Measurements of income inequality: A critique
Inflation and income inequality: A link through the job-search process

RUBENS PENHA CYSNE
(EPGE/FGV)

Data: 19/08/2004 (Quinta-feira)
Horário: 16h

Local:
Praia de Botafogo, 190 – 11º andar
Auditório n° 1

Coordenação:
Prof. Marcelo Fernandes
e-mail: mfermand@fgv.br
Measurements of Income Inequality: A Critique*

Rubens Penha Cysne†

August 14, 2004

Abstract

In this paper I claim that, in a long-run perspective, measurements of income inequality, under any of the usual inequality measures used in the literature, are upward biased. The reason is that such measurements are cross-sectional by nature and, therefore, do not take into consideration the turnover in the job market which, in the long run, equalizes within-group (e.g., same-education groups) inequalities. Using a job-search model, I show how to derive the within-group invariant-distribution Gini coefficient of income inequality, how to calculate the size of the bias and how to organize the data in order to solve the problem. Two examples are provided to illustrate the argument.

1 Introduction

The first two paragraphs of this introduction are not directly related to the main point of the paper.

Measurements of inequality can be based on many different inequality measures. Among such measures, some are decomposable, some are not. A decomposable inequality measure is a measure that can be broken down into a weighted average of inequalities within subgroups of the population and

---

*Key Words: Unemployment, Gini, Inequality, Income Distribution, Critique. JEL: J30, I30, D33.
†Professor at the Getulio Vargas Foundation Graduate School of Economics (EPGE/FGV) and, in 2004, a Visiting Scholar at the Department of Economics of the University of Chicago. E mail: rpcysne@uchicago.edu.
the inequality existing among those groups. Theil’s coefficient is an example of a decomposable measure\(^1\), whereas the Gini coefficient is an example of a measure which is not decomposable\(^2\).

The fact that a measure is not decomposable, though, does not mean that the overall inequality cannot be expressed as a sum of parts, including a within-group and a between-group inequality. Pyatt (1976) and Yao (1999) have shown, for instance, that the Gini coefficient can be decomposed into a within-group, between-group and a residual overlapping inequality, each term being necessarily nonnegative.

Given the popularity of the Gini coefficient, this will be the index that I will use in this work. For this reason I shall refer to Pyatt’s decomposition once more, later in the paper. Note, though, that any other measure could be used as well.

Indeed, the point I want to make here refers to the measurements of inequality, not to measures of inequality. It is robust under all measures.

Last punctual observation, to simplify the exposition of the main argument I assume throughout the whole paper that the only source of income of each worker/consumer is the wage income\(^3\).

Our economy is composed \(N\) different classes (or groups) of homogenous workers. One can think of the economy as a small economy in which workers are employed by a foreign firm. For our purposes, workers should belong to the same class if and only if they face the same distribution of wage offers. In practical terms, this is to say that the different \(N\) classes are supposed to differ by structural parameters such like the level of education or skills of the workers.

As a practical problem, imagine that a researcher gathers income data from the entire population of workers in this economy, in order to measure its degree of income inequality. In each of the \(N\) subgroups of homogenous workers, some have just been laid off, some others have been laid off last period and have not found a job offer this period, others are unemployed because they have found an offer but have turned it down (using the usual terminology, because the offer was below their reservation wage), while, finally, others are employed, but with different wages (distributed in the whole

\(^1\)See Bourguignon (1979).
\(^2\)See Pyatt (1976) and Yao (1979).
\(^3\)Just for the record, transfers and capital income usually represent only a small fraction of most households’ total income. For the United States, for instance, following the 1992 SCF (Survey of Consumer Finances), transfers and capital income account in average for only around 28% of the total income of the households surveyed. This percentage tends to be even lower in developing countries. For this reason, here I will refer to income inequality and labor-income inequality indistinctly.
range between the reservation wage and the upper bound of the distribution of wage offers). The point to note is that the cross-sectional data gathered by the researcher, therefore, will necessarily include a dispersion of wages in each one of the different subgroups.

It happens, though, that by assumption each group is composed of homogenous workers. This implies that, in the long run, given the continuous turnover in the job market, there is no reason for the average wage of workers in the same group to differ. Since each worker will go through the same cycle of being fired, looking for a job etc., there is no reason to assume that he is any different from other workers in the same group. Therefore, the inherent inequality within each class, at least under a long-run perspective, should be made equal to zero.

It happens, though, that this equalization-by-turnover fact does not show in the data gathered by the researcher, due to its cross-sectional nature. The data will show a within-group inequality, even though this within-group inequality vanishes in the long-run. The result is an overestimation of the within-group inequality in each of the $N$ homogenous-workers group and, consequently, of the overall inequality in the economy.

If it is true that the users of the data are interested in a long-run inequality, and if the available statistics have not taken this fact into consideration, all within-group inequality will be source of (a positive) bias. The first point to be investigated here, therefore, is what increases this bias.

In order to formalize this point, suffices concentrating the analysis on one of the $N$ groups of homogenous workers. Proceeding in this way, we are lead to two within-group inequalities: the short run inequality, based upon the cross-sectional distribution, and the long-run inequality, which will always be equal to zero.

Focusing on a certain group, I start by proving the existence and uniqueness of a cross-sectional invariant measure of the wage distribution in the context of a job-search model. The existence and uniqueness of the invariant distribution are the main tools used to show that cross-sectional differences among workers determined by the turnover in the job market vanish in the long-run. Technically, the cross-sectional distribution of wage offers turns out to be the same as the long-run distribution of wages of any specific worker.

Next, I use the invariant distribution of wages to derive a closed-form expression for the Gini coefficient of income inequality. This allows for the determination of which variables are responsible for the within-group inequality, the source of bias, and how they affect it.

The basic framework used here is a variation of McCall's (1970) job-search model. The presentation follows the approach to this model offered in Stokey and Lucas (1989), with the additional feature of allowing for the possibility
that, in each period, the worker does not get a job offer.

The paper proceeds as follows. Section 2 presents the basic model. Section 3 proves the existence and uniqueness of the invariant distribution of wages and derives its density. Section 4 is used to obtain the expression of the within-group invariant-measure Gini coefficient of income inequality, and to investigate which variables determine this inequality. A first example is provided. Section 5 presents the main result of the paper, summarized in Proposition 4, and closes with a second example. Section 6 concludes.

2 The Model

In this section and in the next two ones I concentrate on a single group of homogenous workers. The givens of the model are the distribution of wage offers obtained by the workers, the probability of layoffs and the probability that the worker does not get a job offer next period. The group (of homogenous workers) is populated by a continuum of workers, represented by the measurable space \((0,1], \mathcal{B}[0,1], \mathcal{L})\), \(\mathcal{B}[0,1]\) denoting the borelians in \([0,1]\) and \(\mathcal{L}\) the Lebesgue measure.

For \(0 < D < \infty\), consider also the second measurable space \((\Omega, \mathcal{F}, M)\) and, in this space, the measure \(m_w\) induced by the wage-offer function \(w: \Omega \rightarrow [0,D]\). In the induced space \(([0,D], \mathcal{B}[0,D], m_w)\), denote by \(F_w(t)\) the distribution function that \((m_w-a.e.-uniquely)\) determines the measure \(m_w\): \(F_w(t) = M [w \leq t]\).

The analysis of the job search can be made as a function of just two states regarding the consumer's optimization problem: call it state "w" and state "0". State w corresponds to a job offer of \(w\) at hand, and state 0 to no job offer. In state w the worker can accept or turn down the offer. If he accepts it, by assumption he stays employed with that wage till he is laid off, which can happen, in each period, with probability \(\theta\), \(0 < \theta \leq 1\). If he does not accept the offer or if he gets no offer, he remains in state 0. Being in state zero the only thing he can do is wait again for a job offer next period, which happens with probability \(1 - \alpha\), \(0 \leq \alpha < 1\).

