"Insider Trading, Investment, and Liquidity: A Welfare Analysis"

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INSIDER TRADING, INVESTMENT, AND LIQUIDITY:
A WELFARE ANALYSIS

by

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INSIDER TRADING, INVESTMENT, AND LIQUIDITY: A WELFARE ANALYSIS

ABSTRACT

We compare competitive equilibrium outcomes with and without trading by a privately informed "monopolistic" insider, in a model with real investment portfolio choices ex ante, and noise trading generated by aggregate uncertainty regarding other agents' intertemporal consumption preferences. The welfare implications of insider trading for the ex ante expected utilities of outsiders are analyzed. The role of interim information revelation due to insider trading, in improving the risk-sharing among outsiders with stochastic liquidity needs, is examined in detail.

JEL Classification: D52; D82; G14.
KeyWords: Portfolio Choice, Incomplete Markets, Private Information, Rational Expectations.
I. INTRODUCTION and SUMMARY

Models of trading with privately informed traders in financial markets, and the resulting implications for the informativeness of market prices regarding anticipated risky asset returns in a noisy Rational Expectations Equilibrium (REE), have constituted a major component of new advances in economic theory and finance for more than a quarter century. Major contributions to this literature include the papers of Lucas (1972), Grossman and Stiglitz (1980), and Kyle (1985). More recently, inspired in part by work on the welfare implications of incomplete markets beginning with Hart (1975), attention has turned to the impact of such privately informed trades and noisy REE on real variables in the economy, chief among which are (i) the level of privately chosen aggregate investments in risky technologies, and (ii) the levels of welfare of agents who are less well-informed a priori. Recent work emphasizing some of these issues includes the papers of Ausubel (1990), Dennert (1992), Dow and Rahi (1996), Leland (1992), Biais and Hillion (1994) and Repullo (1994). In some of these analyses, the informational monopoly power of the insider, and its strategic use, have been incorporated along the lines of Kyle (1985).

In much of the above-mentioned work on informed trading and its impact on real variables, it has been customary -- in order not to have unrealistic fully revealing REE and no insider trading profits -- to postulate some portion of the market demand for (or supply of) securities arising from unmodelled "noise traders", whose endowments and preferences for consumption are left unspecified. This makes it difficult to reach a welfare judgement, regarding for example the impact of allowing Insider trading by (informed) managers of a firm, even if its implications for some endogenous variables such as the informativeness of asset prices in a noisy REE -- and the level of aggregate real investment in risky technologies -- can be ascertained. Thus, an important issue in financial regulatory policy, regarding the desirability
of trading by (ex post identifiable) informed Insiders of a traded firm, remains largely unresolved at the conceptual level.

Our major goal in this paper is to rectify this shortcoming, by modelling both noise traders and rational (a priori) uninformed traders together, as agents with well-specified endowments and preferences whose intertemporal consumption preferences, and hence interim trading strategies, are ex ante uncertain. This phenomenon is modelled as interim "shocks" to agents' preferences (or other incomes) affecting their (indirect) utility functions for consumption (withdrawal of savings) at one of two time points that follow the ex ante beginning, when real investment decisions -- the allocation of agents' endowments across risky and riskless technologies -- are made. Thus, our methodology "transplants" modelling techniques from the literature on banking models (Bryant, 1980) to the arena of privately informed insider trading, an innovation also found in the recent related work of Qi (1996).

Our second methodological observation is to note that, when privately observed (conditional on some aggregate shock) and not-separately-insured shocks to agents' intertemporal consumption preferences are postulated -- as a convenient modelling device to capture "noise trading" without abandoning welfare analysis -- we are in a context of incomplete markets (Hart, 1975), even in the absence of privately informed insiders who acquire interim information about future risky asset returns, indeed even with riskless investment technologies. Thus, traded outcomes with endogenous real investment choices, even in a one-commodity (at each time-point) model, may be constrained Pareto-inferior to what could be attained by a planner in terms of agents' ex ante expected utilities, even if she has no information on agents' private liquidity shocks; see Bhattacharya and Gale (1987). Hence, to characterise fully the incremental impact of privately informed (on asset returns) trading in these models, one may consider scenarios in which (owing to the nature of the preference
shocks of non-insiders) traded outcomes are ex ante Pareto inefficient in the absence of the possibility of interim trading based on insider information. Thus, the details of our modelling differ from the usual negative exponential utility and Normal returns and information distributions-based modelling of Grossman-Stiglitz, Dennert, Leland, Repullo, Dow and Rahi etc., who all work with a setup in which the interim traded outcomes are always ex-ante Pareto efficient in the absence of private information on asset returns.¹

The recent literature on insider trading, as well as some earlier work of Allen (1984), makes it clear that the greater interim informativeness of asset market prices brought about by informed trading may benefit other investors' welfare, if real investment-level choices are sufficiently flexible (with low liquidation costs) at the interim stage when the insider acquires her information. Thus, for example, the average level of such interim investment may be higher with than without insider trading (Leland, 1992), since the lower conditional variance of future asset returns with insider trading, in a noisy REE, causes rational outsiders to augment their demand schedules for risky investments. Similarly, as Diamond and Verrecchia (1982) and more recently Holmstrom and Tirole (1993) have pointed out, interim share prices that reflect a greater degree of (otherwise unverifiable) payoff-relevant information regarding future returns, may be useful to construct precise performance measures for incentive schemes for firms' managers, vis-à-vis their ex ante effort and project selection choices. In this paper, we deemphasize these interim investment effects², focusing instead on (a) inflexible ex ante (aggregate) choices across risky long-term, versus riskless short-term, real technologies for

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¹ The reason is, of course, the wealth-invariant demand for the risky asset implied by (intertemporally additively separable) exponential utility preferences.

² Our modelling choice of deemphasizing interim investment choices is justified in an environment in which the time lag between the insiders' information arrival and subsequent public knowledge thereof (e.g., for accounting earnings or tender offers) is short, and/or the nature of interim information makes it costless to disclose ex post, partly to serve as the basis for managerial incentive schemes.
investment, and (b) interim traded prices for early liquidation by a stochastic proportion of non-insiders, which are further affected by the presence of insider trading.

The interim consumption and portfolio allocations of non-insiders are clearly affected by a greater informativeness of interim asset prices brought about by insider trading. The insider, in turn, is a strategic player, with "market power" arising only from her information, who takes others' portfolio choices into account in deciding on her trading strategy, given her private information at the interim stage. Together, these choices—along with the aggregate liquidity shocks for outsiders—determine the informativeness of interim asset prices regarding their ultimate returns. We study these effects of insider trading on interim asset prices, on the outsiders' ex ante optimal allocations of their savings across a riskless and a risky long-term assets, and finally on the outsiders' ex ante expected utility levels with and without insider trading at the interim stage, before risky returns are realized.

