A theoretical connection between inflation and income inequality

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A Theoretical Connection Between Inflation and Income Inequality*

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Abstract

This work investigates the effects of inflation on income distribution. We use a dynamic shopping-time model to show that a differentiated access to transacting technologies by poor and rich consumers is enough to generate a positive link between inflation and the Gini coefficient of income distribution.

1 Introduction

Although there remains a controversy on the empirical literature relating inflation to income distribution (Galli and Hoeven (2001) call it the inflation-income inequality puzzle) several works in the area (e.g., Bulir (2001), Romer and Romer (1998)) present compelling evidences that, at least for high rates of inflation, these variables are positively correlated.

This literature, though, still lacks optimizing dynamic models which deliver such result from a theoretical perspective, formally associating inflation with some measure of income inequality. Our purpose in this paper is filling in this gap.

Elsewhere (Cysne and Maldonado (2004)), we study this problem from a search-theoretic perspective. Here, instead, we concentrate on examining the

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possibility that a link between inflation and income distribution can be generated solely by means of a differentiated access to transacting technologies. The usual connection between inflation and income distribution, regarding the different amount of inflation tax paid by rich\(^1\) and poor consumers, as percentage of their incomes, is kept away from our considerations.

The shopping time chosen by the poor and by the rich differ because the latter, by assumption, have access to a better transacting technology. The underlying intuition connecting inflation to income distribution, in this case, is that the higher the rate of inflation, the more important turns out to be this lack of balance between the more and the less endowed consumers.

Indeed, when the nominal interest rate is equal to zero, both the rich and the poor have the same shopping-time, equal to zero. The higher the rate of inflation and the interest rate, though, the higher the opportunity costs of holding monetary assets, and the higher the substitution away from monetary assets to shopping time. Therefore, one should expect those with access to a better transacting technology to do relatively better when inflation is higher. We shall see that this is indeed what happens in the economy.

2 The model

Our basic model draws upon the homogeneous-agents shopping-time model with interest-bearing deposits presented by Simonsen and Cysne (2001) and Cysne (2003), which, in turn, draws upon Lucas (2000).

Our economy has an infinite number of homogenous-consumers cohorts. The cohorts are classified by the productivity of their consumers in the production of the consumption good, and distributed in the \([0, 1]\) interval. Each cohort has the same (large) number of consumers. There are no transactions between different cohorts. The productivity of consumers in cohort \(j \in [0, 1]\) is \(\delta_j > 0\). We suppose that the productivity is nondecreasing in \(j\). There is a cutoff productivity \(\delta_j, j \in (0, 1)\) such that consumers in cohorts with productivity \(\delta_j < \delta_j\) are called "poor" and consumers in cohorts with productivity \(\delta_j > \delta_j\) are called "rich".

The poor have access only to currency \((M)\), which they use to make their transactions. The rich can use currency and an interest-bearing assets \(X\) to make their transactions\(^2\). \(X\) pays a nominal interest rate equal to \(i_x\). The

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\(^1\)What we mean by rich and poor consumers is defined later in the text.

\(^2\)Mulligan and Sala-i-Martin (1996) have shown that the cost of adopting financial
rich also buy bonds (B) form the government. Bonds pay an interest rate \( i \) and are not used for transacting.

In the description of the model, for the sake of notational clarity, we suppress the subindex \( j \) that characterizes the (homogeneous) consumers in each cohort.

Both types of consumers, rich and poor, gain utility from the consumption of a single non-storable consumption good, and have a separable utility function

\[
\int_0^\infty e^{-st}U(c_t) \, dt, \quad c \in C([0, \infty), [0, \infty)).
\]  

We suppose that \( U \) is continuous, increasing and concave. Consumers in each cohort are endowed with one unit of time that can be used to transact \( s \) or to be used in the production of the consumption good, \( y \), according to the production function:

\[
y = \delta(1 - s)
\]  

2.1 Government and Banks

Our economy is a Fisherian economy with lump sum taxation, where the government can implement any given interest rate vector. We shall therefore refer to the nominal interest rate as a policy variable. The government, here consolidated with the Central Bank, is supposed to issue currency and bonds, and to collect reserve requirements from the banks. Banks buy bonds from the government and issue \( X \). The banking system is competitive. \( k \) (\( 0 < k < 1 \)) stands for the reserve requirement on \( X \). The zero-profit condition implies \( i - i_x = k_i \).