The individual is not allowed to search while in the job. Going to the job market again requires first quitting the job and then waiting for a new offer.

\(^4\) It follows from the developments to be made below that having \(\alpha = 1\) would imply the only ergodic set associated with the transition function defined by the job-search mechanism to be the set \(\{0\}\), and the (only) invariant distribution of the associated Markov process to the degenerated distribution with mass one at this point. I rule out this case by having \(\alpha\) strictly less than one.
next period, which can be easily proved to make this option valueless. The job offers are independent and drawn according to the measure \( m_w \), which is supposed to be known by all workers. The worker is not allowed to borrow or to lend. His consumption, \( c_t \), is equal to his income, \( w_t \), in each period.

Consumers maximize the expected present value of their consumption:

\[
E \left( \sum_{t=0}^{\infty} \beta^t c_t \right)
\]

With \( v(w) \) stating for the value function, and \( A, R \), respectively, for "accept" and "reject", the recursive version of the consumer's problem is given by the maximization of:

\[
v(w) = \max_{A,R} \{ w + \beta [ (1-\theta)v(w) + \theta v(0) ] , v(0) \} \quad (1)
\]

where

\[
v(0) = \beta \left[ (1 - \alpha) \int_{[0,\bar{\omega}]} v(w')dF_w(w') + \alpha v(0) \right]
\]

Making \( X = \frac{1-\alpha}{1-\alpha \beta} \):

\[
v(0) = \beta X \int_{[0,\bar{\omega}]} v(w')dF_w(w')
\]

The reservation wage, \( \bar{\omega} \), which by definition makes the consumer indifferent between \( A \) and \( R \) in (1), is determined by:

\[
\bar{\omega} = \frac{\beta (1-\alpha)}{1-\beta (1-\theta)} \int_{[\bar{\omega},D\bar{\omega}]} (w' - \bar{\omega})dF_w(w') \quad (2)
\]

3 The Invariant Distribution of Wages

This section aims at proving the existence and uniqueness of a stationary distribution of wages in each group of workers of the economy, and at calculating its density as a function of the givens of the model. In order to do this, I start by analyzing the invariant distribution of the wage offers under the rules of the job search. The invariant distribution of wages is then obtained by simply taking into consideration that all wage offers below the reservation wage lead to a wage equal to zero in that period.
The approach here follows very closely Stokey and Lucas (1989, c. 10 and 11). Additional technical details about each one of the developments here can be found in this reference.

The reservation wage $\bar{w}(j)$ divides $[0, D]$ into two regions: the acceptance region $A = [\bar{w}, D]$ and the non-acceptance region $A^c = [0, \bar{w}]$. Consider a new measurable space $([0, D], \mathcal{B}_{[0, D]}, m_o)$. $m_o$ is a measure of the wage offers of a certain worker in the economy. It differs from the measure $m_w$ because, as we are going to see below, it considers the rules of the job search. Denote by $m_{ot}$ its expression at time $t$. At each time $t$, $m_o$ is determined by its initial value and by a transition function $P : [0, D] \times \mathcal{B}_D \to [0, 1]$ (see (3) below) which, in turn, depends upon the rules of the job search process.

The transition function $P$ is determined in the following way: If the current state (given by the wage offer) is $w \in A^c$, the probability of having an offer next period in any borelian $B \subset [0, D]$ is $(1 - \alpha)q(B) + \alpha$, if $0 \in B$, and $(1 - \alpha)q(B)$ if $0 \notin B$.

Alternatively, if the current state is $w \in A$, by the rules of the problem the worker can only lose his job (with probability $\theta$) or keep the same wage next period. Therefore, with probability zero he will have a wage next period in a borelian $B$ that does not contain neither $0$ nor $w$. If the borelian $B$ contains $0$, but not $w$, or $w$ but not zero, the transition probabilities are, respectively, $\theta$ and $1 - \theta$. If it contains both, since these are disjoint events (because $0 \notin A$), $P(w, B) = 1$.

This transition function implies that $[0, D]$ is the only ergodic set of the problem. $m_{ot}$, $t = 0, 1, 2, ...$, relates to the function $P$ by the relation:

$$m_{ot+1}(B) = \int P(w, B)m_{ot}(dw)$$

Denote by $\Psi([0, D], \mathcal{B}_{[0, D]})$ the space of signed measures in $([0, D], \mathcal{B}_{[0, D]})$ and by $\Lambda([0, D], \mathcal{B}_{[0, D]})$ the set of probability measures in $([0, D], \mathcal{B}_{[0, D]})$. It can be shown that $\Psi([0, D], \mathcal{B}_{[0, D]})$ is a vector space. Define in this space a norm (the total variation norm) by:

$$\|\lambda\| = \sup \sum_{i=1}^{k} |\lambda(A_i)|$$

with the supremum above being considered among all finite partitions of $[0, D]$. Since $\Lambda([0, D], \mathcal{B}_{[0, D]})$ is a subset of $\Psi([0, D], \mathcal{B}_{[0, D]})$, (4) defines a metric on the space $\Lambda([0, D], \mathcal{B}_{[0, D]})$. This space, when endowed with the norm $\|\|$, defined by (4), is a complete metric space.

Define in this space the operator $T^*$ by:

$$T^*(m_{ot})(B) = m_{ot+1}(B), \ B \in \mathcal{B}_{[0, D]}$$
In order to talk about an invariant distribution of wages in this economy, it is necessary to show that the distribution of wage offers has one and only one fixed point under the operator \( T^* \). For the demonstration of this important point it suffices proving that, for some \( N \geq 1 \), \( T^{*N} \) is a contraction in the metric space \( \Lambda([0, D], B_{[0, D]}) \). Indeed, since \( \Lambda([0, D], B_{[0, D]}) \) is a complete metric space, if \( T^{*N} \) can be shown to be a contraction, by the \( N \)-stage contraction theorem (Corollary 2 of theorem 3.2 in Stokey and Lucas), \( T^* \) admits one and only one fixed point in \( \Lambda([0, D], B_{[0, D]}) \).

Proving that \( T^{*N} \) is a contraction, therefore, is the only thing we have to do here. This can be easily done with the help of Lemma 11.11 and exercise 11.5a in Stokey and Lucas (1989). Following these results, for \( T^{*N} \) to be a contraction in \( \Lambda([0, D], B_{[0, D]}) \) it suffices to show that there exists a point \( w_0 \in [0, D] \), an integer \( N \geq 1 \), and a number \( \epsilon > 0 \), such that \( P^N(w, \{w_0\}) > \epsilon \) for all \( w \in [0, D] \).

**Proposition 1** The adjoint operator \( T^* \) of the transition function \( P \) defined above has one and only one unique fixed point. This fixed point is the invariant measure of wage offers defined in \( ([0, D], B_{[0, D]}) \) by (7).