We compare agents' ex ante and interim optimal choices, given aggregate resource constraints and/or budget constraints at traded equilibrium prices, as well as their welfare levels across three scenarios: (A) optimising choices by a welfare-maximising planner, (B) interim trading among (early- and late-dying) outsider agents only, and (C) interim trading with possible participation by the informed insider. These comparisons are carried out numerically, for reasons of tractability in the face of possibly binding interim resource constraints, i.e., "corner solutions", which in turn affect the agents' ex ante optimal choices. We find that the non-insider agents' ex ante welfare is always the highest in scenario (A), strictly, which is not surprising since our planner is endowed with the same interim information as the insider, and she can thus adjust early- and late-dying agents' optimal consumption levels to the realised (interim information on the) return on the long-term technology, which is in general desirable. Our results on comparing agents' choices and
welfare levels and also interim equilibrium asset prices, across the scenarios (B) and (C),
generates more subtle and perhaps surprising conclusions.

We find that often the outsider agents’ ex ante expected utility levels are higher in scenario
(C), in which the informed insider may take part in the interim asset trading, as compared to
scenario (B) in which the uninformed outsiders carry out such trading among themselves. This
outcome is more likely to arise when the lowest possible return on the risky technology rises (as
well as the highest), and also as the proportion of agents requiring early consumption increases –
provided it is not so large as to make trading by the insider unprofitable for her. This result - of
outsider agents being ex ante better off with the insider trading - becomes less likely as the range
of possible variation in the proportion of early-dying agents in the economy becomes higher, the
reason being that then the insider is thereby able to carry out sales of higher quantities of the
long-term asset when its anticipated future return is low.

The adverse selection losses incurred by uninformed agents to the insider may be
exceeded for their ex ante expected utility by the main beneficial impact of insider trading,
which arises as follows. Since the insider does not sell the long-term asset when its anticipated
return is high, if in addition the aggregate liquidity shock is low, the selling price on the long-
term asset fully reflects its high return, which enhances the consumption level of early diers,
modulo the purchasing power of late-dier agents. This impact of insider trading on outsider
agents’ consumption profiles is the dominant factor behind the possibility of outsiders’ ex ante
welfare improving with insider trading -- even without any interim flexibility in aggregate real

3 This occurs when a small proportion of late-dier agents requires a high risk premium in interim asset prices to
compensate for the adverse selection losses to the insider.
4 She may still obtain a profitable price whenever the uninformed outsiders are “confused” between the states of
nature in which (i) the aggregate liquidity shock is low but the insider is selling the long-term asset, and (ii)
the aggregate liquidity shock is high but the insider is not selling, because she expects high returns on the long-
term asset.
5 Our insider is endowed with the risky asset, and can only sell it because any borrowing would reveal her
identity.
investment, unlike in Allen (1984), Leland (1992) and Dow and Rahi (1996), for example. It is more difficult to discern the impact, in a definite direction, of insider trading on the outsiders’ ex ante (privately) optimal asset allocation choices, for example to align these more closely to the ex ante optimal choices in scenario (A), than to those in trading scenario (B). The latter possibility is always logically present in an incomplete-markets setting (Hart, 1975) with the uninsured private liquidity shocks, in which interim traded allocations are generally allocationally ex ante inefficient even with a riskless long-term (as well as a short-term) asset (see Bhattacharya and Gale, 1987) – but this ex ante indirect effect does not appear to be playing a major role in our scenario (C).

Our paper is set out as follows. In the next section, we describe the main features of our model, and solution methods for it. Numerical comparisons of investment choices, asset prices and welfare are made in Section III. In Section IV we conclude.

II. ALTERNATIVE ALLOCATIONAL MECHANISM

There are three time points \( t=0,1,2 \). All agents are born at \( t=0 \) and supply inelastically savings/endowments of unity in aggregate. There is a continuum of agents with aggregate Lebesgue measure of unity, and in addition possibly an Insider of strictly positive measure. These endowments can be invested either in a risky two-period technology paying off at \( t=2 \), or in a riskless storage technology paying off at \( t=1 \) and, if reinvested at \( t=1 \), at \( t=2 \). Holdings
of the two period risky technology can, however, be traded in a secondary market at \( t=1 \), with selling by agents who wish to consume early. The storage technology has unit gross returns, and the risky technology with constant returns to scale has final payoffs per unit investment of \( \tilde{\theta} \) distributed as:

\[
\tilde{\theta} = \begin{cases} 
\theta_L & \text{with probability } \pi, \\
\theta_H & \text{with probability } (1-\pi)
\end{cases}
\]

as viewed from the ex ante time point \( t=0 \), where \( \theta_H > \theta_L \). It is assumed that \( \pi \) is common knowledge among all the agents, and that

\[
\pi \theta_L + (1-\pi) \theta_H > 1.
\]

For convenience, we sometimes denote \( \{\pi, (1-\pi)\} \) as \( \{\pi_L, \pi_H\} \).

The non-insider agents' intertemporal preferences for consumption, at \( t=1 \) and/or \( t=2 \), can be described as follows. There are two aggregate states \( l \) and \( h \), and associated conditional probabilities \( 0 < \alpha_i < \alpha_h < 1 \), such that conditional on the aggregate state \( l(h) \), each agents' utility for consumptions at \( t=1 \) and/or \( t=2 \) is an independently identically distributed random variable

\[
U(C_i) \text{ with probability } \{[\alpha_i] \text{ or } [\alpha_h]\}
\]

\[
U(C_i, C_2) = U(C_j) \text{ with probability } \{[1-\alpha_i] \text{ or } [1-\alpha_h]\}
\]

These aggregate "liquidity states" \( l \) and \( h \), are assumed to arise with ex ante probabilities \( q \) and \( (1-q) \), sometimes denoted \( \{q_l, q_h\} \). We assume that \( \{q, \alpha_i, \alpha_h\} \) are common knowledge, but that each uninformed agent only knows her own realized \( U(C_i, C_2) \), but not the aggregate state \( l(h) \). These randomized preferences, coupled with their aggregate variability, have effects on prices that are akin to those arising from "noise traders" in REE models.\(^6\)

\(^6\) In recent papers, e.g., Diamond and Verrecchia (1981) or Dow and Rahi (1996), noise trading has been modelled explicitly via endowment shocks (with Normal distributions) to agents having negative exponential utilities for consumption at one future time-point.

\(^7\)
Agents make per capita real investment choices across the two available investment technologies, in proportions K and (1-K) respectively, initially at t=0. Further investment in or liquidation of the risky technology at the interim date t=1 is assumed to be infeasible. However, individual agents who wish to consume at t=1, and those who wish to postpone their consumption until t=2, can anticipate trading their long-term investment in the risky technology at equilibrium prices \( P(K, \theta, \alpha_i, j \in \{L,H\}, i \in \{l,h\} \), per unit investment. Here, \( P(K, \theta, \alpha_i) \) is the Rational Expectation Equilibrium price mapping from the underlying aggregate state (including the equilibrium investment choice K at t=0) which must be measurable with respect to the information possessed by the collection of trading agents, possibly, including the informed insider when she participates.