2.2 The rich consumer maximization problem

With the price of the consumption good indicated by \( P = P(t) \), the rich consumers in each cohort \( j \) face the budget constraint:

\[ technology is negatively related to the level of education. Such a finding supports our hypothesis of a segmented market for financial assets if we think of productivity as being positively correlated with education.\]
\[ \dot{B} + \dot{M} + X = iB + i_x X + P \left( y - c \right) + H \]

\( H \) indicates the (exogenous) flow of money transferred to the consumer by the government, and the dot over the variable its time derivative. Each consumer takes \( H \) as exogenous in his optimization problem.

Making \( \pi = \dot{P}/P \) (inflation rate), \( m = M/P, x = X/P \) and \( h = H/P \), and taking into account (2), the budget constraint of the rich consumer (meaning, of a consumer in a cohort with productivity \( j > j \)) reads:

\[ m + x = \delta(1 - s) - c + h + (i - \pi)b + (i_x - \pi)x - \pi m \quad (3) \]

In the steady state, the consolidation of the balances of the government and of the banks leads to:

\[ h = \pi m - (i - \pi) b - (i_x - \pi)x \quad (4) \]

Since our purpose here is concentrating on the shopping-time association between inflation and income distribution, we want to rule out the possibility that different (relative) amounts of net real financial income paid by each cohort interfere in the income distribution process. We do this by assuming that \( h \) is determined by the government in such a way that equation (4) holds separately for each cohort \( j \).

Rich consumers have access to a shopping technology \( F(m, x, s) \) where \( m \geq 0, x \geq 0, s \in [0, 1], F_m > 0, F_x > 0, F_s > 0 \). Further conditions on the function \( F(\cdot) \) will be introduced later.

The rich consumer maximizes \( 1 \) subject to (3) and to

\[ 0 \leq c \leq F(m, x, s) \]

The first order conditions for a steady state solution of the maximization problem are given by:

\[ i = \pi + g \quad (6) \]
\[ \delta F_m = iF_x \quad (7) \]
\[ \delta F_x = (i - i_x)F_s \quad (8) \]
2.3 The poor consumer maximization problem

Poor consumers also have access to the technology $F(m, x, s)$ but are not allowed to adopt it fully, since they are constrained to having $x = 0$. The poor consumer maximizes $I$ subject to

$$0 \leq c \leq F(m, 0, s)$$

$$c + m \leq \delta (1 - s) + h - \pi m$$

The first order condition is:

$$\delta F_m(m, 0, s) = iF_s(m, 0, s) \quad (9)$$

3 The Steady State Solutions

From this section onwards, we shall, when necessary, use the subindexes $p$ and $r$, respectively, for poor and rich. We shall also restrict our analysis to the case of a transacting technology weakly separable in shopping time and monetary assets, by making $F(m, x, s) = G(m, x) s^3$. $G(m, x)$ is differentiable, homogeneous of degree one, increasing with respect to each variable and with $G_m/G_x$ an increasing function of $x/m$.