**Proof.** Take \( N = 2 \) and \( w_0 = 0 \). From what we saw about the transition function \( P \), there are two cases to consider: if \( w \in A = [\bar{w}, D] \),

\[
P^2(w, \{w_0\}) = \theta m_w(A^c) + (1 - \theta)\theta.
\]

Alternatively, if \( w \in A^c = [0, \bar{w}) \),

\[
P^2(w, \{w_0\}) = m_w(A^c)m_w(A^c) + m_w(A)\theta.
\]

Take

\[
\epsilon = \frac{1}{2} \min \{ \theta m_w(A^c) + (1 - \theta)\theta, \ m_w(A^c)m_w(A^c) + m_w(A)\theta \}
\]

Then, \( \epsilon > 0 \) and \( P^N(w, \{w_0\}) = P^2(w, \{0\}) > \epsilon \), all \( w \in [0, D] \). This proves that \( T^{*N} \) is a contraction. The result then follows from the \( N \)-stage contraction theorem. The second assertion follows by definition. \( \blacksquare \)

After proving existence and uniqueness of the invariant measure of wage offers, the next step is calculating the invariant distribution. Stokey and Lucas (1989) do a particular case in which \( \alpha = 0 \). I make a similar development here. Note that, for any \( C \subset A \):

\[
m_{\alpha t+1}(C) = m_{\alpha t}(A^c)(1 - \alpha)m_w(C) + m_{\alpha t}(C)(1 - \theta) \tag{5}
\]

The determination of the invariant measure \( m_{\alpha}(C) = \lim_{t \to \infty} m_{\alpha t}(C) \) requires the calculation of \( m_{\alpha t}(A^c) \). Since a worker is unemployed in period \( t + 1 \) if and only if he was already unemployed and drew a wage offer in \( A^c \) or was employed and lost his job, we have:

\[
m_{\alpha t+1}(A^c) = m_{\alpha t}(A^c)[(1 - \alpha)m_w(A^c) + \alpha] + m_{\alpha t}(A)\theta \tag{6a}
\]
Taking limits, equation (6a) trivially implies
\[ m_o(A^c) = \frac{\theta}{\theta + m_w(1 - \alpha)} \cdot \]
Taking limits in (5) and using this result yields, for \( C \subset A \):
\[
m_o(C) = \lim_{t \to \infty} m_{ot}(C) = \frac{(1 - \alpha)m_w(C)}{\theta + (1 - \alpha)m_w(A)}
\]

Make \( f_p(s)ds \) represent the number of people earning income in the range \((s, s + ds)\). Taking into consideration that all wage offers in \( A^c \) imply a wage equal to zero, we can finally write the invariant measure of wages:
\[
f_p(s) = \begin{cases} \frac{\theta}{\theta + (1 - \alpha)m_w(A)} & \text{if} \quad s = 0 \\ \frac{(1 - \alpha)dF_w(s)}{\theta + (1 - \alpha)m_w(A)} & \text{if} \quad \bar{w} \leq s < D \end{cases}
\]

This measure has two possible interpretations. First, by the law of large numbers, since we are considering a large numbers of workers drawing from the same distribution, it represents the cross-sectional distribution of wages in the economy. Second, for a fixed worker, it represents the long-run distribution of wages over time. The former interpretation is the one in which we are interested when we construct the Gini coefficient of (cross sectional) income inequality. The latter interpretation is the one we use to argue that all workers in this group share the same long-run income, thereby making the (long-run within-group) inequality equal to zero. The long-run average wage is calculated as:
\[
s_A(s) = \int_{[\bar{w}, D]} \frac{s(1 - \alpha)dF_w(u)}{\theta + (1 - \alpha)m_w(A)}
\]
where \( \bar{w} \) follows from (2).

4 The Within-Group Income Distribution

In the long run, all workers in each group will have the same average wage, given by (8) and, therefore, the long-run within-group inequality will be equal to zero. Under this perspective, the short-run within-group inequality, on which we are going to concentrate in this section, is a source of (long run) bias. It is interesting, therefore, investigating how the parameters of a specific group of workers generating the job search affect this inequality. We shall conclude in this section that the within-group inequality increases with the probability of layoff and with the probability of not finding a job offer.
Any cross-sectional analysis of (labor) income will find heterogeneous situations among consumers. Some have just been laid off, others have found no offer, others have just turned down a wage offer, while others are employed with different wages, ranging from the reservation wage to the wage at the top of the distribution. To measure this (short-run, within-group) inequality I use the Gini coefficient of income distribution calculated under the stationary distribution of wages. The Gini coefficient \( G \) is a ratio between two areas. The first area is the one between the the curves \( k(j) = j \) and the Lorenz curve \( L(j) \), to be defined below. The second area is the one between the curves \( k(j) = j \) and \( k_1(j) = 0 \). In all cases, \( j \) runs from 0 to 1. By integrating:

\[
G = 1 - 2 \int_{[0,1)} L(j) dj
\]

The Lorenz curve expresses the fraction of income earned by a fraction of the population, when this population is ordered from the poorer to the richer. Given the income density function (7), the fraction of the population earning income less or equal to \( s^* \) is given by the distribution function:

\[
F_p(s^*) = \int_0^{s^*} f_p(u)du = \begin{cases} 
\frac{\theta}{\theta + (1-\alpha)m_w(A)} & \text{if } 0 \leq s < \bar{w} \\
\frac{\theta}{\theta + (1-\alpha)m_w(A)} + \int_{[\bar{w},s^*)} \frac{(1-\alpha)dF_w(u)}{\theta + (1-\alpha)m_w(A)} & \bar{w} \leq s \leq D
\end{cases}
\]

(10)

and the fraction of income earned by workers with income less or equal to \( s^* \) by:

\[
F_s(s^*) = \frac{1}{\bar{w}} \int_0^{s^*} s f_p(u)du = \begin{cases} 
0 & \text{if } 0 \leq s < \bar{w} \\
\int_{[\bar{w},s^*)} \frac{(1-\alpha)dF_w(u)}{\theta + (1-\alpha)m_w(A)} & \bar{w} \leq s \leq D
\end{cases}
\]

(11)

The Lorenz curve given by is given by the function \( F_s(F_p) \) when \( s^* \) runs from 0 to \( D \).

Note in (10) that if the reservation wage is less than the lower bound of the distribution \( F_w \), then \( m_w(A) = 1 \). If, additionally, \( \alpha = 0 \) and \( \theta = 1 \), the proportion of the population with a wage equal to zero is 1/2. This happens because, when the probability of unemployment is equal to one, the worker is unemployed once in each two periods (since in the first period, under the conditions here stated, he accepts the wage offer). This observation will be helpful in the future when we analyze the behavior of the Gini coefficient when theta tends to one, and compare it with the case in which alpha tends to one.
From (9), it is easy to see that an increase of the income inequality can be characterized by a decrease of the area under the Lorenz curve.

The next lines pursue an expression for the Lorenz curve as a function of \( f_p \), which is known. This will allow us to characterize what happens with the Gini coefficient of income inequality when the parameters \( \theta \) and \( \alpha \) change, given the measure \( m_w \).

Taking the derivative in (10) and (11) above, one concludes that the slope of the Lorenz curve is given by \( s/s_A \), which by integration with respect to the Lebesgue-Stieltjes measure \( F_p(s^*) \) yields \( F_s(s^*) = \frac{1}{s_A} \int_0^{s^*} udF_p(u) \). Using integration by parts and (10):

\[
F_s(s^*) = \frac{1}{s_A} \left[ s^* \int_0^{s^*} f_p(u)du - \int_0^{s^*} \left( \int_0^{s^*} f_p(v)dv \right)du \right] \tag{12}
\]

We are interested in the area under the Lorenz curve between 0 and a certain wage \( W \in [0, D] \). Call it \( A_L(0, W) \).

\[
A_L(0, W) = \int_{[0,W]} F_s(F_p(s^*))f_p(s^*)ds^* \]

Using \( F_s(F_p(s^*)) \) given by (12):

\[
A_L(0, W) = \frac{1}{s_A} \int_{[0,W]} f_p(s^*)ds^* \left[ s^* \int_0^{s^*} f_p(u)du - \int_0^{s^*} \left( \int_0^{s^*} f_p(v)dv \right)du \right] \tag{13}
\]

Finally, by integrating the last double integral by parts:

\[
A_L(0, W) = \frac{1}{s_A} \int_{[0,W]} f_p(s^*)ds^* \int_{[0,s^*]} uf_p(u)du \tag{13}
\]

**Proposition 2** If an economy follows the rules described in Section 2, the area under the Lorenz curve associated with the long-run wage distribution is given by:

\[
A_L(0, 1) = \frac{(1 - \alpha) \int_{[0,1]} ds^* \int_{[0,s^*]} udF_w(u)}{(\theta + (1 - \alpha)(1 - F(\bar{w}))) \int_{[0,D]} udF_w(u)} \tag{14}
\]

\[\text{[\textsuperscript{5}]} \text{The convexity of the Lorenz curve.}\]
and the Gini coefficient of income distribution by:

\[
G = 1 - 2 \left( \frac{1 - \alpha}{\theta + (1 - \alpha)(1 - F(w))} \right) \int_{[\theta, l]} udF_w(u) \]

(15)

Proof. The first part follows from (8), (7) and (13). The second part follows from (9).