The insider is assumed to have an exogenously given ex ante endowment of the risky technology only at t=0, from which she may choose to (partially) sell and reinvest in the riskless short-term technology at t=1, for consumption at t=2. The interim and final payoffs and consumption allocations, arising from a rationally anticipated \( P(K, \theta, \alpha_i) \) mapping, are taken into account by the atomistic outsiders in their optimal, and identical, ex ante choices K and (1-K) to respectively invest in the short-term riskless and risky long-term asset.

**A. Ex Ante Optimal Allocations**

The central planner, endowed with interim information about the risky asset payoff, chooses \( C_{ij} \) and K to maximise:

\[
\sum_{i=1}^{h} \sum_{j=L,H} q_i \pi_j \left[ \alpha_i U(C_{ij}) + (1 - \alpha_i) U(C_{2ij}) \right]
\]

subject to the resource constraints that, for each \( i, j \in \{l,h\} \times \{L,H\} \):

\[
\alpha_i C_{ij} \leq K
\]

7 In Bhattacharya and Gale (1987) it is shown that, even when \{\alpha, \theta\} are deterministic, interim traded allocations at t=1 are ex-ante Pareto inefficient unless \( U(C) = \log(C) \).
\[ (1-\alpha_i) C_{2ij} = (\theta_j (1-K) + K - \alpha_i C_{1ij}) \quad (5b) \]

**B. Traded Equilibria Without Inside Information**

If at \( t=0 \) agents choose (per capita) investments of \( K \) and \( (1-K) \) in the storage and risky two-period technologies, respectively, then those who wish to consume at \( t=1 \) *ex post* obtain consumption:

\[ C_{ij} = \left[(1-K)P_{i} + K\right] \quad (6a) \]

whereas those who wish to consume late at \( t=2 \) obtain the consumption level:

\[ C_{2ij} = \left[(K - P_{j}X_{j}(P_{i})) + \theta_j (1-K + X_{j}(P_{i}))\right] \quad (6b) \]

where the subscripts \( \{i,j\} \) refer to the aggregate states of liquidity \( I (h) \) and risky asset return \( L (H) \), respectively and \( X_{j}(P_{i}) \) is the per capita trade of “late diers” buying the long-term asset, at \( t=1 \), from the “early diers”. In an equilibrium without inside information about \( \tilde{\theta} \), \( \{P_{i}, X_{j}\} \) can only depend on the liquidity state \( i \). Furthermore, in equilibrium we must have market clearing:

\[ (1-\alpha_i)X_i(P_i) = \alpha_i (1-K) \quad (7a) \]

and since the "late diers" wishing to consume (only) at \( t=2 \) have, in the aggregate, no agents to borrow from\(^8\), we must also have:

\[ K-P_i X_i(P_i) \geq 0. \quad (7b) \]

Equations (7a) and (7b) together imply the aggregate liquidity constraint on market-clearing prices \( \{P_i\} \):

\[ P_i \alpha_i (1-K) \leq (1-\alpha_i) K \quad (8) \]

In their ex ante choice of \( K \), agents maximise their ex ante expected utility

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\(^8\) In other words, the equilibrium borrowing rate at \( t=1 \) must be such that no late-dier outsider wishes to borrow.
\[
\text{Max } \sum_{i=1}^{\alpha_i} \sum_{j=L,H} q_{ij} \left[ \alpha_i U(C_{ij}) + (1 - \alpha_i)U(C_{ij}) \right] 
\]

whereas at \( t=1 \), given \( P_i \) (which in equilibrium will reveal state \( l \) or \( h \) to traders without private information about \( \tilde{\theta} \)), the "late diers" choose \( X_i \) to

\[
\text{Max } \sum_{j=L,H} \pi_j U(C_{2ij}) \left[ X_i \right] 
\]

leading to a uniquely maximal \( X_i (P_i) \) which, in interim equilibrium (7a), must satisfy (7b), given the ex ante optimal \( K \) choice which \textit{anticipates} this equilibrium (and optimal) evolution \( \{X_i, P_i\} \) at \( t=1 \). The equilibrium prices \( P_i(K) \) are to be found among the positive real roots of a non-linear equation in \( P_i \), unless the no-borrowing constraint (7b) binds in which case \( P_i = \frac{(1 - \alpha_i)K}{\alpha_i (1 - K)} \), from equation (8).

The program \textsc{Mathematica®} is used to compute the solution to the first-order condition relative to \( K \) and \( X_i \), the market clearing conditions (7a) and the no-borrowing conditions (7b). The resulting computed \textit{ex ante} optimal \( K^* \) (liquid asset investment) choices at \( t=0 \), as well as our agents' \textit{ex ante} expected utility, which depend on the parameters \( \{\alpha_i, \alpha_h, \theta_L, \theta_H, q, \pi\} \) of the model, are numerically tabulated in Section III below. These solutions are then compared to those arising in a scenario in which an additional agent, the \textit{insider}, with perfect information about the realised value of \( \theta \) at \( t=1 \), participates strategically in the trading (anticipating the trading by the uninformed) at the interim stage, as well as with the allocations that are \textit{ex ante} optimal, as described in the subsection above.

\textbf{C. Noisy REE with Insider Trading and Market Orders}

\footnote{The non-linear equation is quadratic with logarithmic utility and of degree 4 when \( U(C) = -1/(2C^2) \). Three out of the four roots can be dismissed (two are complex; one exceeds the value of \( \theta \)).}
We now postulate that, in addition to the agents we have already modelled, there is an insider endowed at $t=0$ with $n \geq [\alpha_h - \alpha_i]$ units of the risky, and illiquid, technology only, which she may sell at time $t=1$ and invest in the riskless storage technology. This insider only wishes to consume at time $t=2$, and she knows perfectly at $t=1$ if the return on the risky technology at $t=2$ would be $\theta_H$ or $\theta_L$. Only for simplicity in computing her expected utility, and thus her decision to participate in any selling at $t=1$ or not, we assume that the insider is risk-neutral.

The imperfectly informed outsider late diers’ trades are assumed to depend on the partitions of the aggregate state space, $\{\alpha_i, \alpha_h\} \times \{\theta_L, \theta_H\}$, that are revealed to them by the equilibrium prices with the insider trading. The outsider agents take the market-clearing REE prices in these partitions as given parametrically, and the late diers submit demand functions $\{X(P)\}$ with domain restricted to these prices only; the early diers supply their long-lived asset inelastically. The insider chooses her trading rule strategically to take the outsiders’ behaviour into account. However, we assume that the insider can condition its trade only on its realized information about $\theta$ ($\theta_L$ or $\theta_H$), but not on the aggregate liquidity shock among non-insiders, i.e., $\alpha_i$ or $\alpha_h$. Since it is in the interest of insiders to “mask” their private information about $\tilde{\theta}$, such strategic trading by insiders will (be shown to) result in a noisy REE in which the following three partitions of the aggregate state space are revealed by equilibrium prices:

$$a: \{h,L\} \quad (11a)$$

$$b: \{(l,L), \{h,H\}\} \quad (11b)$$

$$c: \{l,H\} \quad (11c)$$
with the associated (weakly increasing) set of interim prices \( \{P_a, P_b, P_c\} \) respectively. In such an equilibrium, the insider sells a quantity \( Q > 0 \) of the risky asset in states \( \{h, L\} \) and \( \{l, L\} \), and does not trade otherwise. In particular, we rule out any borrowing at \( t=1 \) by the insider from "late dier" outsiders, who are identical and know that "early diers" have no wish to borrow for the future.