• Rich:

Equations (7) and (8) now read:

$$iG = G_m \delta s \quad (10)$$

$$kG = G_x \delta s \quad (11)$$

In equilibrium, since the consumption good is non-storable and the government transfers to each cohort match the net amount of real interests paid or received:

$$\delta (1 - s) = c = G(m, x) s. \quad (12)$$

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3This transacting technology is a particular case of the one used by Simonsen and Cysne (2000). Lucas (1993, 2000) presents the link between first degree homogenous transacting technologies and the classical inventory-theoretic literature. For the importance of weak separability, in another context, see Cysne (2001).
Given the hypotheses about $G(m, x)$, the marginal rate of substitution is an increasing function of the asset ratio $x/m$. Taking the inverse function and using (10) and (11):

$$\frac{x_r}{m_r} = J \left( \frac{G_m}{G_x} \right) = J \left( \frac{1}{K} \right) \quad J'(-) > 0 \quad (13)$$

Equations (10), (11) and (12), with $k$ fixed, determine $s_r, m_r$ and $x_r$ as a function of the policy variable $i$:

$$s_r(i) = \frac{-i(1+kJ(1/k))}{2G(1,J(1/k))} + \sqrt{\frac{\delta^2(1+kJ(1/k))^2 + i(1+kJ(1/k))}{4G(1,J(1/k))}}$$

$$x_r(i) = \frac{i(1+kJ(1/k))}{i(1+kJ(1/k))} \cdot \frac{J(1/k)}{\delta s_r}$$

$$m_r(i) = \frac{\delta s_r}{i(1+kJ(1/k))}.$$  

This determination proceeds as follows: Since $G(m, x) = mG_m + xG_x$ (Euler’s theorem), from (10) and (11):

$$s_r = \frac{i(m_r + kx_r)}{\delta} \quad (14)$$

To obtain the equilibrium variables use (13) to get $x_r$ as a function of $m_r$. Then use (14) to get $m_r$ (and $x_r$) as a function of $s_r$. Finally, by taking into consideration that $G(m, x) = mG(1, J(1/k))$, one can obtain $s_r$ by using the expressions for $m_r$ and $x_r$ in (12).

- Poor:

In this case equations (10) and (12) are still valid, however, with $x = 0$. The first degree homogeneity of $G$ implies $G(m, 0) = mG(1, 0)$ and $G_m = G(1, 0)$. Therefore the first order equation (10) can be rewritten as:

$$\delta s_p = i \cdot m_p$$
Following the same procedure previously outlined, the solutions for the poor are given by:

$$s_p(i) = \frac{-i}{2G(1,0)} + \sqrt{\frac{i^2}{4G(1,0)^2} + \frac{i}{G(1,0)}}$$

$$m_p(i) = \frac{\delta s_p}{i}.$$ 

### 4 Main Results

Note that $s_p(0) = s_r(0) = 0$, since in this case there is no private cost associated with the use of money. Lemma 1 establishes necessary and sufficient conditions for the shopping time of the poor to be greater than the shopping time of the rich.

**Lemma 1** $s_p > s_r$ if the transaction technology and the parameter $k$ are such that

$$G(1, J(1/k)) > G(1, 0)(1 + kJ(1/k))$$

**Proof.** $s_r$ and $s_p$ are determined, respectively, as roots of the quadratic equations:

$$f_r(s) = s^2 + \frac{(1 + kJ(1/k))i}{G(1, J(1/k))} s - \frac{(1 + kJ(1/k))i}{G(1, J(1/k))}$$

and

$$f_p(s) = s^2 + \frac{i}{G(1, 0)} s - \frac{i}{G(1, 0)}$$

The family of quadratic equations $g(x; b) = x^2 + bx - b, b > 0$ always presents a real root $x_1$ such that $0 < x_1 < 1^4$. Besides, since this root satisfies $x_1^2 + bx_1 - b = 0$, it follows from the implicit function theorem that:

$$\frac{dx_1}{db} = \frac{1 - x_1}{\sqrt{b^2 + 4b}} > 0$$

4Indeed, $g(0; b) = -b < 0$ and $g(1; b) = 1 > 0$. The other root is negative and can be disregarded, because $s > 0$. 

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The demonstration is complete once one notices that (15) is equivalent to having
\[
\frac{(1+\beta J(1/\beta))i}{i} \leq \frac{1}{i} \quad \text{in (16) and (17).}
\]
Condition (15) is satisfied if the productivity of the transacting technology
with respect to \(x\) is high enough for all values of \(x\) and \(m\). Indeed, this
enhances the disadvantage of the poor due to not having access to this asset,
thereby leading them to spend more time shopping. Example 1 below shows
that this condition is satisfied, for instance, when \(G(m, x)\) is a CES function
with an elasticity of substitution \(\sigma \in (1, \infty)\).