Proposition 1 allows for a direct calculation of the (short run) income distribution within a certain group, once the distribution of wage offers is known.

Below, denote by \( a_L \) the lower bound of the support of the distribution of wage offers. The reservation wage \( w \) is usually a function of theta and alpha. In the case in which:

\[
1 + \beta \theta (1 - \alpha) E_{\text{eq}} w < a_L
\]

(16)

though, that does not happen. where. Under such circumstances, for small variations of \( \theta \) and \( \alpha \), one has:

Proposition 3 If a group of workers follows the rules described in Section 2, and obeys condition (16), then, regardless of the initial distribution of wage offers, the (within-group, short run) Gini coefficient of income distribution is an increasing function of the probability of layoff (\( \theta \)) and an increasing function of the probability that the worker does not get a job offer (\( \alpha \)).

Proof. This is a consequence of (15).

Remark 1 When condition does not apply, a qualitative analysis of the problem shows the following. The y-coordinate of the Lorenz remains at zero till the population reaches mass \( \frac{\theta}{\theta + (1 - \alpha)(1 - F(w))} \). Since this is an increasing function of theta, for the Gini coefficient not to be an increasing function of theta, it is necessary that the Lorenz curve with a lower theta crosses the Lorenz curve with the higher theta from above (note that the slope of the Lorenz curve is given by \( w/w_A \) and \( w_A \) is a decreasing function of theta). Example 1 below shows that condition (16) is not a necessary condition for Proposition 1.

Next, I illustrate the result of Propositions 1 and 2 with an example of the calculation of the within-group income inequality. The example regards a certain group of (homogenous) workers and goes as follows. First, I calculate
the area under the Lorenz curve by the usual method\(^6\). Then I show that one gets the same answer for the area under the Lorenz curve when (14) is used.

**Example 1** Suppose the measure \(m_w\) is given by the Lebesgue measure in \([0,1]\). Note that this measure does not obey condition (16). Using (7):

\[
f_p(s) = \begin{cases} 
\frac{\theta}{\theta+(1-\alpha)(1-w)}, & w = 0 \\
0, & 0 < s < \bar{w} \\
\frac{1-\alpha}{\theta+(1-\alpha)(1-w)}, & \bar{w} \leq s
\end{cases}
\]

which leads to the expression for the fraction of the population with income less or equal than \(s\):

\[
F_p(s) = \begin{cases} 
\frac{\theta}{\theta+(1-\alpha)(1-w)}, & 0 \leq s < \bar{w} \\
\frac{\theta+(s-\bar{w})(1-\alpha)}{\theta+(1-\alpha)(1-w)}, & \bar{w} \leq s \leq 1
\end{cases}
\]

From (11):

\[
F_s(s) = \frac{1}{s_A} \int_0^s u f_p(u)du = \begin{cases} 
0, & 0 \leq s < \bar{w} \\
\frac{(s^2-\bar{w}^2)}{1-\bar{w}^2}, & \bar{w} \leq s \leq 1
\end{cases}
\]

Solve for \(s\) in the second term in (18) and substitute into (19) to get the expression for the Lorenz curve:

\[
L(j) = \begin{cases} 
0, & 0 \leq j < \frac{\theta}{\theta+(1-\alpha)(1-w)} \\
\frac{j(\theta+(1-\alpha)(1-w)+\bar{w}(1-\alpha)-\theta)^2 - \bar{w}^2}{(1-\bar{w}^2)} \frac{\theta}{\theta+(1-\alpha)(1-w)} \leq j \leq 1
\end{cases}
\]

To calculate the area under the Lorenz curve, make:

\[
U = \int_0^1 L(j) dj = \int_0^1 \frac{j(\theta+(1-\alpha)(1-w)+\bar{w}(1-\alpha)-\theta)^2 - \bar{w}^2}{(1-\bar{w}^2) \frac{\theta}{\theta+(1-\alpha)(1-w)}} dj
\]

\(^6\)The usual method, when feasible, uses a parameter \(s\) to write the fraction of the population that earns income less or equal than \(s\), does the same regarding the fraction of total income earned by workers with income less or equal than \(s\), and then proceeds to the elimination of the parameter. See, e.g., Kendall and Stuart (1963).
Making \( u = \frac{\theta + (1-\alpha)(1-\bar{w}) + \omega(1-\alpha)-\theta}{1-\alpha} \), the above integral reads:

\[
U = \int_{\omega}^{1} \frac{(1-\alpha)(u^2 - \bar{w}^2)}{(1 - \bar{w}^2)(\theta + (1 - \alpha)(1 - \bar{w}))} du
\]

By integration:

\[
U = \frac{(1 - \alpha)(1 - 3\bar{w}^2 + 2\bar{w}^3)}{3(1 - \bar{w}^2)(\theta + (1 - \alpha)(1 - \bar{w}))}
\]

(21)

By using (2) and (9), (21) leads to the closed-form solution to the Gini coefficient. To compare this expression with the one given by (14), and show that both expressions deliver the same result, note that, in this case, in (14), \( 1 - F(\bar{w}) = 1 - \bar{w} \) and:

\[
\int_{[\omega, D]} udF_w(u) = \frac{1 - \bar{w}^2}{2}
\]

\[
\int_{[\omega, 1]} ds^* \int_{[\omega, s^*]} udF_w(u) = \frac{1 - 3\bar{w}^2 + 2\bar{w}^3}{6}
\]

from which (21) follows trivially.

Let's proceed to find the Gini coefficient in this case, as a function of both theta and alpha. Using (21), the only thing we have to do is calculating the reservation wage as a function of both alpha and theta. From (2) we get:

\[
\bar{w}(\theta, \alpha) = \frac{1 - \beta \alpha + \beta \theta}{\beta - \beta \alpha} - \sqrt{\frac{1 - \beta \alpha + \beta \theta}{\beta - \beta \alpha}} - 1
\]

(22)

The final expression for the Gini coefficient as a function of the parameters theta and alpha can be obtained by substituting (22) in (21) and using (9). Figure 1 and 2 present the (short run, within-group) Gini coefficient, for \( \beta = 0.98 \), when theta and alpha are allowed to vary in \((0, 1)\). Note that, in this case, the Gini coefficient is an increasing function of both theta and alpha, a concave function of theta, and a convex function of alpha. Note also that the rate of unemployment of this group of workers is given by \( \frac{\theta + (1-\alpha)(1-\bar{w})}{\theta + (1-\alpha)(1-\bar{w})} \), which is an increasing function of both theta and alpha. This is to say that one can expect the rate of unemployment to be positively correlated with the within-group inequality. When alpha is very close to one (but not one), the Gini is very close to one as well, because a very small percentage of the population happens to get job offers. All the remaining workers have no offers and a wage equal to zero. Having theta close to one, though, does not imply a Gini coefficient tending towards one. The reason is that, even when theta is equal to one, those workers who were not employed last period are allowed (with probability \( 1 - \alpha \)) to get job offers and, possibly, to accept them.
5 The Overall Income Inequality