The insider’s choice of \( Q \) is made subject to the knowledge that late-dier outsiders would choose their net purchases (per capita) of the risky asset \( X_{ij} \), in aggregate state \( \{i,j\} \), to maximize their conditional expected utility:

\[
\text{Max}_{X_{ij}} \sum_{i \neq k \neq h \neq l} \hat{\pi}_{ij} \log \left( (K - P_{ij} X_{ij}) + \theta_i \left( 1 - K + X_{ij} \right) P_{ij} \right)
\]

where \( P_{ij} \) is the noisy REE equilibrium price at \( t=1 \) in state \( \{i,j\} \) per unit of the risky technology, \( \hat{\pi}_{ij} \) is the outsiders’ revised beliefs about \( \Theta \), and \( X_{ij} \) must satisfy:

\[
X_{ij} = X_{kl}, \quad i \neq k \text{ and/or } j \neq l, \quad \text{if } P_{ij} = P_{kl}
\]

The outsiders’ trades at \( t=1 \) must also satisfy a no-borrowing constraint:

\[
P_{ij} X_{ij} \leq K \forall_{ij}
\]

Equivalently, the REE must meet the aggregate liquidity constraint:

\[
P_{ij} \alpha_i (1-K) \leq (1-\alpha_i) K
\]

The revised beliefs \( \{\hat{\pi}_{ij}\} \) of outsiders depend, of course, on the partitions of the aggregate state space generated by the trading of (themselves and) the insider. Finally, the outsiders’ ex ante optimal choice of investment pattern, \( K \), is made to maximise in equation (9) as above, taking into account the \( \{X_{ij}, P_{ij}\} \) configurations in interim equilibrium at \( t=1 \) for each \( K \) choice ex ante. Finally, in examining the existence of any equilibrium with \( Q > 0 \) (in states \( \{l, L\} \) and \( \{h, L\} \)) trades by the insider, we must compare her expected utility in such an
equilibrium versus one in which -- as in Section II.b above -- it desists from trading, and thus one obtains an equilibrium in which prices are $P_i$ in states $\{1, L\}$ and $\{1, H\}$ and $P_h \leq P_i$ in states $\{h, L\}$ and $\{h, H\}$.

We are now in a position to describe fully the noisy REE in our setting with a strategic insider at the interim stage:

**PROPOSITION.** If condition (16) below is satisfied, then there exists a noisy REE in which the insider sells $Q > 0$ in states $\{1, L\}$ and $\{h, L\}$ where $Q$ satisfies

\[(1-K)\alpha_h = (1-\alpha_h)X(P_b)\]  \hspace{1cm} (14a)

\[(1-K)\alpha_i + Q = (1-\alpha_i)X(P_b)\]  \hspace{1cm} (14b)

where $X(P_b)$ is the late dier's per capita demand for trade in the risky technology in states $\{(1, L)\}$ and $\{(h, H)\}$ given equilibrium price $P_b$ therein, chosen to maximize in (12a) given their revised beliefs:

(i) \[p_H | P_b = \frac{q_H \pi_H}{(q_H \pi_H + q_H \pi_L)}\]  \hspace{1cm} (14c)

with the complementary conditional probability $\hat{p}_L = 1 - \hat{p}_H$. In the other states, equilibrium prices and beliefs satisfy:

(ii) in state $\{h, L\}$ price $P_a$ with $\left(\hat{p}_H | P_a \right) = 0$, and

\[(1-\alpha_h) X(P_a) = (1-K)\alpha_h + Q\]  \hspace{1cm} (14d)

where $X(P_a)$ maximizes in (12a) given $P_a$ and $\left(\hat{p}_H | P_a \right)$;

(iii) in state $\{1, H\}$ price $P_c$ with $\left(\hat{p}_H | P_c \right) = 1$, and

\[(1-\alpha_i) X(P_c) = (1-K)\alpha_i\]  \hspace{1cm} (14e)

where $X(P_c)$ maximizes in (12a) given $P_c$ and $\left(\hat{p}_H | P_c \right)$. 

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The demands of outsiders must further satisfy the conditions:

\[ X(P_a) = \frac{K}{P_a} \text{ if } P_a < \theta_c \]  
(15a)

\[ X(P_a) \in \left[ 0, \frac{K}{P_a} \right] \text{ otherwise} \]  
(15b)

and, similarly,

\[ X(P_c) = \frac{K}{P_c} \text{ if } P_c < \theta_H \]  
(15c)

\[ X(P_c) \in \left[ 0, \frac{K}{P_c} \right] \text{ if } P_c = \theta_H \]  
(15d)

Together, the outsiders’ ex ante optimal investment choice \( K \) and the interim equilibrium prices must satisfy the aggregate liquidity constraint (13). Finally, in order to satisfy the condition for profitability of this insider trading strategy we must have that, in equilibrium, given the ex ante optimal choice of \( K \) by non-insiders,

\[ q_i(P_a - \theta_L) + q_m(P_c - \theta_H) \geq 0. \]  
(16)

Remark: violation of equation (16) is possible since \( P_a < \theta_L \) is feasible.

III. NUMERICAL RESULTS ON INVESTMENTS, PRICES, AND WELFARE.

The complexity of the above set of models, in particular the possibility of “corner solutions” vis-à-vis interim \( \{X_i\} \) trades at \( t=1 \) over a (possibly proper) subset of the parameter space, appears to rule out a fully analytic solution procedure for our programs of finding the equilibrium \( \{P_i(K)\} \) functions and the ex ante optimal \( K \) choices as delineated above. Hence, even for our agents with additively separable (over time) power utilities, we
have to resort to numerical simulations\textsuperscript{10} -- of equations of the type embodied in the Proposition above -- in order to compare equilibrium outcomes across alternative informational "regimes". In particular, the scenarios to be compared are those of:

(A) \textit{First Best, or Unconstrained Optimal Outcomes};

(B) \textit{Uninformative Equilibria}, with no-one having (revised) information about \( \theta \) at \( t=1 \);

(C) \textit{Insider Trading Equilibria}, with trading by insiders affecting prices, and hence outsiders' information about realised \( \theta \).

In Tables I through IV below, we compare the equilibrium outcomes in these different regimes, focusing in particular on the non-insider agents' ex ante optimal liquid asset investment (\( K \)) choices, and their equilibrium ex ante expected utilities\textsuperscript{11}. We seek to understand under what circumstances one would expect to see one trading regime (no insider) do better than another (insider trading) for ex ante welfare, in order in particular to establish guidelines for desirable regulatory restrictions on insider trading (which is ex-post identifiable and punishable with non-zero probability).

We have done our numerical simulations using the following sets of parameter values:

(i) \( \{ q, \pi \} = \{ \frac{1}{2}, \frac{1}{2} \} \);

(ii) \( \{ \alpha_p, \alpha_h \} = \{ .1, .15 \}, \{ .9, .95 \}, \{ .48, .53 \}, \{ .45, .55 \}, \{ .4, .6 \} \);

(iii) \( \{ \theta_L \} \in \{ .75, .8, .85, .9, .95 \} \) with \( \{ \theta_H \} \in \{ 1.25, 1.3, 1.35, 1.4, 1.45, 1.5 \} \).

For most of our simulations, we have worked with \( U(C)=\frac{1}{2}C^2 \), with a relative risk aversion coefficient of 3, though other \( U(C) \) were tried as well. We have taken \( n=1 \), i.e., an insider with at least equal shareholdings as that of non-insiders. However, it is the equilibrium extent

\textsuperscript{10} The \textsc{mathematica} programs are available from the authors upon request.
of selling of the long-term risky technology in some states of nature at t=1 by the insider
(Q>0) that impact on interim prices, and Q is bounded above by the difference in the
aggregate trades among type II (late dier) and type I (early dier) outsiders across the states
{1,L} and {h,H}, a difference which the insider "masks" via her trading.

From the comparisons in Table I, we see that (1) the first-best solution (A) always
dommates the uninformed only trading (B) and insider trading (C) scenarios in ex ante
welfare, (2) that for \( \{ \alpha_n - \alpha_i \} = .05 \), the outsiders' welfare is higher with insider trading (C)
than without in 23 of the 30 cells of the matrix in the \( \{ \theta_L, \theta_H \} \) space \(^{12} \), and (3) this outcome
arises only in 10 cells when \( \{ \alpha_n - \alpha_i \} = .1 \) and in only 4 cells if \( \{ \alpha_n - \alpha_i \} = .2 \). Note also that
insider trading is more likely to improve outsiders' welfare when \( \theta_L \) is high (above .85), and
the degree to which it does so is greater when \( \theta_H \) goes up. However, as the gap \( \{ \alpha_n - \alpha_i \} \)
widens -- allowing the amount of equilibrium insider trading (selling) Q to increase -- insider
trading equilibria (C) become more likely worse for outsiders than traded equilibria (B)
without such trading.

In Table II, we look at outsiders' ex ante investment (K) choices, across scenarios (A),
(B) and (C) -- focusing on the case \( \{ \alpha_n, \alpha_i \} = \{ .48, .53 \} \). No clear pattern of comparison
emerges, except to note that K(B)>K(C)>K(A) when \( \{ \theta_L, \theta_H \} \) are low, whereas
K(A)>K(B)>K(C) or K(A)>K(C)>K(B) when \( \{ \theta_L, \theta_H \} \) are high. Hence, there appears to be
no discernible pattern of investment choice with insider trading, K(C), being closer to the ex

\(^{11} \) Qi (1996) works with risk-neutral outsider agents, hence his model does not capture the impact of insider
trading on risk-sharing among (late- and early-dier) outsider agents that our calibrations do.
\(^{12} \) When both \( [ \theta_L, \theta_H ] \) are high the individual borrowing (12c) and the aggregate liquidity (13) constraints are
violated in state \( [ \alpha_L, \alpha_H ] \) in the insider trading case. We therefore compute the solution imposing \( P_x = (1-\alpha)K[\alpha, \alpha \ast(1-K)] \). The resulting equilibrium values are reported in the tables, under the heading (C2). When the
aggregate liquidity constraint starts binding in states \( [ \alpha_L, \alpha_H ] \) and \( [ \alpha_i, \alpha_H ] \) as well, we further impose \( P_y = (1-
\alpha_n)K[\alpha_n (1-K)] \) and report solutions under the heading (C3).
ante optimal choice $K(A)$ than is $K(B)$, the ex ante choice in the interim traded equilibrium without the insider.

In Table III, we look at interim prices -- in the two partitions $\{\alpha_i, \alpha_h\}$ for trading scenario (B) and in the three partitions $\{(\alpha_i, \theta_H), (\alpha_i, \theta_L) \cup (\alpha_h, \theta_H), [\alpha_h, \theta_L]\}$ in scenario (C) -- for different values of $\{\theta_L, \theta_H\}$. Note that in the partition $[\alpha_i, \theta_H]$ the equilibrium with insider trading (often) has the interim long-term asset price equalling $\theta_H$, which leads to consumption gains for early diers, that are beneficial in terms of the ex ante welfare of outsider agents. For $\theta_L \in \{.8, .85\}$ and $\theta_H$ not so high late diers’ expected utility falls (slightly) but early-diers’ increases by more, leading to an increase in ex ante welfare relative to (B). For both $[\theta_L, \theta_H]$ large, the aggregate liquidity constraint binds and price is accordingly lower than $\theta_H$ in state $[\alpha_i, \theta_H]$, which increases the return obtained by the late diers on the long-term technology. The rise in ex ante welfare is now associated to an increase in both early and late diers’ expected utility.

Finally, in Table IV, we present some welfare comparisons for lower and higher average levels of $\alpha$, i.e., $\{\alpha_i, \alpha_h\} = \{.1, .15\}$ and $\{.9, .95\}$. In the former case, the insider trading solution (C) is welfare superior to the traded solution (B) without the insider only when $\theta_L \geq .9$, as compared to $\theta_L \geq .8$ when $\{\alpha_i, \alpha_h\} = \{.48, .53\}$ - but the insider chooses not to trade when $\theta_L = .95$ and $\theta_H \geq 1.4$, so that insider trading effectively aids outsiders’ welfare in only 9 of the 30 cells. The reasons for this are that (i) with lower $\alpha$, fewer early-diers gain from the price improvement in the $[\alpha_i, \theta_H]$ state brought about by insider trading, and (ii) with high $[\theta_L, \theta_H]$ the insider’s losses in the state $[\alpha_h, \theta_H]$ overwhelm her gains in $[\alpha_i, \theta_L]$. With $\{\alpha_i, \alpha_h\} = \{.9, .95\}$, the insider chooses not to trade whenever $\theta_L \geq .9$ and $\theta_H \geq 1.35$, or
$\theta_l=.95$, so trading scenario (C) improves outsiders' welfare - as compared to scenario (B) - only in 5 of the 30 $\{\theta_l, \theta_h\}$ cells.$^{13}$

IV. CONCLUDING REMARKS

We have shown, with an intertemporal model of individual as well as aggregate liquidity shocks to uninformed agents, that interim trading by informed insiders can improve outsiders' ex ante welfare, even when aggregate investment choices can not (technologically) respond to any partial information revelation brought about by such insider trading via prices. The rationale behind our finding is the beneficial impact of insider trading on outsiders' selling prices and consumption in some states, which more than compensates for

$^{13}$ We have carried out comparisons analogous to those in Table I for $U(C)=\log(C)$, with relative risk-aversion of unity, and $U(C)=-C^\gamma$, with relative risk-aversion of five. When $\{\alpha_e, \alpha_s\} = \{.4, .6\}$, insider trading
their adverse selection losses in other states of nature. We find these results to be particularly interesting because the impact of insider trading on equilibrium interim real investment choices by a firm, an “alternative channel“ for its beneficial effect, is artificial at best—is it not the same insiders who are supposed to be choosing the firm’s investments in the first place 14(with their private information)?

This beneficial impact of insider trading on outsiders’ ex ante welfare, which we have documented, is particularly likely to arise when (1) the insider’s (equilibrium) trades are small, relative to outsiders’ liquidity-based trades, and (2) the riskiness (lower bound) of return on the investment technology, about which the insider is privately informed at the interim date is also not too high (low). Otherwise, as conventionally thought, insider trading is harmful to the outsiders’ (ex ante) welfare.

---

14 This is assuming the presence of adequate incentive schemes for the insider that are contingent on their firm’s realised ex post total return, when the insider is (a manager) tempted by shirking or private benefits.
Table I
Ex Ante Optimal Expected Utilities of Non-Insider
\( \alpha_i = 0.48, \alpha_n = 0.53 \)

(A) First Best

<table>
<thead>
<tr>
<th>( \theta_H / \theta_L )</th>
<th>0.75</th>
<th>0.8</th>
<th>0.85</th>
<th>0.9</th>
<th>0.95</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.25</td>
<td>-0.5</td>
<td>-0.497939</td>
<td>-0.489502</td>
<td>-0.477079</td>
<td>-0.46507</td>
</tr>
<tr>
<td>1.3</td>
<td>-0.498621</td>
<td>-0.493293</td>
<td>-0.482096</td>
<td>-0.469815</td>
<td>-0.45792</td>
</tr>
<tr>
<td>1.35</td>
<td>-0.495351</td>
<td>-0.487462</td>
<td>-0.475247</td>
<td>-0.463101</td>
<td>-0.451316</td>
</tr>
<tr>
<td>1.4</td>
<td>-0.491052</td>
<td>-0.481171</td>
<td>-0.468892</td>
<td>-0.456876</td>
<td>-0.445198</td>
</tr>
<tr>
<td>1.45</td>
<td>-0.486223</td>
<td>-0.475106</td>
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<tr>
<td>1.5</td>
<td>-0.481159</td>
<td>-0.469443</td>
<td>-0.457461</td>
<td>-0.445693</td>
<td>-0.434229</td>
</tr>
</tbody>
</table>

(B) Without Insider Trading

<table>
<thead>
<tr>
<th>( \theta_H / \theta_L )</th>
<th>0.75</th>
<th>0.8</th>
<th>0.85</th>
<th>0.9</th>
<th>0.95</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.25</td>
<td>-0.5</td>
<td>-0.498982</td>
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<td>-0.484341</td>
<td>-0.470024</td>
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<td>-0.478592</td>
<td>-0.464251</td>
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<tr>
<td>1.35</td>
<td>-0.497705</td>
<td>-0.493809</td>
<td>-0.486493</td>
<td>-0.473364</td>
<td>-0.459077</td>
</tr>
<tr>
<td>1.4</td>
<td>-0.495582</td>
<td>-0.490697</td>
<td>-0.482415</td>
<td>-0.468667</td>
<td>-0.454418</td>
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<tr>
<td>1.45</td>
<td>-0.493197</td>
<td>-0.487538</td>
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<td>-0.464299</td>
<td>-0.450206</td>
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<tr>
<td>1.5</td>
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<td>-0.484431</td>
<td>-0.474934</td>
<td>-0.460591</td>
<td>-0.446384</td>
</tr>
</tbody>
</table>

(C1) With Insider Submitting Market Orders

In the shaded area it does not pay the insider to trade. In the shaded area the aggregate liquidity constraint binds in state \( \text{IH} \). Equilibrium values for these areas coincide with those in (C2) or (C3) below and are marked with *.

<table>
<thead>
<tr>
<th>( \theta_H / \theta_L )</th>
<th>0.75</th>
<th>0.8</th>
<th>0.85</th>
<th>0.9</th>
<th>0.95</th>
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<tbody>
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</tr>
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<td>-0.482554</td>
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<td>-0.464265</td>
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</tr>
<tr>
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<td>-0.460893</td>
<td>-0.447461</td>
</tr>
</tbody>
</table>

(C2) and Liquidity Constraint Imposed in State \( \text{IH} \)

In the shaded area the aggregate liquidity constraint binds in states \( \text{IL} \) and \( \text{hH} \).

<table>
<thead>
<tr>
<th>( \theta_H / \theta_L )</th>
<th>0.85</th>
<th>0.9</th>
<th>0.95</th>
</tr>
</thead>
<tbody>
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<td>1.25</td>
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<td>-0.485648</td>
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<tr>
<td>1.3</td>
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<td>-0.485648</td>
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</tr>
<tr>
<td>1.35</td>
<td>-0.486943</td>
<td>-0.485648</td>
<td></td>
</tr>
<tr>
<td>1.4</td>
<td>-0.486943</td>
<td>-0.485648</td>
<td></td>
</tr>
<tr>
<td>1.45</td>
<td>-0.473491*</td>
<td>-0.465908*</td>
<td></td>
</tr>
<tr>
<td>1.5</td>
<td>-0.468716*</td>
<td>-0.465908*</td>
<td></td>
</tr>
</tbody>
</table>

(C3) and Liquidity Constraint Imposed in \( \text{IH}, \text{IL} \) and \( \text{hH} \).

<table>
<thead>
<tr>
<th>( \theta_H / \theta_L )</th>
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<th>0.95</th>
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<tr>
<td>1.3</td>
<td>-0.47523*</td>
<td>-0.46094*</td>
</tr>
<tr>
<td>1.35</td>
<td>-0.46933*</td>
<td>-0.45517*</td>
</tr>
<tr>
<td>1.4</td>
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<td>-0.44999*</td>
</tr>
<tr>
<td>1.45</td>
<td>-0.45926*</td>
<td>-0.44533*</td>
</tr>
<tr>
<td>1.5</td>
<td>-0.45494*</td>
<td>-0.44112*</td>
</tr>
</tbody>
</table>
Table I (continues)  Ex Ante Optimal Expected Utilities of Non-Insider

\( \alpha_i = 0.45, \alpha_n = 0.55 \)

(A) First Best

<table>
<thead>
<tr>
<th>( \theta_i / \theta_b )</th>
<th>0.75</th>
<th>0.8</th>
<th>0.85</th>
<th>0.9</th>
<th>0.95</th>
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</thead>
<tbody>
<tr>
<td>1.25</td>
<td>0.5</td>
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<td>-0.436494</td>
</tr>
</tbody>
</table>

(B) Without Insider Trading

<table>
<thead>
<tr>
<th>( \theta_i / \theta_b )</th>
<th>0.75</th>
<th>0.8</th>
<th>0.85</th>
<th>0.9</th>
<th>0.95</th>
</tr>
</thead>
<tbody>
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<td>1.25</td>
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</tr>
<tr>
<td>1.35</td>
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<tr>
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<td>-0.46078</td>
<td>-0.44755*</td>
</tr>
</tbody>
</table>

(C) With Insider Submitting Market Orders

In the area it does not pay the insider to trade. In the area the aggregate liquidity constraint binds in state IH. Equilibrium values for these areas coincide with those in (B) above or in (C) below, and are marked with *. Cells are marked with \( \square \) when (C)> (B).