- The Gini Coefficient and the Rate of Inflation

To measure the inequality in the income distribution we use the Gini
coefficient of income distribution. The Gini coefficient \(G\) is given by:

\[
G = 1 - 2 \int_0^1 L(j) dj
\]

where
\[
L(j) = \frac{\int_0^j c_u du}{\int_0^1 c_u du}
\]
stands for the Lorenz curve. The Lorenz curve measures the proportion of
the total income of the economy that is received by the lowest \(100j\%\) of the
consumers. It can be easily shown that the Gini coefficient expresses the
area between the Lorenz curve and the Lorenz curve for an economy where
everyone receives the same income.

We proceed to calculate the Gini coefficient for our economy. Note that \(s_r\)
and \(s_p\) do not depend on the productivity coefficient \(\delta_j\). It will be notationally
convenient to define
\[
\Delta_j = \int_0^j \delta_u du
\]

and
\[
\Gamma_j = \int_0^j \Delta_u du
\]
The first step is to calculate \( \int_0^j c_u du \). If \( j < j \),
\[
\int_0^j c_u du = \int_0^j c_u^* du = \int_0^j \delta_u (1 - s_p) du = (1 - s_p) \Delta_j
\]
If \( j > j \),
\[
\int_0^j c_u du = \int_0^j c_u^* du + \int_j^j c_u^* du = (1 - s_p) \Delta_j + (1 - s_r) (\Delta_j - \Delta_j)
\]
Thus, from (18):
\[
G = 1 - 2 \int_0^j (1 - s_p) \Delta_j dj + \int_j^j ((1 - s_p) \Delta_j + (1 - s_r) (\Delta_j - \Delta_j)) dj (1 - s_p) \Delta_j + (1 - s_r) (\Delta_1 - \Delta_j)
\]
\[
G = 1 - 2 (1 - s_p) \Gamma_j + (1 - s_r) \Delta_j (1 - j) + (1 - s_r) (\Gamma_1 - \Gamma_j) - (1 - s_r) (1 - j) \Delta_j
\]
\[
G = 1 - 2 (1 - s_p) \Gamma_j + (1 - s_r) \Delta_j (1 - j) + (1 - s_r) (\Gamma_1 - \Gamma_j) - (1 - s_r) (1 - j) \Delta_j
\]
Proposition 2: If condition (15) is satisfied, then the Gini coefficient is a non-decreasing function of the nominal interest rate (or, equivalently, of the rate of inflation), at the point \( i = 0 \).

Proof. We proceed by calculating the derivative of the Gini coefficient with respect to the interest rate at \( i = 0 \).
\[
G(0) = 1 - 2 \frac{\Gamma_j + \Delta_j (1 - j) + \Gamma_1 - \Gamma_j - \Delta_j (1 - j)}{\Delta_1} = 1 - 2 \frac{\Gamma_1}{\Delta_1}
\]
\[
\frac{G(i) - G(0)}{2} = \frac{-\Delta_1 (1 - s_p) (\Gamma_j + \Delta_j (1 - j)) - \Delta_1 (1 - s_r) (\Gamma_1 - \Gamma_j - \Delta_j (1 - j))}{\Delta_1 [\Delta_j (1 - s_p) + (\Delta_1 - \Delta_j) (1 - s_r)]} + \frac{\Gamma_1 \Delta_j (1 - s_p) + \Gamma_1 (\Delta_1 - \Delta_j) (1 - s_r)}{\Delta_1 [\Delta_j (1 - s_p) + (\Delta_1 - \Delta_j) (1 - s_r)]}
\]
\[
\Gamma_1 = \Gamma_j + \int_j^1 \Delta_u du < \Gamma_j + \Delta_1(1-j)
\]

we have

\[
\left[ \Gamma_1 \Delta_j - \Delta_1 \left( \Gamma_j + \Delta_j (1-j) \right) \right] < 0
\]