So far I have analyzed the within-group inequality in detail. Now, let's go back to our researcher trying to measure the degree of income inequality in an economy in which there are \( N \) groups of homogeneous workers. Suppose this researcher collects the income data of each group, separately, and uses the Gini coefficient (for our purposes, any other measure could be used) to calculate, using Pyatt's (1976) and Yao's (1999) decomposition, the within-group, the between-group and the overlapping parts of this index:

\[
G = G_W + G_B + G_O \quad (23)
\]

In (23), \( G_W \), \( G_B \) and \( G_O \) (all nonnegative) denote, respectively, the within-group, the between-group and the overlapping part of the Gini coefficient. \( G_B \) is zero if the mean incomes of all classes are identical, and \( G_O \) is zero if the richest person in each low income group is not better off than the poorest person in the next higher income group (a condition which is not met in our case, since in each of the \( N \) groups there will be workers with income equal to zero - unless \( e = 0 \) in that group). The main point of this paper is summarized in the Proposition below:

**Proposition 4** Suppose that all workers facing the same distribution of wage offers in the economy have been grouped together, and that the economy is composed of \( N \) of such homogenous-workers groups. Then, in a long-run perspective, the within-group part of the measure of inequality (\( G_W \) in (23) above, in case the Gini coefficient is the one being used) should not be considered for the purpose of measuring income inequality. Adding \( G_W \) leads to a bias in the measurement of inequality. This bias is an increasing function of the probability of layoff and of the probability of not getting a job offer, in each group. Since the level of unemployment in each group is an increasing function of these two variables as well, one can expect the level of unemployment to be positively correlated with the bias generated by including \( G_W \).

**Proof.** The first part of the proof follows from Proposition 1. The second part, from Propositions 2 and 3. ■

The correct way of calculating the inequality, in the long run, would be using (8) to calculate the average wage of each group and, in a second step, using \( N \) points to construct a measure of inequality. If the Gini coefficient

\footnote{If the researcher used the Theil index, the overlapping part would be equal to zero.}
were to be used, a $N$-point Lorenz curve based on the average wage and population of each of the $N$ groups is all that would be necessary.

**Example 2** Suppose that $N = 2$, that the first group of workers in the economy is described by example 1, and that the second group has $\alpha = \theta = 0$ and all the mass of the distribution of wage offers concentrated on the average wage of the first group. Then, the long run inequality in this economy should be equal to zero, because under the convergent measures, both groups have the same average wage, and in each group all workers will have the same wage as well. However, the data collected by the researcher will necessarily show some workers with income equal to zero (since $\theta \neq 0$ in the first group) and some others with a positive wage (since the average wage of group one is strictly positive). Therefore, the overall income inequality calculated by the researcher will be strictly positive, even though all workers are homogenous from a long-run perspective.

### 6 Conclusions

In this paper I have claimed that measurements of income inequality, under any of the usual inequality measures used in the literature, are upward biased, when considered from a long-run perspective. The reason is that such measurements are cross-sectional by nature and, therefore, do not take into consideration the turnover in the job market which, in the long run, equalizes within-group (e.g., same-education groups) inequalities.

From a technical perspective, I have proved the existence and uniqueness of the stationary distribution of wages, shown how to deduce the invariant-distribution within-group Gini coefficient of income inequality, and concluded that this coefficient, possibly a bias under our perspective here, can be expected to move positively correlated with the rate of unemployment.

Under the view of the paper, if the final number regarding inequality is to be used in a long-run perspective, its measurement should start by grouping all workers facing the same distribution of wage offers. In practical terms, this means controlling the overall population by variables which are not cyclically offsetting, such as unemployment. Groups should differ by structural parameters such as the level of education, skills etc. Proceeding from this point on, only the inequality between such groups should be considered. This way, transitory inequalities based on short-term variables are duly left apart, based on the fact that the within-group income of all workers in the same group converges in the long run to the same real number.

Now suppose that the only inequality measurements available have not taken the precaution described here, that they are available in a time series,
and that the objective of the user of the data is long-run oriented. Then, the results developed here suggest that the researcher should contemplate the possibility of working with the residuals (under the appropriate norm) of the inequality data over the subspace generated by the unemployment and any other short-run cyclically offsetting variables.

References


Figure 1

Gini versus Theta, Alpha=0, Beta=.98

Gini versus Alpha, Theta=.02, Beta=.98
The Gini Coefficient as a Function of Theta and Alpha; Beta=0.98

Figure 2
Inflation and Income Inequality: A Link Through the Job-Search Process*

Rubens Penha Cysne†

August 14, 2004

Abstract

In this paper I devise a new channel by means of which the (empirically documented) positive correlation between inflation and income inequality can be understood. Available empirical evidence reveals that inflation increases wage dispersion. For this reason, the higher the inflation rate, the higher turns out to be the benefit, for a worker, of making additional draws from the distribution of wages, before deciding whether to accept or reject a job offer. Assuming that some workers have less access to information (wage offers) than others, I show that the Gini coefficient of income distribution turns out to be an increasing function of the wage dispersion and, consequently, of the rate of inflation. Two examples are provided to illustrate the mechanism.

1 Introduction

Several works in the economic literature link inflation to income inequality from an empirical perspective. Bulir (1998), Romer and Romer (1998)) and Cardoso et alli (1995) are examples of this type.

Despite the fact that such distributional effects are an important issue in public policy, though, the theoretical literature on the subject is surprisingly

*JEL: J 30, E50, E60. Keywords: Inflation, Inequality, Gini, Income Distribution, Search.
†Professor of Economics at the Getulio Vargas Foundation Graduate School of Economics (EPGE/FGV) and, in 2004, a Visiting Scholar at the Department of Economics of the University of Chicago. E-mail: rpcysne@uchicago.edu.
scarce. In particular, this literature still lacks new ideas and theoretical arguments illustrating how correlations between inflation and inequality can be generated, in the long run, under dynamic settings in which individual consumers maximize the discounted value of their utilities.

Analyses of the link between inflation and inequality usually explore, descriptively, how relationships between capitalists and workers, or between debtors and creditors, are affected by inflation. The usual explanation that poor consumers have less access to interest-bearing money and thereby end up paying a larger share of their income as inflation tax can be included under the debtor/creditor classification as well. A different argument, linking inflation and income distribution through the sharing of the welfare costs of inflation, rather than through distributional effects, has been provided by Cysne, Monteiro and Maldonado (2004).

In this paper I devise a new channel by means of which inflation can provoke income inequality\(^1\). The main idea is that consumers with more information can benefit relatively more, in the job-search process, from the increase of wage dispersion, than consumers with less information. Given the stylized fact that wage dispersion is an increasing function of the rate of inflation\(^2\), the Gini coefficient of income inequality turns out to increase when inflation increases. Two examples are offered to illustrate the proposed mechanism.

The basic model used for the argument draws on Stokey and Lucas’s (1989) version of McCall’s (1970) job-search model. The paper proceeds as follows. Section 2 presents the basic model and assumptions. Subsection 2.1 formalizes the unconstrained consumer problem, in which the number of draws is a choice variable, depending upon an idiosyncratic cost. Subsection 2.2 simplifies the analysis by assuming a cost function that makes the first

\(^1\)More rigorously, this paper deals with long-run wage inequality. However, transfers and capital income usually represent only a small fraction of most households’ total income. For the United States, for instance, following the 1992 SCF (Survey of Consumer Finances), transfers and capital income account on average for only around 28% of the total income of the households surveyed. This percentage tends to be even lower in developing countries.

\(^2\)Wage dispersion is particularly high when inflation reaches a certain level and leads to staggered (lagged) indexation. Under a (mandatory) fixed frequency of adjustments, the ratio of nominal wages of a certain category, just after and before the adjustment, is given by \(1 + \pi\), \(\pi\) standing for the rate of inflation. By these means, under staggered indexation, the higher the rate of inflation, the higher the ratio of existing wages. Brazil in the seventies and early eighties is an example of an economy facing such circumstances. Simonsen (1970) and Dornbusch and Simonsen (1986) are usual sources on this issue. Cardoso (1993), Cardoso et alli (1995) and Souza (2003) present more recent empirical evidence that high rates of inflation increase wage dispersion.
draw free for all workers, whereas other draws are free for a subgroup of works, and prohibitively expensive for the remaining workers.

Under this constrained setting, which is going to be the one used to deliver the main result of the paper, I solve for the reservation wage and (in subsection 2.3) for the long-run average wage of each of the two groups of workers. The long-run average wage is calculated under the invariant distribution of the Markov process determined by the constrained optimization problem. Subsection 2.4 is used to show how to calculate the Gini coefficient of income distribution under a given level of inflation and wage dispersion. The main result of the paper, as well as two examples, are delivered in section 3. Section 4 concludes.

2 The Model

I start by formalizing the unconstrained problem faced by the worker, when he is allowed to choose the number of offers he can draw from the distribution of wages. The givens of the model are the distribution of wage offers faced by the workers, the distribution of technology/cost of acquiring additional job offers and the probability that a certain worker faces, each period, of losing his job.

In the measurable space \(([0,1), B_{[0,1)}, \mathcal{L})\), standing for the Borelians in \([0,1)\) and \(\mathcal{L}\) for the Lebesgue measure in this space, consider a continuum of workers. Each worker has certain technology/cost to get draws from the exogenous distribution of wages. Other than that, workers are all equal. Such a technology leads to a cost, for worker \(j\), \(j \in [0,1)\), of acquiring \(n\) draws from the distribution of wages, given by \(k_j(n)\).

For \(0 < D < \infty\), consider the second measurable space \((\Omega, \mathcal{F}, p)\) and, in this space, the measure \(q\) induced by the (real) wage function \(w: \Omega \rightarrow [0, D]\). In the space induced by \(w\), \(([0, D], B_{[0,D]}, q)\), denote by \(F(t)\) the distribution function that \((q-a.e.\) uniquely) determines the measure \(q : F(t) = p(w \leq t)\).

In the remaining, subindex \(j\) is only introduced when strictly necessary. The consumer is not allowed to borrow or to lend. His consumption \(c_t\) is equal to his income \(w_t\) in each period. The consumer maximizes:

\[
E \left( \sum_{t=0}^{\infty} b^t c_t, \quad 0 < b < 1 \right)
\]
2.1 The Unconstrained Optimization

Once the consumer chooses \( n \), he only considers, in the beginning of the next period, the best (maximum) offer \( w \) among the \( n \) offers. At this point, the consumer can accept or turn down the best offer. If he accepts, he stays employed one period for sure. At the end of this period either he is laid off, with probability \( \theta \), of he keeps his job and wage for sure for one more period, with probability \( 1 - \theta \). The worker is never allowed to voluntarily quit his job or to search while working. If he does not accept the offer or if he is laid off, he restarts the problem by choosing another number of offers \( n \) for the next period. The job offers are drawn independently from \([0, D]\) according to the measure \( q \). \( q \) is known by all workers.

The formal analysis of the unconstrained problem starts backwards, by assuming that the consumer has already decided about \( n \). The decision at this point, to which I turn now, is resolving about accepting or rejecting the best offer at hand. The states of the problem are given by the wage offers at hand, as well as by the status \( E \) (employed) and \( U \) (unemployed).

When already employed with wage \( w \), the value function of any consumer is:

\[
v(w, E) = w + (1 - \theta)bv(w, E) + \theta bV
\]

where \( V \) is the optimum present value for the consumer when he follows the whole course of choosing the optimal strategy, starting with the choice of \( n \).

If unemployed, but with a (best) wage offer \( w \) at hand (\( A \) for accept, \( R \) for Reject):

\[
v(w, U) = \max_{A, R} \{ w + (1 - \theta)bv(w, E) + \theta bV - k(n), bV - k(n) \} \quad (2)
\]

Solving for \( v(w, E) \) in (1) and using (2):

\[
v(w, U) = \max_{A, R} \left\{ \frac{w + \theta bV}{1 - b(1 - \theta)} - k(n), bV - k(n) \right\} \quad (3)
\]

Since the optimization above is carried out under a fixed value of \( n \), and since \( b \) and \( \theta \) are given parameters, the above equation implies that the optimum strategy is of a reservation-wage type. The reservation wage \( \bar{w} \) can then be expressed as a function of \( V \) by the equalization of the two terms in the second member of (3):

\[
\bar{w} = bV(1 - b(1 - \theta) - \theta) \quad (4)
\]
Since $V$ is the value function when $n$ assumes its optimal value, $\bar{w}$ is not a function of $n$ and the value function (3) can be determined by:

\[
v(w, U) = \begin{cases} 
    bV - k(n) & \text{if } w < \bar{w} \\
    \frac{w+\theta bV}{1-\theta} - k(n) & \text{if } w \geq \bar{w}
\end{cases}
\]

(5)

Make $F_{(r,n)}$ stand for the distribution function of the order statistics of order $r$, of a sample of size $n$, and $E_{(n,n)}$ for the respective expectation operator. Define:

\[
G(n) = E_{(n,n)}v = \int_0^D v(w, U)dF_{(n,n)}(w)
\]

The unconstrained consumer problem reads:

\[
V = \max_n G(n)
\]

Note in the equation above that the optimum $n$ is equal to plus infinity when $k(n) = 0$. Indeed, since the consumer makes his decision based on the highest offer, the higher the number of offers he gets the better, because all additional information can be simply disregarded.

### 2.2 The Constrained Optimization

From now on I want to incorporate into the model, in the easiest possible way, a usual real-world situation in which some consumers end up with more wage offers than others, irrespective of their efforts to change it.

The most direct way of capturing this occurrence, without introducing unnecessary calculations that would not add to the main point of the paper, is by postulating that the cost of the first draw is zero for all consumers, and that the cost of any quantity of additional draws is zero for a first group of consumers (say, all $j$ in $[0, j]$) and infinite for the remaining consumers (all $j$ in $[j, 1]$). From the analysis of the precedent subsection, assuming this technology is equivalent to assuming that consumers in cohorts 0 to $j$ will always end up with one job offer, and consumers in cohorts from $j$ to 1 will have a number of offers tending to infinity.

From this point on, I will denominate workers in $[0, j]$ by "group $P$" ($P$ for poor) and workers in $[j, 1]$ by group $R$ ($R$ for rich).

Given the above construction, it is an easy guess that consumers in group $R$ will be better off than consumers in group $P$. The formal point I want to make, though, does not concern the level of the ratio between the wages in group $R$ and group $P$. It concerns the variation of this level with the rate of inflation.
The constrained problem solved by the consumer under the cost function postulated above is of a simpler nature. He only has to decide whether to accept or reject the best wage offer at hand, given the number of draws from the distribution allowed by nature. There is no previous decision about \( n \). The constrained value function of consumers making \( n \) draws from the distribution of wages, when a best offer \( w \) is at hand is:

\[
v(w) = \max_{A,R} \left\{ w + (1 - \theta)bw + \theta b \int_{[0,D]} v(w')dF(n_j,n_j), b \int_{[0,D]} v(w')dF(n_j,n_j) \right\}
\]

in which case the reservation wage is given by:

\[
\bar{w}(j) = \frac{b}{1 - b(1 - \theta)} \int_{[\bar{w}(j),D]} (w - \bar{w}(j))dF(n_j,n_j)
\]

where consistently with the hypothesis made above:

\[
n_j = \begin{cases} 
1, & 0 \leq j < \tilde{j} \\
\infty, & \tilde{j} \leq j \leq 1
\end{cases}
\]

2.3 The Stationary Distribution and the Long-Run Real Average Wage

Make \( \lambda_t \) in \( ([0,D],[B_{[0,D]}) \), represent the measure of the wage offers received by a certain worker at time \( t \). This measure is determined by \( P : [0,D] \times [0,1] \rightarrow [0,1] \), the transition function of the problem, as shown below by (8) and (9).

The transition function \( P \) is determined in the following way. For elements of the (induced) sample space \( w \) in \( [0,\bar{w}] \), assign a probability measure for sets \( B \) in \( B_{[0,D]} \) equal to \( q(B) \). Otherwise, for \( w \) in \( [\bar{w},D] \), assign, measure 0, \( \theta \), 1 - \( \theta \) or 1 to any \( B \) in \( B_{[0,D]} \), depending, respectively, if neither \{0\} or \( w \) is in \( B \), if \( w \in B \) but \( 0 \notin B \), if \( w \in B \) but \( 0 \notin B \) or if both \( w \in B \) and \( 0 \in B \).

In order to talk about an invariant distribution of wages in this economy, it is necessary to show that the distribution of wage offers has one and only one fixed point under the operator \( T^* \) defined by:

\[
m_{o,t+1}(B) = \int P(w,B)m_{o,t}(dw)
\]
\[ T^*(m_{a(t)})(B) = m_{a(t+1)}(B), B \in \mathcal{B}_{[0, D]} \]  

Cysne (2004) provides this demonstration for a more general model.

The next step is finding the invariant distribution \( \lambda \), which happens to be the fixed point of \( T^* \) in the space of probability measures in \( ([0, D], \mathcal{B}_{[0, D]}). \)

As shown in Stokey and Lucas (1989, c. 10), for sets \( C \subset N \) (of employed workers) this invariant measure is given by the solution to:

\[ \lambda_{t+1}(C) = \lambda_t(N^c)q(C) + \lambda_t(C)(1 - \theta) \]  

where:

\[ \lambda_{t+1}(N^c) = \lambda_t(N^c)q(N^c) + \lambda_t(N)\theta \]  

Taking limits in (11a) and (10) yields, for any \( C \subset N \):

\[ \lambda(C) = \frac{q(C)}{\theta + q(N)} \]

Since all mass of wage offers in \( N^c \) implies a wage equal to zero, the long-run average wage of a certain worker \( j \) (which coincides with a cross-sectional average of wages along the whole economy) is then given by:

\[ A(j) = \int_{\bar{w}(j), D} \bar{w}d\lambda = \int_{\bar{w}(j), D} \frac{\bar{w}dq_{n_j,n_j}}{\bar{w} + q_{n_j,n_j}(N(j))} \]  

2.4 Income Distribution

The existence of different numbers of draws from the distribution of wages among workers leads to different income patterns. To measure income inequality I use the Gini coefficient of income distribution. The Gini coefficient \( G \) is a ratio between two areas. The first area is the one between the the curves \( f(j) = j \) and the Lorenz curve \( L(j) \), to be defined below. The second area is the one between the curves \( f(j) = j \) and \( g(j) = 0 \). In all cases, \( j \) runs from 0 to 1. By integrating:

\[ G = 1 - 2 \int_{[0,1]} L(j) dj \]  

where:

\[ L(j) = \frac{1}{1 - A(0)} \int_{0}^{j} A(u) dm(u) \]  

\( m \) denoting the measure in the measurable space \( ([0, 1], \mathcal{B}_{[0,1]}) \) determined by (7) and (12). The formula above uses the fact that the long-run average wage \( A \) is an increasing function of \( j \).
3 Main Result

Consider two different economies, say, $L$ and $H$, with different rates of inflation, $\pi_L$ and $\pi_H$ (for "low" and $H$ for "High"). The only difference between economy $L$ and economy $H$, provoked by the rate of inflation, is that the dispersion of the wage offers in economy $H$, to be defined precisely below, is greater than in economy $L$. Other than that, the economies are the same. In each economy, a fraction $j$ of the consumers (which I have called group $P$ in section 2) has access to only one wage offer, whereas the remaining fraction, $1 - j$, (group $R$) can have as many draws from the distribution of wages as they desire. Other than that, consumers in each group, and in each economy, are the same.

The exogenous distribution of wage offers is given in each period, respectively, in economies $L$ and $H$, by the arbitrary measures $q_L$, with support in $[a_L, b_L]$ and $q_H$, with support in $[a_H, b_H]$. By assumption, due to the higher rate of inflation in economy $H$:

\[ b_L < b_H \]

where $E_{q_L}w$ and $E_{q_H}w$ stand for the expected value of the distribution of wages in each economy, respectively, under the measures $q_L$ and $q_H$. The main result of the paper is given by Proposition 1 below. To simplify the calculations I work under the assumption that the parameters of the model satisfy, in both economies:

\[ \frac{\beta}{1 + \beta \theta} E_{q_L}w < a_H \]

and

\[ \frac{\beta}{1 + \beta \theta} E_{q_H}w < a_L \]

Proposition 1 Consider two economies as described above. Then, the Gini coefficient of income distribution in the economy with high inflation is higher than the one in the economy with low inflation.

Proof. Consider, first, group $R$. In each economy, following (6), making $n_j \to \infty$ in both cases, the reservation wages of workers of group $R$, in economies, $L$ and $H$ converge in measure, respectively, to $\bar{w}_{LR} = b_L$ and $\bar{w}_{HR} = b_H$, and the average wages (using (12)) to $A_{LR} = \frac{b_L}{1 + \theta}$ and $A_{HR} = \frac{b_H}{1 + \theta}$.

Now consider group $P$. Following (6) and (16), their reservation wage is given, respectively, in economy $L$ and $H$, by $\bar{w}_{LP} = \frac{\beta}{1 + \beta \theta} E_{q_L}w$ and $\bar{w}_{HP} =$
\[ \frac{1}{1+\delta} E_{qH} w \] and (by (12) and (16)) the average wage by \( A_{LP} = \frac{E_{qL} w}{1+\delta} \) and \( A_{HR} = \frac{E_{qH} w}{1+\delta} \).

Next, consider the Gini coefficient of income distribution, initially in economy \( L \). Using (13) and (14), the Lorenz curve \( L(j) \) reads:

\[
L(j) = \begin{cases} \\
\frac{j A_{LP}}{A_{LP} + (1-j) A_{LR}} & 0 \leq j < \bar{j} \\
\frac{j A_{LP} + (j-\bar{j}) A_{LR}}{A_{LP} + (1-j) A_{LR}} & \bar{j} \leq j \leq 1
\end{cases}
\]

and the Gini coefficient:

\[
G_L(Z_L) = 1 - \frac{\bar{j}(2-\bar{j}) + (1-\bar{j})^2 Z}{\bar{j} + (1-\bar{j}) Z}
\]

where \( Z_L = \frac{A_{LR}}{A_{LP}} \). Note that \( G_L'(Z_L) > 0 \). Therefore, since the \( \bar{j} \) is the same for both economies \( L \) and \( H \):

\[
G_L < G_H \Leftrightarrow \frac{A_{LR}}{A_{LP}} = Z_L < Z_H = \frac{A_{HR}}{A_{HP}}
\]

This condition is equivalent to having:

\[
\frac{b_L}{E_{qL} w} < \frac{b_H}{E_{qH} w}
\]

which is guaranteed by (the higher dispersion hypothesis) (15).

In Proposition 1 used the assumption that group \( R \) can draw an infinite number of points of the distribution given by \( q_H \), as well as assumption (16). The two examples below show that neither of these assumptions is actually necessary. Example one drops the first assumption, and example 2 drops both assumptions.

It is necessary for the main result of the paper, in general, only assuming that group \( R \) has one more draw from the distribution than group \( P \), and that wages are more dispersed in the economy where inflation is higher.

**Example 1** Here I drop the assumption, used in Proposition 1, that the number of wage offers in group \( R \) goes to infinity. Assume that \( q_L \) and \( q_H \) have support in just two points each, \( q_L \) in \( \{a_L, b_L\} \) and \( q_H \) in \( \{a_H, b_H\} \), with \( 0 < a_H < a_L < b_L < b_H \leq D \). Masses (by assumption, all strictly positive) in these points are denoted, respectively, by \( q_H(a_H) = q_L(a_L) = q_a \) and \( q_L(b_L) = q_H(b_H) = q_b \). Suppose the transformation from \( L \) to \( H \) is mean preserving, meaning that \( q_a a_L + q_b b_L = q_a a_H + q_b b_H \).
In economy L the rate of inflation is $\pi_L$ and, in economy $H$, $\pi_H$, with $\pi_L < \pi_H$. In each economy, in each period, consumers in $[0, \bar{j})$, when unemployed, have one draw from the distribution of wages (given, respectively, by the measures $q_L$ and $q_H$), whereas consumers in $[\bar{j}, 1]$ have two draws from the distribution of wages. Other than that, the economies and the respective groups are the same.

Take economy L. In this economy, workers draw wages $a_L$ and $b_L$ with masses $q_a$ and $q_b$, respectively. First note that if the distribution were degenerated ($a_L = b_L$), then there would be no distinction, in economy L, between those who make one draw and those who make two draws from the distribution of wages. Indeed, in this case the marginal amount of information provided by the second draw is null. In this economy the Gini coefficient of income distribution would be zero, since all workers would have the same average income. It is trivial, though, that the Gini coefficient in economy $R$, which by assumption would be characterized by a nondegenerated distribution of wage offers, would be greater than the Gini of economy L.

Now suppose, more interestingly, that $a_L = b_L$. Still regarding only economy L, the expected values of the distribution of wages, for groups $P$ and $R$, are given, respectively, by:

$$E_{Pw} = \int_{\{a_L, b_L\}} wdq_{L(1,1)}$$  \hspace{1cm} (19)

and

$$E_{Rw} = \int_{\{a_L, b_L\}} wdq_{L(2,2)}$$  \hspace{1cm} (20)

Note that $E_{Pw} \leq E_{Rw}$. In a variation of assumption (16), I assume here that the parameters $b$ and $\theta$ are such that:

$$\frac{bE_{Rw}}{1 + b\theta} < a_L$$

(an equivalent hypothesis also applying to economy $H$). This is a necessary and sufficient condition for the reservation wage of both groups, $P$ and $R$, in each economy, to be below the lower bound $a$.

The reservation wages in economy L of groups $P$ and $R$ are given, respectively, by $\bar{w}_{LP} = \frac{b}{1 + b\theta} E_{LPw}$ and $\bar{w}_{LR} = \frac{b}{1 + b\theta} E_{LRw}$, with $\bar{w}_{LP} \leq \bar{w}_{LR}$ (the reservation wage of workers in group R is higher because workers in this group can always simply disregard one of the two draws). From (12):

$$A_{LP} = \frac{E_{Pw}}{1 + \theta} < \frac{E_{Rw}}{1 + \theta} = A_{LR}$$  \hspace{1cm} (21)
Using (14) and (13), the Gini coefficient of income distribution (still in economy L), as above, is given by (17). Also as in the demonstration of Proposition 1:

\[ G_L < G_H \Leftrightarrow \frac{A_{LR}}{A_{LP}} = Z_L < Z_H = \frac{A_{HR}}{A_{HP}}. \]

From (21), dividing both the numerator and the denominator by \( a_L \):

\[ \frac{A_{LR}}{A_{LP}} = \frac{q_1^2 + \frac{b_L}{a_L}(1 - q_1^2)}{q_1 + \frac{b_L}{a_L}(1 - q_1)}. \]

Since \( \frac{A_{LR}}{A_{LP}} \) is an increasing function of \( b_L \), the relative range of the distribution of wage offers in the country with low inflation, and since (15) in this case implies \( \frac{b_L}{a_L} < \frac{b_H}{a_H} \), it follows that \( \frac{A_{LR}}{A_{LP}} < \frac{A_{HR}}{A_{HP}} \) and that \( G_L < G_H \).

**Example 2** Regarding Proposition 1, in this second example I drop the assumption that \( n \) tends to infinity (here, \( n = 2 \)), as well as assumption (16). Assume that \( q_L \) and \( q_H \) are given, respectively, by the uniform distribution in \([2,3]\) and \([1,4]\). In the respective supports, this leads to the distribution functions

\[ F_L(1,1)(s) = -2 + s, \quad F_H(1,1)(s) = -1/4 + (1/4)s, \]

\[ F_L(2,2)(s) = 4 - 4s + s^2, \quad F_H(2,2)(s) = 1/16 - (1/8)s + (1/16)s^2. \]

Using (6), and (12), after some tedious calculations, the reservation wages and the average wages, in each case, can be shown to assume the values given by Table 1 below:

<table>
<thead>
<tr>
<th></th>
<th>Economy</th>
<th>Ratio H/L</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low</td>
<td>High</td>
<td></td>
</tr>
<tr>
<td>Rich</td>
<td>2.71</td>
<td>3.44</td>
</tr>
<tr>
<td>Poor</td>
<td>2.61</td>
<td>3.25</td>
</tr>
<tr>
<td>Rich</td>
<td>2.81</td>
<td>3.64</td>
</tr>
<tr>
<td>Poor</td>
<td>2.75</td>
<td>3.51</td>
</tr>
</tbody>
</table>

The most important point in Table 1 is that the ratio of the average wages, between economy \( H \) and economy \( L \), is higher for the rich than for the poor. By (18), this implies that income is more concentrated in economy \( H \) than in economy \( L \). The reason for \( \frac{A_{LR}}{A_{LP}} < \frac{A_{HR}}{A_{HP}} \), as presented in the demonstration of Proposition 1, is that the rich are more able to take advantage of the (mean-preserving) increase of uncertainty than the poor.

It is also interesting to note that (as one would expect) the rich always have a higher reservation wage and a higher average wage than the poor, in both economies, \( L \) and \( H \).
And that, for both groups, R and L, the reservation wage and the average wage in economy H is higher than in economy L. This fact shows that both groups are able to take advantage of the increase of uncertainty, because of the option-nature of the job-search mechanism (bad draws can always be discarded).

4 Conclusion

In this paper I formalize a link between inflation and the Gini coefficient of income inequality, assuming that higher rates of inflation lead to an increase of the dispersion of wage offers. Under this setting, the higher the inflation rate, the higher turns out to be the benefit, for a worker, of making additional draws from the distribution of wages. Assuming that some workers have less access to information (wage offers) than others, I show that the Gini coefficient of income distribution turns out to be an increasing function of the rate of inflation. Two examples are provided to illustrate the mechanism.

References