<table>
<thead>
<tr>
<th>( \theta_i / \theta_b )</th>
<th>0.75</th>
<th>0.8</th>
<th>0.85</th>
<th>0.9</th>
<th>0.95</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.25</td>
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<td>-0.499713</td>
<td>-0.495645</td>
<td>-0.483396</td>
<td>-0.46839</td>
</tr>
<tr>
<td>1.3</td>
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<td>-0.497637</td>
<td>-0.490946</td>
<td>-0.476624</td>
<td>-0.46196</td>
</tr>
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<td>-0.458871</td>
<td>-0.44194</td>
</tr>
</tbody>
</table>

(C) and Liquidity Constraints imposed in states IH, bH and IL

\footnote{\textsuperscript{15} When the liquidity constraint is imposed in state IH only, it is violated in the other states.}
Table I (continues)

Ex Ante Optimal Expected Utilities of Non-Insider

\( \alpha_l = 0.4, \alpha_s = 0.6 \)

(A) First Best

<table>
<thead>
<tr>
<th>( \theta_{H}/\theta_{L} )</th>
<th>0.75</th>
<th>0.8</th>
<th>0.85</th>
<th>0.9</th>
<th>0.95</th>
</tr>
</thead>
<tbody>
<tr>
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<td>-0.442372</td>
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</tbody>
</table>

(B) Without Insider Trading

<table>
<thead>
<tr>
<th>( \theta_{H}/\theta_{L} )</th>
<th>0.75</th>
<th>0.8</th>
<th>0.85</th>
<th>0.9</th>
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</thead>
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<td>1.35</td>
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<td>-0.48684</td>
<td>-0.474815</td>
<td>-0.463463*</td>
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<tr>
<td>1.4</td>
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<td>-0.490943</td>
<td>-0.482353</td>
<td>-0.47042</td>
<td>-0.45916*</td>
</tr>
<tr>
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<td>-0.479068</td>
<td>-0.466407</td>
<td>0.455219*</td>
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<tr>
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<td>-0.484825</td>
<td>-0.475517</td>
<td>-0.462728</td>
<td>0.451597*</td>
</tr>
</tbody>
</table>

(C) With Insider Submitting Market Orders

In the shaded area it does not pay the insider to trade. Equilibrium values for this area coincide with those in (B) above, and are marked with *. The aggregate liquidity constraint never binds in state IH. Cells are marked with \( \Box \) when (C)>(B).
<table>
<thead>
<tr>
<th>$\theta_H / \theta_h$</th>
<th>0.75</th>
<th>0.8</th>
<th>0.85</th>
<th>0.9</th>
<th>0.95</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.25</td>
<td>-0.499007</td>
<td>-0.500455</td>
<td>-0.498212</td>
<td>-0.487293</td>
<td>-0.477177</td>
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<td>-0.499562</td>
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<td>-0.480712</td>
<td>-0.465396</td>
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<tr>
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<td>-0.497387</td>
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<td>-0.459631</td>
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<tr>
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<td>-0.481145</td>
<td>-0.464201</td>
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<td>-0.487923</td>
<td>-0.47678</td>
<td>-0.459555</td>
<td>-0.444385</td>
</tr>
</tbody>
</table>

**Table II**

Ex Ante Investment Choices

(A) First Best

<table>
<thead>
<tr>
<th>$\theta_H / \theta_h$</th>
<th>0.75</th>
<th>0.8</th>
<th>0.85</th>
<th>0.9</th>
<th>0.95</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.25</td>
<td>1.</td>
<td>0.8341</td>
<td>0.5842</td>
<td>0.5712</td>
<td>0.5592</td>
</tr>
<tr>
<td>1.3</td>
<td>0.8892</td>
<td>0.7264</td>
<td>0.5885</td>
<td>0.5751</td>
<td>0.5628</td>
</tr>
<tr>
<td>1.35</td>
<td>0.8115</td>
<td>0.653</td>
<td>0.5927</td>
<td>0.5789</td>
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<tr>
<td>1.4</td>
<td>0.755</td>
<td>0.6113</td>
<td>0.5967</td>
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<tr>
<td>1.45</td>
<td>0.713</td>
<td>0.6155</td>
<td>0.6005</td>
<td>0.5862</td>
<td>0.5728</td>
</tr>
<tr>
<td>1.5</td>
<td>0.6811</td>
<td>0.6194</td>
<td>0.6042</td>
<td>0.5896</td>
<td>0.5783</td>
</tr>
</tbody>
</table>

(B) Without Insider Trading

<table>
<thead>
<tr>
<th>$\theta_H / \theta_h$</th>
<th>0.75</th>
<th>0.8</th>
<th>0.85</th>
<th>0.9</th>
<th>0.95</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.25</td>
<td>1.</td>
<td>0.9181</td>
<td>0.7857</td>
<td>0.5425</td>
<td>0.5193</td>
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<tr>
<td>1.3</td>
<td>0.9453</td>
<td>0.8649</td>
<td>0.7375</td>
<td>0.5283</td>
<td>0.5177</td>
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<tr>
<td>1.35</td>
<td>0.9069</td>
<td>0.8286</td>
<td>0.7063</td>
<td>0.5273</td>
<td>0.5164</td>
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<tr>
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<td>0.879</td>
<td>0.8031</td>
<td>0.6857</td>
<td>0.5267</td>
<td>0.5156</td>
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<td>1.45</td>
<td>0.8583</td>
<td>0.7847</td>
<td>0.672</td>
<td>0.5262</td>
<td>0.515</td>
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<td>0.7712</td>
<td>0.663</td>
<td>0.5259</td>
<td>0.5146</td>
</tr>
</tbody>
</table>

(C1) With Insider Submitting Market Orders

In the shaded area it does not pay the insider to trade. In the shaded area the aggregate liquidity constraint binds in state IH. Equilibrium values for these areas coincide with those in (C2) or (C3) below, and are marked with *. Cells are marked with when (C)>(B).

<table>
<thead>
<tr>
<th>$\theta_H / \theta_h$</th>
<th>0.75</th>
<th>0.8</th>
<th>0.85</th>
<th>0.9</th>
<th>0.95</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.25</td>
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<td>0.9056</td>
<td>0.7336</td>
<td>0.5228</td>
<td>0.5175</td>
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<tr>
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<td>0.8348</td>
<td>0.6666</td>
<td>0.5299</td>
<td>0.5246</td>
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<tr>
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<td>0.7856</td>
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<td>0.5125</td>
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<td>0.5079</td>
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<td>1.45</td>
<td>0.822</td>
<td>0.7235</td>
<td>0.5634</td>
<td>0.5012</td>
<td>0.4962</td>
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<tr>
<td>1.5</td>
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<td>0.7036</td>
<td>0.5533</td>
<td>0.4915</td>
<td>0.4863</td>
</tr>
</tbody>
</table>

(C2) and Liquidity Constraint Imposed in State IH

(C3) and Liq. Constraint Imposed
In the shaded area \[\text{III}\] the aggregate liquidity constraint binds in states \(lL\) and \(hH\).

\[
\theta_{H}/\theta_L = 0.85 \quad 0.9 \quad 0.95 \\
1.25 & 1.45 & 0.5265^* & 0.5205^* \\
1.3 & 1.45 & 0.527^* & 0.5209^* \\
1.35 & 1.45 & 0.5275^* & 0.5212^* \\
1.4 & 1.45 & 0.5279^* & 0.5216^* \\
1.45 & 1.5 & 0.5283^* & 0.5219^* \\
1.5 & 1.5 & 0.5287^* & 0.5222^* \\
\]

Table III

Interim Equilibrium Prices

\(\alpha_i = 0.48, \alpha_h = 0.53\)

(B) Without Insider Trading

Equilibrium price when \(\alpha_i = 0.48\).

\[
\epsilon\theta_{H}/\theta_L = 0.75 \quad 0.8 \quad 0.85 \quad 0.9 \quad 0.95 \\
1.25 & 1 & 1.001 & 1.002 & 1.003 & 1.044 \\
1.3 & 1.001 & 1.002 & 1.003 & 1.007 & 1.051 \\
1.35 & 1.002 & 1.003 & 1.004 & 1.011 & 1.057 \\
1.4 & 1.004 & 1.004 & 1.005 & 1.014 & 1.061 \\
1.45 & 1.005 & 1.005 & 1.006 & 1.015 & 1.064 \\
1.5 & 1.006 & 1.006 & 1.006 & 1.017 & 1.065 \\
\]

Equilibrium price when \(\alpha_i = 0.53\).

\[
\theta_{H}/\theta_L = 0.75 \quad 0.8 \quad 0.85 \quad 0.9 \quad 0.95 \\
1.25 & 1 & 0.9988 & 0.9976 & 0.997 & 0.9579 \\
1.3 & 0.9987 & 0.9976 & 0.9967 & 0.9933 & 0.9517 \\
1.35 & 0.9976 & 0.9965 & 0.9958 & 0.9894 & 0.9471 \\
1.4 & 0.9964 & 0.9956 & 0.9951 & 0.9867 & 0.9438 \\
1.45 & 0.9954 & 0.9947 & 0.9944 & 0.9849 & 0.9415 \\
1.5 & 0.9944 & 0.9939 & 0.9939 & 0.9839 & 0.94 \\
\]
Table III

Interim Equilibrium Prices

In the shaded area it does not pay the insider to trade. In the shaded area the aggregate liquidity constraint binds in state IH. Equilibrium values for these areas coincide with those in (C2/C3) on the right, and are marked with *. Cells are marked with when (C)> (B).

(C1) With Insider Submitting Market Orders

(C2/C3) and Liquidity Constraints

<table>
<thead>
<tr>
<th>$\theta_H / \theta_L$</th>
<th>0.75</th>
<th>0.8</th>
<th>0.85</th>
<th>0.9</th>
<th>0.95</th>
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<tbody>
<tr>
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</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$\theta_H / \theta_L$</th>
<th>0.85</th>
<th>0.9</th>
<th>0.95</th>
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</thead>
<tbody>
<tr>
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<td>1.176*</td>
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<tr>
<td>1.3</td>
<td>1.207*</td>
<td>1.178*</td>
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<td>1.209*</td>
<td>1.179*</td>
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<td>1.181*</td>
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<tr>
<td>1.45</td>
<td>1.445*</td>
<td>1.213*</td>
<td>1.182*</td>
</tr>
<tr>
<td>1.5</td>
<td>1.457*</td>
<td>1.215*</td>
<td>1.184*</td>
</tr>
</tbody>
</table>

Price when $\alpha_i = 0.48$ and $\theta_j = \theta_H$

Price when $\alpha_i = 0.53$ and $\theta_j = \theta_L$

Price when $\alpha_i = 0.48$ and $\theta_j = \theta_L$ and when $\alpha_i = 0.53$ and $\theta_j = \theta_L$
Table IV
Ex Ante Optimal Expected Utilities of Non-Insider

In the shaded area it does not pay the insider to trade, and equilibrium values are identified with *. Cells are marked with ■ when (C)>(B).

\( \alpha_i = 0.1, \alpha_u = 0.15 \)  
(B) Without Insider Trading

\[
\begin{array}{cccccc}
\theta_u / \theta_h & 0.75 & 0.8 & 0.85 & 0.9 & 0.95 \\
1.25 & -0.5 & -0.498198 & -0.490806 & -0.472434 & -0.445991 \\
1.3 & -0.498794 & -0.494136 & -0.483527 & -0.462049 & -0.435614 \\
1.35 & -0.495933 & -0.489038 & -0.476085 & -0.452728 & -0.426346 \\
1.4 & -0.492177 & -0.483527 & -0.468866 & -0.444369 & -0.41803* \\
1.45 & -0.487965 & -0.477935 & -0.462028 & -0.436843 & -0.410539* \\
1.5 & -0.483527 & -0.472434 & -0.455625 & -0.43004 & -0.403765* \\
\end{array}
\]

(C) With Insider Submitting Market Orders

\[
\begin{array}{cccccc}
\theta_u / \theta_h & 0.75 & 0.8 & 0.85 & 0.9 & 0.95 \\
1.25 & -0.499967 & -0.498549 & -0.491278 & -0.472414 & -0.445112 \\
1.3 & -0.499094 & -0.494612 & -0.483866 & -0.461618 & -0.434383 \\
1.35 & -0.496334 & -0.489489 & -0.476197 & -0.451917 & -0.424767 \\
1.4 & -0.492657 & -0.483866 & -0.468707 & -0.443192 & -0.416111 \\
1.45 & -0.488389 & -0.478109 & -0.46158 & -0.435314 & -0.408559 \\
1.5 & -0.483866 & -0.472414 & -0.454885 & -0.428173 & -0.401195 \\
\end{array}
\]

\( \alpha_i = 0.9, \alpha_u = 0.95 \)  
(B) Without Insider Trading

\[
\begin{array}{cccccc}
\theta_u / \theta_h & 0.75 & 0.8 & 0.85 & 0.9 & 0.95 \\
1.25 & -0.5 & -0.499862 & -0.499297 & -0.497967 & -0.496592* \\
1.3 & -0.499908 & -0.499552 & -0.498736 & -0.497263 & -0.495917* \\
1.35 & -0.49969 & -0.499161 & -0.498159 & -0.496621* & -0.495298* \\
1.4 & -0.499402 & -0.498736 & -0.497595 & -0.49603* & -0.494727* \\
1.45 & -0.499078 & -0.498303 & -0.497037 & -0.495484* & -0.494195* \\
1.5 & -0.498736 & -0.497874 & -0.496548 & -0.494978* & -0.4937* \\
\end{array}
\]

(B) With Insider Submitting Market Orders


Dow J.; and Rahi R., "Informed Trading, Investment, and Welfare". European University Institute, October 1996