Therefore,

\[
\frac{G(i) - G(0)}{2} > 0 \iff s_p(i) > s_r(i)
\]

Taking the limit as \( i \to 0 \),

\[
s_p(i) > s_r(i) \Rightarrow G'(i) \big|_{i=0} \geq 0
\]

The Proposition follows from Lemma 1. □

**Example 1** We consider the transacting technology \( F(m, x, s) = G(m, x)s = A(m^a + x^a)^{1/a}s \), \( A > 0 \), \( 0 < a < 1 \) and productivities \( \delta_j = \delta \) if \( j \leq j' \), \( \delta_j = \lambda \delta \), if \( j > j' \) (\( \lambda > 1 \)). Note that in this case:

\[
kJ(1/k) = k^{\alpha/(a-1)}
\]

\[
G(1, J(1/k) = A(1 + k^{\alpha/(a-1)})^{1/a}
\]

It can be easily checked that the condition (15) and the previous assumptions about \( G \) are satisfied. In this case (\( \delta \) can be taken as equal to one in the calculations of \( G \) because it cancels out):

\[
s_r(i) = -\frac{i}{2A} \left( 1 + k^{a-1} \right)^{1-1/a} + \sqrt{\frac{i^2}{4A^2} \left( 1 + k^{a-1} \right)^{2(1-1/a)} + \frac{i}{A} \left( 1 + k^{a-1} \right)^{1-1/a}}
\]
\[
\Delta_j = \begin{cases} 
\frac{j}{\lambda j - (\lambda - 1)j} & \text{if } j \leq \bar{j} \\
\frac{j^2}{2} & \text{if } j > \bar{j}
\end{cases}
\]

\[
\Gamma_j = \begin{cases} 
\frac{j^2}{2} & \text{if } j \leq \bar{j} \\
(j - \bar{j})^2 + (j - \bar{j}) + \bar{j}^2 / 2 & \text{if } j > \bar{j}
\end{cases}
\]

The value of the Gini coefficient for different values of the interest rate can be obtained using (19) and the above expressions. Table 1 presents the values of \(s_r\), \(s_p\), and of the Gini coefficient for the parameter values \(a=0.3\), \(A=40\), \(k=0.25\), \(\bar{j}=0.75\), \(\lambda=10\) and interest rates equal to 0, 100%, 200% and 500%.

Table 1

<table>
<thead>
<tr>
<th>Interest Rate (100%)</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>(s_r)</td>
<td>0</td>
<td>0.046</td>
<td>0.065</td>
<td>0.100</td>
</tr>
<tr>
<td>(s_p)</td>
<td>0</td>
<td>0.146</td>
<td>0.200</td>
<td>0.296</td>
</tr>
<tr>
<td>Gini Coefficient</td>
<td>0.5192</td>
<td>0.5383</td>
<td>0.546</td>
<td>0.560</td>
</tr>
</tbody>
</table>

Figure 1 presents the evaluation of the Gini coefficient when interest rates assume values between zero and a thousand percent.

5 Conclusions

We have developed a simplified model, based on a shopping-time rationale, to investigate the effects of inflation on the Gini coefficient of income distribution. A basic assumption of the model is that some (cohorts of) consumers have access to a better transacting technology than others. Our basic conclusion is that a formal link between inflation and the Gini coefficient of income distribution based solely on this fact can be theoretically proved. For transacting technologies satisfying a certain condition, established in the text, an increase of the inflation rate unequivocally leads to a deterioration of the income distribution. Finally, we have provided an example that satisfies such condition and in which the Gini coefficient is a monotonically increasing function of the rate of inflation.

Note that, as one should expect, \(G = 0\) for \(\bar{j} = 0\) or \(\bar{j} = 1\).
 References


The Gini Coefficient as a Function of the Interest Rate

Figure 1: