenience, it really does not work well in practice, there being many evidences of its incapability to account for stylized facts. The equity and the forward premium puzzles are two of the most famous and reported empirical failures of this consumption-based approach.

5In what follows we shall consider that preferences and the environment faced by representative agent do not depend on the location of her residence.
To give a summary account of the theoretical and empirical issues related to these puzzles, let this representative agent be an American investor who can freely trade American and foreign assets, besides having access to forward and spot exchange rate markets.

When analyzing the FPP, the relevant assets are the covered and uncovered trade of a foreign government bond. The real returns on these foreign assets in terms of the representative investor's numeraire can be written respectively as

$$R^{C}_{t+1} = \frac{F_{t+1}(1 + \tau^F_t)P_t}{S_tP_{t+1}}$$

and

$$R^{U}_{t+1} = \frac{S_{t+1}(1 + \tau^U_t)P_t}{S_tP_{t+1}},$$

where $F_{t+1}$ and $S_t$ are the forward and spot prices of foreign currency in terms of domestic currency, $P_t$ is the dollar price level and $\tau^F_t$ represents nominal net return on a foreign asset in terms of the foreign investor's preferences. It is, then, possible to express the consumption Euler equation of the excess returns of uncovered over covered operation with foreign bonds as

$$0 = \mathbb{E}_t \left\{ u'(c_{t+1}) \frac{P_t}{u'(c_t)} \left( R^C_{t+1} - R^U_{t+1} \right) \right\},$$

along the lines of Lucas' (1978) model.

Working with the power utility, equation (4) can be rewritten as

$$0 = \mathbb{E}_t \left[ \left( \frac{c_{t+1}}{c_t} \right)^{-\gamma} \left( R^C_{t+1} - R^U_{t+1} \right) \right].$$

The consumption Euler equation (8) is useful to access the EPP, if one takes assets $i$ and $j$ to be S&P500 index and the short-term American government bond, and the FPP, if, instead, the assets are covered and uncovered purchasing of foreign government bonds.

Most tests reported in the literature are based on over-identifying restrictions on estimations that use Hansen's (1982) Generalized Method of Moments (GMM). With regards to the FPP, the results obtained in Mark (1985) and Hodrick (1989), using quarterly observations, report values of the coefficient of relative risk aversion, $\gamma$, equal to 40 and 61, respectively. The findings in the case of EPP are similar, in the sense that in both cases the estimated values are above 10, which is viewed as unacceptable and unreasonable, according to Krugman (1981), Mehra and Prescott (1985) and Romer (1995).

Many reasons have been given to try and account for this empirical failure. Many research agendas have been developed with this aim. Some economists argue that this dismal performance is only due to the poor quality of available individual consumption data. Under this condition, equation (8) estimated directly from consumption data is flawed with measurement error. However attempts to handle this problem along this line have pointed out that measurement error per se cannot account for the EPP.\^6

\^6Lettau (2002), using income data — which is of superior quality than consumption data — evaluates the volatility bounds at the individual level. This strategy is appropriate if individual consumption is smoother than individual income, a theoretical possibility that finds strong support in the empirical literature. His study shows that, even if agents are forced to consume their labor income, the model is not able to produce sufficiently large risk premiums.
As pointed out by Kocherlakota (1996), to solve these puzzles one must abandon at least one of the three basic assumptions used to derive the consumption Euler equations: agents have preferences representable by standard utility functions, asset trading is costless and these markets are complete.

It is beyond the scope of this paper to give a full account of the large body of research produced in the area. Our point here is simply that, once one recalls that a forward contract is itself a financial asset one should be suspicious about using (7) to account for the behavior of the forward premium. Since consumption-based models cannot account for these puzzles, as the literature has extensively reported, they should be of little help in our attempt to narrow the association between the EPP and the FPP. What we need, therefore, is a model that does work for the EPP to try and see if it also works for the FPP.

Finding such a model is a surmountable task. As we have already mentioned, the choice of an economic model capable to understand the economic forces that drive a SDF that accounts for all asset pricing puzzles is certainly the biggest challenge in modern asset pricing theory. We shall not attempt it here. Our goal here is to answering the following question. If we were able to somehow recover a SDF, $M_{t+1}$, that satisfies (2), when of concern is the excess return of the S&P500 over the US short term bond, would this SDF also account for the FPP? In other words, were we capable of writing down a model that consistently generated this SDF, would this model explain the FPP?

Our strategy for answering this question consists in estimating an SDF using a purely statistical methodology and evaluating whether this SDF accounts for both puzzles.

3 Predictability of exchange rates, aggregate stock returns and interest rates

The EPP and the FPP are similar with regards to the incapacity of traditional consumption models to account for the risk premiums involved in these different markets, but different since the strong predictability power of the forward premium is pointed as a specificity of the foreign exchange market. Then, a departure point in this narrowing process can be, based on a more general framework as the Asset Pricing Equation, to derive an equation that describes the currency depreciation capable to support a discussion about the problems related to the empirical evidences of this predictability, and then to establish a comparison, mentioning the empirical works that tend to support the relevant predictability of interest rates and aggregate stock returns.

3.1 FPP: the conventional regression and evidences

Most empirical studies that report the FPP, evidence this puzzle through the finding of $\hat{\alpha}_1 < 0$, when one runs the regression,

$$s_{t+1} - s_t = a_0 + a_1(f_{t+1} - s_t) + u_{t+1},$$  (9)
with $u_{t+1}$ denoting the regression error.

This regression is a commonly used test of efficiency of foreign exchange market. The test of the null hypothesis of absence of risk premium and consequently the efficiency generally involves testing the joint hypothesis $\hat{\alpha}_1 = 1$, $\hat{\alpha}_0 = 0$ and the residuals from the estimated regression are serially uncorrelated.

Even though the rejection of this null hypothesis should not be surprising in a world where agents are risk averse — where, consequently, there exists risk premiums — one would at least expect $\hat{\alpha}_1 > 0$. This finding would confirm the intuition of economists and the implication of most economic models that the domestic currency is expected to depreciate when domestic nominal interest rates exceed foreign interest rates. Instead, in most empirical works, the opposite obtains: $\hat{\alpha}_1 < 0$, which implies an expected domestic currency appreciation when domestic nominal interest rates exceed foreign interest rates. According to Froot (1990) the average value of $\hat{\alpha}_1$ is $-0.88$ for over 75 published estimates across various exchange rates and time periods.

### 3.2 Currency depreciation theoretical framework

Since the null hypothesis of the regression (9), often rejected, represents the equilibrium condition in a world where markets are efficient and the agents are risk neutral, have rational expectations and value returns in nominal terms, there is a literature that attempts to describe the exchange rate movements assuming the conditional joint lognormality of some specific variables and relying on less restrictive assumptions with the intent to identify the reasons of the counterintuitive empirical findings reported. Along these lines, consider an economy where assumptions 1 and 2 are valid.

**Assumption 1:** The Asset Pricing Equation (1) holds;

**Assumption 2:** There are no arbitrage opportunities;

We have many reasons to think that asset markets can be described by these assumptions. They are mild and underlie most of the recent and fundamental insights in finance; see, e.g., Hansen and Singleton (1982, 1983, 1984), Mehra and Prescott (1985) and Mulligan (2002).

In order to derive an equation that describes the currency depreciation movements, we shall add some structure on the stochastic process for the SDF and the real returns, specifying the conditional joint distribution of $M_{t+1}R_{t+1}^R$ through the next assumption. Despite its prominent role in both theoretical and empirical studies of asset pricing due to its analytical convenience, the next assumption is unduly restrictive and we shall only use it for ease of presentation. In our empirical work we shall dispense with it and work in a more general framework.

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7 For details of the methodology of this kind of efficiency test, see Fama (1975).
8 See, for example, Lucas (1982) and Bekaert (1996).
9 Very good references are the surveys by Hodrick (1987) and Engel (1996).
**Assumption 3:** The conditional joint distribution of $M_{t+1}R_{t+1}^i$ is lognormal

Taking logs in both sides of (1) and further applying a Taylor expansion yields, for every asset $i$ in the economy,

$$r_{t+1}^i = -m_{t+1} - \frac{1}{2}(\sigma_{t+1}^i)^2 + \varepsilon_{t+1}^i,$$

(10)

where the variables in small letters are the logs of the variables in capital letters, $\varepsilon_{t+1}^i = \ln \left(M_t R_{t+1}^i\right) - \mathbb{E}_t(\ln(M_{t+1}R_{t+1}^i))$ denotes the innovation in predicting $\ln(M_t R_{t+1}^i)$ and $(\sigma_{t+1}^i)^2 \equiv V_t(\ln M_{t+1} + \ln R_{t+1}^i)$, the conditional variance given the available information at time $t$.

From (10) one verifies that asset returns are decomposed in three terms. The first one is the logarithm of the SDF, $m_{t+1}$, which is common to all returns and is a random variable. The second one, $(\sigma_{t+1}^i)^2$, is idiosyncratic and, given past information, also deterministic, but not necessarily constant. The third one is $\varepsilon_{t+1}^i$. Idiosyncratic, as well, and unforecastable, which means that it presents no serial correlation. Hence, disregarding deterministic terms, the only source of serial correlation is $m_t$.

Because equation (10) must hold for any asset traded in an economy where assumptions 1 to 3 are valid, including foreign government bonds, it is straightforward to show that

$$s_{t+1} - s_t = \frac{1}{2}[(\sigma_C)^2 - (\sigma_U)^2] + (\pi_{t+1} - s_t) + \varepsilon_{t+1}^U - \varepsilon_{t+1}^C,$$

(11)

where $\sigma_C \equiv V_t(\ln M_{t+1} + \ln R_{t+1}^C)$ and $\sigma_U \equiv V_t(\ln M_{t+1} + \ln R_{t+1}^U)$.

Consequently,

$$s_{t+1} - s_t = -\left[\frac{1}{2}V_t(s_{t+1}) + \text{cov}_t(s_{t+1}, m_{t+1} - \pi_{t+1})\right] + (\pi_{t+1} - s_t) + \varepsilon_{t+1}^U - \varepsilon_{t+1}^C,$$

(12)

where $\pi_{t+1}$ is the (log of) domestic price variation.

This equation (12) describes the movement of currency depreciation and is useful in drawing implications for the SDF and in deriving empirical tests of FPP.

### 3.3 The predictability of exchange rate movements: a non-shared specificity of the FOREX market?

How are we to formally access whether the estimates reported in the literature of the FPP are really off the mark in a world where agents are risk averse? Compare (9) with (11). It is apparent that the latter is a particular case of the former, where term $(\sigma_C^2 - (\sigma_U^2)^2)$ is assumed to be time-invariant. Time invariance does not seem to be a sound assumption, given that asset returns have clear signs of conditional heteroskedasticity — see Bollerslev et al. (1988), Engle et al. (1990) and Engle and Marcucci (2005). Even though the heteroskedasticity of (the log of) assets returns is neither necessary nor sufficient to break time invariance of $(\sigma_C^2 - (\sigma_U^2)^2)$, it makes

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10This equation is slightly different from the one reported in Backus et al. (1995), since they disregard the $\text{cov}_t(s_{t+1}, \pi_{t+1})$ term, which is a valid procedure for advanced economies as the American.
this invariance very unlikely since movements in \((\sigma^U)^2\) and \((\sigma^V)^2\) would have to exactly cancel out period by period.

Based on (12), it is apparent that the disappointing empirical findings reported when regression (9) is used may be due to a problem of omitted variable which necessarily makes the estimator more inefficient and creates bias and inconsistency if the omitted term is correlated with the forward premium.

At this point, we have some relevant issues: \(i)\) whether this bias may account for the strong predictability power of forward premium associated to \(\alpha_1 < 0\), \(ii)\) what could be said about the behavior of a SDF that accounts for the FPP and \(iii)\) if still remaining an uncomfortable predictability power of forward premium, we may consider this forecast power as a specificity of the FOREX market.

According to Backus et al. (1995), the \(V_t(s_{t+1})\) and \(\text{cov}_t(s_{t+1}, m_{t+1})\) terms should exhibit considerable variation if one is to accommodate the evidence regarding the forward premium. Since the \(V_t(s_{t+1})\) term appears only because we defined the expected rate of depreciation in logarithms and following Beakert and Hodrick (1993), omitting this term from regression (9) is not responsible for finding \(\alpha_1 < 0\), the \(\text{cov}_t(s_{t+1}, m_{t+1})\) term must play the big role in generating the FPP, which supports that a promising path, would be to model the exchange rate movements as a bivariate asymmetric GARCH-in-mean process capable of accommodating the empirical evidences reported in the literature. This approach is applied in a companion paper — da Costa et al. (2006).

With regards to the second issue, taking the conditional expectation on (12) and on (9), it is straightforward to see that

\[
\mathbb{E}_t(s_{t+1} - s_t) = - \left[ \frac{1}{2} V_t(s_{t+1}) + \text{cov}_t(s_{t+1}, m_{t+1} - \pi_{t+1}) \right] + (t f_{t+1} - s_t)
\]

and consequently,

\[
(t f_{t+1} - s_t)(1 - \alpha_1) = \alpha_0 + \left[ \frac{1}{2} V_t(s_{t+1}) + \text{cov}_t(s_{t+1}, m_{t+1} - \pi_{t+1}) \right].
\]

Denoting by \(\varphi_t(m_{t+1}) = \left[ \frac{1}{2} V_t(s_{t+1}) + \text{cov}_t(s_{t+1}, m_{t+1} - \pi_{t+1}) \right]\) and by \(\mu_i(x) \equiv \text{i}th\) unconditional central moment of a univariate probability function \(P(x)\), we can conclude that, under lognormality, to account for the FPP, i.e. to account for negative values of the parameter \(\alpha_1\) extensively reported in this literature, it is necessary and sufficient that the SDF satisfies

\[
\begin{cases}
\mu_i(\varphi_t(m_{t+1})) > \mu_i(t f_{t+1} - s_t), & \text{for } i \text{ even} \\
|\mu_i(\varphi_t(m_{t+1}))| > |\mu_i(t f_{t+1} - s_t)| & \text{for } i \text{ odd,}
\end{cases}
\]

which, for instance, has the necessary conditions reported in Backus et al. (1995) as an implication.
Finally, regarding the last issue, it is possible to observe an enormous literature documenting the predictability of aggregate stock returns from many variables as recent changes in short-term interest rates and of the interest rates from its forward rates. According to Campbell (2000), most financial economists seem to have accepted that aggregate returns contain an important predictable component and that its predictability appears more striking at long horizons than short ones and according to Fama and Bliss (1987), there is little evidence that forward rates can forecast near-term changes in interest rates for annual U.S. Treasury, but when the forecast horizon is extended, this forecast power improves — one-year forward rates can explain 8%, 24% and 48% of the variation of the change in the one-year interest rate 2, 3 and 4 years ahead, respectively —, empirical findings confirmed by Campbell and Shiller (1989).

To summarize. The strong predicting power of the forward premium, pointed thought to be a specificity which has no counterpart in the literature on stock returns, may not be considered as such specificity, since its magnitude may be a consequence of the bias that is present when using the conventional regression (9) and, as reported, interest rates and aggregate stock returns are also predictable. In some sense we part ways from most of the international literature by thinking that this might be the case. Our empirical findings will also provide evidence toward this direction.

4 Empirical Application

In this section we construct a SDF using two methods: i) AFI's methodology, and ii) a statistical multifactor model. The idea is to use the pricing kernels thus constructed to try and see if we are able to account for both the EPP and the FPP. That is, working in a very general framework, Asset Pricing Equation, we try and establish an association between the two puzzles based on an empirical exercise that is implemented using these two techniques to estimate the SDF.

We first describe the details and respective sources of the data that will be used. Next, we present the consistent estimator of the SDF derived in AFI and the procedure used to extract the factor realizations from the observable returns and to generate a SDF as a linear function of these portfolio factors. Finally, we test the "pricing power" of both SDF estimators for a data set that contains real returns on foreign bonds.

4.1 Data

When Asset Pricing Equation (2) is used, we are concerning the co-movements of real excess returns and the SDF.

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11 For more details, see Campbell (1987) and Hodrick (1992).
12 Unfortunately, it is possible to find divergent conclusions. According to the first empirical studies on this question used to be restricted to maturities less than a year, as Hamburger and Platt (1975) and Shiller et al. (1983), the simple theory that the slope of the term structure can be used to forecast the direction of future changes in the short and long terms interest rate — three and six-month Treasury-bill rates and thirty-year Treasury bond yield, respectively — seems worthless.
In the empirical applications 1 and 2, to apply both techniques to estimate the SDF we will consider returns available to the average U.S. investor, who has increasingly become more interested in global assets over time. Our data base covers U$ real returns on G7-country stock indices and short-term government bonds, where exchange-rate data was used to transform returns denominated in foreign currency into U$ and and real returns were computed using the consumer price index of services and nondurable goods in the U.S.. In addition to G7 returns on stocks and bonds, we also use U$ real returns on gold, U.S. real estate, bonds on AAA U.S. corporations, Nasdaq and Dow Jones and the S&P 500. The U.S. government bond is chosen to be the 90-day T-Bill, considered by many to be a "riskless asset." Regarding the sources, the real returns on G7 government bonds were extracted from IFS/IMF, the real returns on Nasdaq and Dow Jones Composite Index were extracted from YahooFinance, the real returns on real-estate trusts were extracted from the National Association of Real-Estate Investment Trusts in the U.S., while the remaining series were extracted from the DRI database. Overall, we averaged the real U$ returns on these 20 portfolios or assets, which are, in turn, a function of thousands of assets. These are predominantly U.S. based, but we also cover a wide spectrum of investment opportunities across the globe. This is important element of our choice of assets, since diversification is recommended for both methods used.

In the third empirical application, concerning about the representativeness of American domestic assets — in the sense of generating a SDF able to price correctly risk premium in international markets —, to estimate the SDF we use a data base that covers the period 1990:1 - 2004:3 for U$ real returns on American short-term government bond and on the two hundred American stocks that presented the biggest transaction volume in this period according to CRSP database. Unfortunately, the main limitation imposed to empirical works related to the FPP is due to the forward rate series, since sufficiently large series are hardly available. The remainder of the data set covers U$ real returns on short-term British, Canadian, German, Swiss and Japanese government bonds, where both spot and forward exchange-rate data were used to transform returns denominated in foreign currency into U$. For these five economies, the data base covers the periods 1977:1 - 2004:3.

4.2 The SDF in AFI's methodology

The SDF estimator proposed in AFI, is a simple function of the arithmetic and geometric means of asset returns alone, which means that there is no need for one to characterize preferences or use consumption data, a desirable property of an estimator of SDF to answer our question.

We now present two assumptions that guarantee the consistency of AFI's estimator.

\[ \text{Data on the return on real estate are measured using the return of all publicly traded REITs – Real-Estate Investment Trusts.} \]

\[ \text{The forward rate series were extracted from Chicago Mercantile of Exchange database, while the spot rate series were extracted from Bank of England.} \]
Assumption 2': There is a stochastic discount factor such that $M_t > 0$. The same holds for the mimicking portfolio $M_t^*$.  

Assumption 4: Let $R_t = (R_1^t, R_2^t, \ldots, R_N^t)'$ be an $N \times 1$ vector stacking all asset returns in the economy. The vector process $\{\ln (M_t R_t)\}$ is assumed to be covariance-stationary with finite first and second moments. Let $\varepsilon_t^i = \ln (M_t R_t^i) - \mathbb{E}_{t-1} \{\ln (M_t R_t^i)\}$ denote the innovation in predicting $\ln (M_t R_t^i)$. Then, we further assume that  

$$
\lim_{N \to \infty} \mathbb{E} \left[ q_t \times \frac{1}{N} \sum_i \varepsilon_t^i \right] = 0,
$$

where $q_t = m_t - \mathbb{E}_{t-1} [m_t]$.  

Under no arbitrage, one can further guarantee that is a positive SDF ($M_t > 0$) that prices all assets, so the first part of Assumption 2' is valid under no-arbitrage. The second part of the assumption, however, requires that the SDF that belongs to the span of assets has this property. If markets are not complete this cannot be taken for granted. We take it to be a mild assumption and note that this assumption is also made in virtually all studies about the equity-premium puzzle; see, inter alia, Hansen and Singleton (1982, 1983, 1984), and Mehra and Prescott (1985) since the EPP is based on equation (5).  

As for Assumption 4, it replaces Assumption 3 in that log-normality is no longer imposed, yet it restricts the degree of time-series and cross-sectional dependence. It is a necessary condition for the identification of $m_t$.  

AFI show that in an economy where the assets traded are such that, the vector process $\ln (M_t R_t)$ satisfies Assumptions 1, 2' and 4, the realized SDF at time $t$, denoted $\mathcal{M}_t$, can be consistently estimated as $N, T \to \infty$, using  

$$
\mathcal{M}_t = \frac{\bar{R}_G}{\frac{1}{T} \sum_{j=1}^T (\bar{R}_G \bar{R}_A)},
$$

where $\bar{R}_G$ denotes the geometric mean of the reciprocal of all asset returns and $\bar{R}_A$, the arithmetic mean of all asset returns.  

It is important to bear in mind that there is no need to assume complete markets. Then, if markets are complete the realization of the SDF at time $t$, denoted $M_t$, can be consistently estimated using (15), while if markets are incomplete the mimicking portfolio will be consistently estimated.  

The approach developed in AFI, provides a fully non-parametric way of consistently estimating the realizations of the SDF by the simple observation that a SDF is the serial correlation common feature of asset returns. For any two economic series, a common feature exists if it is present in both of them and can be removed by linear combination. An early example of common features is co-integration, where the feature is a common unit root component that is removed by the co-integrating vector.$^{15}$  

$^{15}$Serial-correlation common features are discussed in great detail in Vahid and Engle (1993, 1997).
4.3 Statistical multifactor model

AFI's method is very attractive for its simplicity and easy interpretation. Yet, being it a novel methodology one might be concerned about the meaning of the underlying assumptions and the consequent robustness of the results herein. To address this issue, we also estimate the SDF using an unconditional linear multifactor pricing model, the dominant model in discrete-time empirical work.

Because these two methods only require financial data, it is possible to use the same data set of asset returns in both procedures and establish a comparison between the two estimators of the SDF.

When someone uses a multi-factor approach, the first step is to specify the factors by means of statistical or theoretical method and then to consider the estimation of the model with known factors. Given our purposes, we are not particularly concerned about identifying the factors themselves but rather about being able to construct a SDF by collapsing the factors into a single variable. We shall, then, rely on a purely statistical factor model, in particular, factors that may be viewed as traded portfolios.

There are two alternative statistical approaches for constructing the model factors, factor analysis and principal component analysis. We adopt the principal component analysis method\textsuperscript{16} due to its very low computational cost, to the possibility of accommodating serially correlated or heteroskedastic residuals and to the financial interpretation of the factors as portfolios.

Since there is an extensive literature reporting the traditional principal component analysis — TPCA\textsuperscript{17}, we will not give a full account of the theoretical and econometric issues associated with it. It is just important to note that the principal components, which are linear combinations of the returns, are constructed to be orthogonal to each other and to be normalized to have a unit length and ordered so that the first principal component explains the largest portion of the sample covariance matrix of returns, the second one explains the next largest portion, and so on. In terms of a multifactor model, the $K$ principal components are the factor realizations.

As for the number of factors, while for the theory to be useful, $K$ should be reasonably small, we still have significant flexibility in this choice. In both empirical exercises, we repeated the estimation and testing of the model for a variety of values of $K$ and observed if the tests were sensitive to increasing the number of factors. In the first and second empirical exercises, our results displayed minimal sensitivity when the number of factors increased from three to five, which suggests that three factors are adequate. In the third exercise, the results displayed high sensitivity when the number of factors increased from three to six, but minimal sensitivity when increased from six to eight, which is in accordance with Connor and Korajczyk (1993) that examined returns from

\textsuperscript{16}Although both procedures can be justified in large samples, in the case of a finite sample, there is no clear answer to the question of which technique is optimal in the sense of providing the most precise measures of the population factors given a fixed sample of returns.

\textsuperscript{17}Campbell, Lo and MacKinlay (1997) and Zivot and Wang (2003) are excellent references.
stocks listed on the New York Stock Exchange and the American Stock Exchange and concluded that there are up to six pervasive factors.

Finally, with regards to the choice of an unconditional model instead of a conditional one, this latter approach is usually cumbersome, typically requires one to assume that investors use the same model of conditioning information that the econometrician does and that the former one is just a special case of the latter that happens to have fixed weights.

4.4 Pricing test

In this subsection, we test the "pricing power" of the two SDF's estimated.

Initially, notice that, from the definition (5) in equation (7) and using the Law of Iterated Expectation, we may write

\[ 0 = E \left\{ M_{t+1} \frac{P_{t+1} (1 + i_t^* - S_{t+1} (1 + i_t^*))}{S_t P_{t+1}} \right\}. \] (16)

A direct test of the existence of FPP using a SDF's estimator consists in verifying if equation (16) holds. These zero-mean tests are straightforward to implement, but it is important to take into account the possible existence of heteroskedasticity or serial correlation. In this case, the estimate proposed by Newey and West (1987) will be used.

In the case of the EPP, we follow the same procedure to test whether equation

\[ 0 = E \left\{ M_{t+1} \frac{(1 + i_{t+1}^{S&P_500}) P_t}{P_{t+1}} - \frac{(1 + i_{t+1}^b) P_t}{P_{t+1}} \right\}. \] (17)

holds. Here, \( i_{t+1}^{S&P_500} \) is nominal return on S&P500 and \( i_{t+1}^b \) nominal return on American Treasury bill.

Three exercises are performed and all of them test the existence of the EPP and the FPP, by verifying if equations (17) and (16) hold for each estimator of the SDF: first the SDF obtained from AFI's methodology, denoted \( \tilde{M}_t \), and second the SDF obtained from a factor model, denoted \( \tilde{M}_t \). The difference between these exercises is the data set used to estimate the SDF's. The reason why we proceed this way shall be clear in the next section when the results will be analyzed.

In the first exercise, the database used to estimate the SDF's — \( \tilde{M}_t^{bench} \) and \( \tilde{M}_t^{bench} \), which will serve as benchmark for the others estimated in the exercises 2 and 3 — covers the period 1977:1 - 2001:3 for US$ real returns on gold, US real estate, bonds on AAA US corporations, Nasdaq, Dow Jones, S&P 500, G7-country stock indices and short-term government bonds. The results of the sample means are reported in table 3. The other two tests will use the same SDF estimate techniques but will rely on a different data base.

Regarding the second test, the data set will also be quarterly, covering the same period but for only a subset of the US$ real returns already listed. For instance, if the foreign bonds are from British government, \( S_{t+1}^{UK} \) is the spot price of British pounds in terms of US dollars and in this case, the data set used to obtain the SDF's (\( \tilde{M}_t^{bench,UK} \) and \( \tilde{M}_t^{bench,UK} \)) is composed of US$ real returns on gold, US real estate, bonds on AAA US corporations, Nasdaq, Dow Jones, S&P 500,
short-term G7-country (except the United Kingdom) government bonds and G7-country (except the United Kingdom) stock indices. We follow this same procedure to Canadian, German, Swiss and Japanese economies. The respective new SDF's estimated in this second exercise (\( \tilde{M}^{exc, CA}, \tilde{M}^{exc, GE}, \tilde{M}^{exc, SW}, \tilde{M}^{exc, JP}, \tilde{M}^{exc, UK} \)) and the SDF's estimated in the first one (\( \tilde{M}^{enc} \) and \( \tilde{M}^{enc}_2 \)), are showed in figures 1 and 2 and its respective summary statistics are reported in table 2. The results of the sample means obtained in this over-fitting test are reported in table 4.

The third and more important test is similar in spirit to the second one, in the sense of being also an over-fitting test, but different since, to test the existence of the FPP in these economies and of the EPP, we will estimate only two SDF's, labeled \( \tilde{M}^{US, fin.mk} \) and \( \tilde{M}^{US, fin.mk}_2 \), using a quarterly data set covering 1990:1 - 2004:3 for US$ real returns on American short-term government bond and on the two hundred American stocks that presented the biggest transaction volume in this period according to CRSP database.

5 Analysis of the results

5.1 SDF estimates

Observing the figures 1 and 2 and their statistics, it is apparent that for both econometric methods, the removal of two assets of the original data set used did not change significantly the estimate of the SDF in any case. A surprising result? In fact, not. One might be (mis-)lead to think that this result is a sign of low sensitivity of the estimators to changes in the data set, but the reason for these little changes observed is associated to which specific assets which are removed.

It is important to note that the fact that we have excluded the assets from our sample does not mean that the agents did not trade on these assets. This is something often forgotten in studies involving the financial markets but the point is that the "parameter" \( M \) we want to estimate is not altered by our restricting the information we use in estimating it! The relevant question is whether we are capable of identifying the realized SDF from the observation of a sample of assets. The problem, therefore, is not one of what space is spanned by each set of asset, as one might have been lead to think, but whether the observed assets are representative in the sense of satisfying the assumptions presented in the previous subsections.

Once again, from figure 3 and the statistics of \( \tilde{M}^{US, fin.mk} \) and \( \tilde{M}^{US, fin.mk}_2 \), it becomes clear that for both methods, these estimators are much more volatile than the respective benchmark ones. Naturally, the set of contingencies spanned using only these American assets is different from the one obtained from the set of 20 assets used in the first exercise, however, this is not the relevant issue since agents do trade with these other assets whether we have included them in our sample or whether we have not. As of this moment, we do not have a good interpretation of what drives the difference. Comparing the graphs of \( \tilde{M}^{enc} \) and \( \tilde{M}^{enc}_2 \) and of \( \tilde{M}^{US, fin.mk} \) and \( \tilde{M}^{US, fin.mk}_2 \) in figure 3, it is possible to evidence that in both cases: i) the multifactor model generates a SDF...
much more volatile than the one obtained using AFI method, but ii) they are seemingly correlated, which is evidenced by the correlations between them, 0.740 and 0.799 respectively.

Finally, with regards the multifactor model, since the factors may be viewed as traded portfolios, it is interesting to observe which assets played the biggest role in the sense of the size of its respective weights. When the data set is composed of the global assets, these assets are: German, British and American stock indices, German, Japanese and British government bonds and Nasdaq, while when the two hundred American stocks that presented the biggest transaction volume are used, the most relevant stocks are: Informix corporation, ARM corporation DEL, Emulex corporation, Ericsson L M Telephone corporation, Iomega corporation, LSI corporation, Lam Resch corporation, Advanced Micro Revices inc. and 3 Com corporation.

5.2 Pricing test

The results of the first test reported in table 3 show that the excess returns between assets traded in the American and international financial markets are both correctly priced in the sense of satisfying equations (17) and (16), respectively, when $\tilde{M}_t^{enc}$ and $\tilde{M}_t^{enc}$ are used. The first interesting question is wether this result was expected.

Consider the set of asset returns used; all measured in US$ real terms. The US$ real return on S&P500 and on uncovered purchasing of British, Canadian, German and Japanese government bonds are in the data set used in this case but there is no US$ real return on an operation that involves any derivative in the data set used in AFI. However, when American and foreign assets include default free government bonds, the covered interest rate parity, which can be written as

$$(1 + i^F_t) = (1 + i^P_t) \frac{P_{t+1}}{S_t};$$

must hold and since the US$ real return on American Treasury bill is included, this first test is, in some sense, "in the sample". Consequently, one might be expecting that, for both methods, the sample mean of

$$M_{t+1} P_t (1 + i^P_t) (t_{F_t+1} - S_{t+1})$$

and

$${{M}_{t+1} P_t (i^{SP} - i^{F}_{t+1})}$$

not to be significantly different from zero.

Despite its not being theoretically surprising, the result is important because the estimated SDF's needed not fit the difference between real return on assets of the data set used to estimate it. This shows that the set of assets we used is "representative" in some sense.

Let us now consider the second test. About the FPP, its results are reported in tables 4. They show that, regarding the FPP, once more the excess returns of uncovered over covered operation with foreign bonds are also "correctly priced" in the sense of satisfying the equation (16). Regarding the EPP test, according to the results\textsuperscript{18}, the excess return of S&P500 over the US short term bond

\textsuperscript{18}The results of the EPP test are not reported here, since our purpose in this second test is to make an "out-of-sample" analysis and also due to similarity between them and the results reported in table 3.
is "correctly priced" in the sense of satisfying the equation (17) when $\tilde{M}^{exc.CA}_t$, $\tilde{M}^{exc.CA}_t$, $\tilde{M}^{JP}_t$ and $\tilde{M}^{exc.JP}_t$, $\tilde{M}^{exc.GE}_t$ and $\tilde{M}^{exc.GE}_t$, $\tilde{M}^{exc.UK}_t$ and $\tilde{M}^{exc.UK}_t$ are used.

For instance, the data set used to obtain $\tilde{M}^{exc.UK}_t$ and $\tilde{M}^{exc.UK}_t$, does not include UK-based assets but cover a spectrum of investment opportunities across the globe except the United Kingdom. Theoretically, the fact that the sample mean of (20) are not significantly different from zero should only be expected if we were successfully identifying the realized $M_{t+1}$ from the observation of returns on the assets of assets that exclude the UK-based ones.

What the results show is that the new estimated SDF's are "pricing correctly" the difference between real returns on assets out of the data set used to obtain it, which is a good performance of an over-fitting exercise.

Using two estimation methods, the estimator fits excess returns assets in and out of the data set which is an evidence that, to solve FPP one need not know the behavior of the assets of the specific country for which the puzzle applies. More precisely, these results show that, using the both techniques: i) the estimators have a strong power of fitting returns on assets out of the data set and ii) to solve the FPP from the US perspective for the assets trade in a specific developed economy we do not necessarily need to observe the assets traded with this economy.

Finally, we observe the relevant results of the third test reported in table 5. It is important to note this third application consists in an over-fitting exercise for both puzzles, since neither real returns on uncovered purchasing of foreign government bonds nor on S&P500 compose the data set used to estimate the SDF and $\tilde{M}^{US..fin.mk}$ and $\tilde{M}^{US..fin.mk}$. With regards both puzzles, according to our results, it is apparent that both estimators can price correctly the excess returns between assets traded in the American and international financial markets, in the sense of satisfying equation (17) and equation (16) for both economies.

Even though we have provided a hint as of the theoretical reasons to expect these results, a question remains: under which conditions would the estimators, $\tilde{M}_t$ and $\tilde{M}_t$, fit real returns on assets out of the data set? The answer depends on the technique used, since this result would not be expected for every SDF estimator.

When the multifactor is being used, the good performance is associated with the capacity of the factors to capture systematic risks of the economy, while applying the AFI method, this answer is related to the consistency of $\tilde{M}_{t+1}$, obtained as $N, T \to \infty$. Taking into account the fact that the data set used to estimate the SDF consists of a representative and relatively long quarterly series, if we have complete markets and the process $M_tR_t$ satisfies assumptions 1, 2' and 4 along
with the ones required to apply the Weak Law of Large Numbers, for the british case, for instance, the sample mean of

\[
\frac{\hat{M}_{t+1}^{UK} P_t (1 + r_t^{UK}) (F_{T+1}^{UK} - S_{T+1}^{UK})}{S_t^{UK} P_{t+1}}
\]

converges to zero as \( N, T \to \infty \).

5.3 Discussion

As we have emphasized, our purpose in this paper was to narrow the association between the EPP and the FPP and therefore to see if a model that worked for the EPP would also work for the FPP. Still, one might wonder whether any stochastic discount factor would price excess return correctly. Because we have performed an unconditional test one may wonder whether any SDF would satisfy equation (17). This is not the case. If we use the Law of Iterated Expectation on the consumption Euler equation (8) and apply this to test the existence of the EPP in an unconditional context, along the lines of Kocherlakota (1996), we find that it is not possible to price correctly the excess return of S&P500 over US short term bond with \( \gamma \) less than 28, which is an evidence that the problem is related to the consumption models.

6 Conclusions

After characterizing, along the lines of Backus et al. (1995), the bias that is present in the regression used in most studies of forward premium and also identifying necessary and sufficient conditions for the SDF to account for the FPP, we apply first the technique advanced by AFI and also the widely used multifactor model to quantify this bias from the time series of the pricing kernel.

We are, then able to show that the forward risk premium is not distinct from domestic equity premium in the sense that the same pricing kernel that explains the latter also accounts for the former, at least with regards to US data. In this sense we believe to have pointed out the possibility of a strong association between these puzzles, with the intention to draw parallels between their empirical failures and to motivate the narrowing of the two research agendas.

References


[31] Lettau, M. (2001), "Idiosyncratic risk and volatility bounds, or, can models with idiosyncratic risk solve the equity premium puzzle?," Federal Reserve Bank of New York and CEPR, Staff reports nº 130.


### Table 1: Data summary statistics

<table>
<thead>
<tr>
<th>Country</th>
<th>Mean Covered Operation (percent)</th>
<th>Mean Uncovered Operation (percent)</th>
<th>Mean Excess Return (percent)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Canada*</td>
<td>2,273</td>
<td>4,493</td>
<td>2,494</td>
</tr>
<tr>
<td>Germany*</td>
<td>2,069</td>
<td>5,381</td>
<td>2,944</td>
</tr>
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<td>Japan*</td>
<td>2,593</td>
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<td>Switzerland*</td>
<td>7,931</td>
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</tr>
<tr>
<td>United Kingdom*</td>
<td>1,767</td>
<td>4,498</td>
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</tr>
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</table>

<table>
<thead>
<tr>
<th>Country</th>
<th>Mean 90-day Return (percent)</th>
<th>Mean S&amp;P500 Return (percent)</th>
<th>Mean Excess Return (percent)</th>
</tr>
</thead>
<tbody>
<tr>
<td>United States*</td>
<td>1,774</td>
<td>3,253</td>
<td>8,952</td>
</tr>
</tbody>
</table>

* Observations from 1977:I to 2004:III - 111 observations

** Quarterly data.
Figure 1: SDF's estimate using AFI's methodology with the global assets

**Canada: \( \hat{M}_t^{\text{benc}} \) and \( \hat{M}_t^{\text{exc.CA}} \)**

**Germany: \( \hat{M}_t^{\text{benc}} \) and \( \hat{M}_t^{\text{exc.GE}} \)**

**Japan: \( \hat{M}_t^{\text{benc}} \) and \( \hat{M}_t^{\text{exc.JP}} \)**

**United Kingdom: \( \hat{M}_t^{\text{benc}} \) and \( \hat{M}_t^{\text{exc.UK}} \)**
Figure 2: SDF's estimate using a Multifactor model with the global assets

Canada: $\tilde{M}_t^{\text{ben}}$ and $\tilde{M}_t^{\text{exc, CA}}$

Germany: $\tilde{M}_t^{\text{ben}}$ and $\tilde{M}_t^{\text{exc, GE}}$

Japan: $\tilde{M}_t^{\text{ben}}$ and $\tilde{M}_t^{\text{exc, JP}}$

United Kingdom: $\tilde{M}_t^{\text{ben}}$ and $\tilde{M}_t^{\text{exc, UK}}$
Figure 3: SDF's estimate using AFI's methodology and Multifactor model

Table 2: SDF's estimate statistics

<table>
<thead>
<tr>
<th></th>
<th>AFI methodology</th>
<th>Multifactor model</th>
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<tbody>
<tr>
<td></td>
<td>( \tilde{M}_t^{\text{benc}} )</td>
<td>( \tilde{M}_t^{\text{benc}} )</td>
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<td>0.995</td>
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<td>( \tilde{M}_t^{\text{exc.CA}} )</td>
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<td>( \tilde{M}_t^{\text{exc.JP}} )</td>
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<td>( \tilde{M}_t^{\text{exc.UK}} )</td>
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<td>( \tilde{M}_t^{\text{US.fin.mk}} )</td>
<td>0.977</td>
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Table 3: Results of the empirical application n°1

FPF: Zero mean estimate of the test (16)
EPP: Zero mean estimate of the test (17)

<table>
<thead>
<tr>
<th>Methodology</th>
<th>Multifactor Model</th>
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<tbody>
<tr>
<td></td>
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</tr>
<tr>
<td></td>
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<tr>
<td>AFI</td>
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<tr>
<td></td>
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</tr>
<tr>
<td>(16) Canada</td>
<td>-0.002¹</td>
</tr>
<tr>
<td></td>
<td>(0.365)</td>
</tr>
<tr>
<td>(16) Germany</td>
<td>-0.003⁴</td>
</tr>
<tr>
<td></td>
<td>(0.685)</td>
</tr>
<tr>
<td>(16) Japan</td>
<td>0.002⁴</td>
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<tr>
<td></td>
<td>(0.785)</td>
</tr>
<tr>
<td>(16) Switzerland</td>
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</tr>
<tr>
<td></td>
<td>(0.253)</td>
</tr>
<tr>
<td>(16) United</td>
<td>-0.006⁴</td>
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<tr>
<td>Kingdom</td>
<td>(0.417)</td>
</tr>
<tr>
<td>(17) United</td>
<td>-0.015¹</td>
</tr>
<tr>
<td>States</td>
<td>(0.056)</td>
</tr>
</tbody>
</table>

* Reject the null hypothesis at the 5% significance level. P- values in the parenthesis
¹ OLS estimate.
² ARCH LM test (5 lags). The null hypothesis is the homoskedasticity.
³ Breusch-Godfrey serial correlation LM test (5 lags). The null hypothesis is the absence of serial correlation.
⁴ OLS estimate proposed by Newey and West (1987), taking into account serial correlation and heteroskedasticity.
Table 4: Results of the empirical application n°2

FPP: Zero mean estimate of the test (16)

<table>
<thead>
<tr>
<th></th>
<th>API Methodology</th>
<th>Multifactor Model</th>
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</thead>
<tbody>
<tr>
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</tr>
<tr>
<td></td>
<td>Mean</td>
<td>Test²</td>
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<tr>
<td>(16) Canada</td>
<td>-0.002¹</td>
<td>(0.430)</td>
</tr>
<tr>
<td>(16) Germany</td>
<td>0.001¹</td>
<td>(0.868)</td>
</tr>
<tr>
<td>(16) Japan</td>
<td>0.002⁴</td>
<td>(0.757)</td>
</tr>
<tr>
<td>(16) United Kingdom</td>
<td>0.005⁴</td>
<td>(0.422)</td>
</tr>
</tbody>
</table>

¹ Reject the null hypothesis at the 5% significance level. P-values in the parenthesis

²ARCH LM test (5 lags). The null hypothesis is the homoskedasticity.

³ Breusch-Godfrey serial correlation LM test (5 lags). The null hypothesis is the absence of serial correlation.

⁴ OLS estimate proposed by Newey and West (1987), taking into account serial correlation and heteroskedasticity.
Table 5: Results of the empirical application

<table>
<thead>
<tr>
<th>AFI Methodology</th>
<th>Multifactor Model</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Mean</strong></td>
<td><strong>Test²</strong></td>
</tr>
<tr>
<td><strong>(16)</strong> Canada</td>
<td>0.000&lt;sup&gt;1&lt;/sup&gt;</td>
</tr>
<tr>
<td><strong>(16)</strong> Germany</td>
<td>-0.007&lt;sup&gt;4&lt;/sup&gt;</td>
</tr>
<tr>
<td><strong>(16)</strong> Japan</td>
<td>0.000&lt;sup&gt;4&lt;/sup&gt;</td>
</tr>
<tr>
<td><strong>(16)</strong> Switzerland</td>
<td>-0.017&lt;sup&gt;4&lt;/sup&gt;</td>
</tr>
<tr>
<td><strong>(16)</strong> United Kingdom</td>
<td>-0.009&lt;sup&gt;4&lt;/sup&gt;</td>
</tr>
<tr>
<td><strong>(17)</strong> United States</td>
<td>0.010&lt;sup&gt;1&lt;/sup&gt;</td>
</tr>
</tbody>
</table>

* Reject the null hypothesis at the 5% significance level. P-values in the parenthesis.

1 OLS estimate.

2 ARCH LM test (5 lags). The null hypothesis is the homoskedasticity.

3 Breusch-Godfrey serial correlation LM test (5 lags). The null hypothesis is the absence of serial correlation.

4 OLS estimate proposed by Newey and West (1987), taking into account serial correlation and heteroskedasticity.