"Welfare Cost of Inflation with Heterogeneous Agents"

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Welfare Cost of Inflation with Heterogeneous Agents

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1 Introduction

The common belief that inflation is the most regressive form of taxation is widely accepted and appears to be supported by empirical evidence. The welfare costs and the distributive impacts of inflation have not yet been studied in a general equilibrium framework with heterogeneous agents.

Defining the inflation tax as the revenue raised by the government through seigniorage, the average annual growth rate of 16.13% (1980), 47.41% (1985) and 146% (1990) of the monetary concept $M_1$, reported by the Brazilian Central Bank, clearly represent an inflationary bias for revenue raising in Brazil.

Recent studies by Barros and Mendoça (1995), Mendoça and Urani (1994), Bonelli and Ramos (1994) among others, clearly show the alarmingly high level of income concentration in Brazil that had increased dramatically over the last 30 years. In particular the latter authors reveal a striking fact that despite the substantial upgrade in the educational level of the Brazilian labor force during the period 1977-1989 \(^2\), the concentration of income had been rising. This suggests that there is no systematic way that education could have affected the dynamic pattern of the Brazilian income distribution as the theory of human capital formation would explain. An important insight from

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\(^2\)During this period the share of workers with less than intermediate schooling went down from 59% to 44% and the share of those that at least started attending high school increased from 19% to 29%
their research, however, is that the variable *position in occupation* 3, which is closely related to one's degree of control over capital, accounts for the highest contribution in the variation of the Theil T index of inequality. Hence, the ownership of capital appears to be an important means of protecting income from inflation in Brazil. This strongly suggests that inflation could be the driving force behind the worsening of the Brazilian income distribution.

The present study is based on the definition of recursive competitive equilibrium (RCE) for a heterogeneous agents model as suggested by Hansen and Prescott (1995). The agents of the model economy are divided in two types, type 1 capital owners and type 2 employees, to match the model with the Brazilian empirical evidence. According to the RCE definition, a linear quadratic (LQ) approximation of the return function is numerically computed around the steady state variable values for each type of agent. This enables to translate the agent’s problem into a LQ dynamic programing framework. Iterating simultaneously over the LQ value functions, a set of consistent and optimal (linear) decision rules and the aggregate pricing function are derived from the first order conditions.

This study has four goals. First, to quantitatively asses the impact of the inflation tax on aggregate welfare as well as on the income distribution based on a general equilibrium model economy with heterogeneous agents, calibrated for the Brazilian economy, in which money is introduced via cash-in-advance constraint. Second, to compare the outcomes of the inflation tax regime with alternative revenue neutral tax policy schemes. Policy 1 replaces the seigniorage revenue by increasing the tax rates on labor and capital income keeping a constant relationship between them, Policy 2 institutes an increase only on labor income, Policy 3 substitutes the inflation tax by an increase on capital income tax only and Policy 4 consists of levying an uniform tax rate on both sources of income. Third, to analyze the transition path to the steady state implied by the optimal decision rules and the pricing function associated with the RCE. Finally, to compare the effects of temporary vs. permanent policy changes in terms of welfare and equity.

The main findings show the regressive nature of the inflation tax and an apparent trade off between Pareto superior and equity improving policy choices. Furthermore, the patterns of the transition paths (which are

3according to this variable the labor force is divided among employees, self employers and employers.
obviously policy dependent) display a noticeable dominance of sequences derived from permanent tax policies relative to those obtained from temporary changes in policy.

The article is organized as follows. Section 2 describes the standard model economy with heterogeneity introduced based on the Brazilian empirical evidence on position in occupation (heterogeneous) groups. Section 3 presents the computational procedure used to find a recursive competitive equilibrium for the model economy based on the algorithm introduced by Hansen and Prescott (1996). Section 4 introduces the parameterization used to calibrate the model economy and the obtained results. Section 5 contains concluding remarks.

2 The Model

According to Bonelli and Ramos (1994) the variable “position in occupation” divides the Brazilian work force among employees 75%, self employers 20% and employers 5%. These shares are shown to be constant over the period 1977-1989. To introduce this fact in a model economy with infinitely lived heterogeneous agents, let \( i = 1, 2 \) denote the groups such that type \( i = 1 \) agents control (own) capital, representing the self employers and employers, whereas type \( i = 2 \) agents do not have access to capital ownership, representing the employees. In other words, type 1 agents supply labor \( h_1 \) and accumulate productive capital \( k_1 \) that is rented to the firm and, type 2 agents only supply labor \( h_2 \) to the single firm that produces output according to a constant return to scale technology.

There is also a government in the economy that supplies money (currency), taxes labor and capital income, and transfers a lump-sum amount of currency to both types of households. Money is valued in this economy for it is required to purchase consumption goods.

Type 1 Agents’ Problem.

Consider a large number of identical type 1 agents endowed with capital \( k_0 \) in period 0, one unit of time per period that can be spent working or enjoying leisure and money balances \( m_{1,0} \) at period 0. The problem of a representative agent of type 1 is to choose optimal sequences for consumption \( \{c_{1t}\} \), labor \( \{l_{1t}\} \), investment \( \{x_{1t}\} \) and money holding \( \{m_{1t}\} \) such that his/her expected
discounted utility is maximized subject to the per period budget, cash-in-advance constraint, and feasibility. Formally, a type 1 agent’s problem is given by:

\[
\max_{c_{1t}, h_{1t}} E_0 \left\{ \sum_{t=0}^{\infty} \beta^t (u(c_{1t}, 1 - h_{1t})) \right\}
\]

such that,

\[
c_{1t} + x_{1t} + \frac{m_{1,t+1}}{p_t} \leq (1 - t_w)w_t h_{1t} + (1 - t_k) r_t k_{1t} + t_k \delta k_{1t} + \frac{m_{1t}}{p_t} + \frac{tr_t}{p_t}
\]

\[
p_t c_{1t} \leq m_{1t} + M_{1,t+1} - M_{1,t}
\]

\[
h_{1t} + l_{1t} = 1, \quad t \geq 0
\]

The law of motion for capital formation is \( k_{1t} = (1 - \delta) k_{1,t-1} + x_{1t} \), the initial capital stock is \( k_0 \),

\( 0 < \beta < 1 \) is the discount rate,

\( c_{1t} \) is the consumption of type 1 agent at time \( t \),

\( 0 \leq h_{1t} \leq 1 \) denotes the units of working time of type 1 agent,

\( x_{1t} \) denotes the type 1 agent’s investment decision at time \( t \),

\( t_w \) is the tax rate on labor income,

\( w_t \) is the wage rate at time \( t \),

\( t_k \) is the tax rate on capital income,

\( r_t \) is the rate of return for capital services at time \( t \),

\( k_{1t} \) is the capital stock at time \( t \) held by type 1 agent,

\( \delta \) is the capital depreciation rate, and

\( tr_t \) is the lump sum transfer from government at time \( t \).

**Type 2 Agents’ Problem.**

Similarly, consider a large number of identical type 2 agents endowed with one unit of time per period and money balances \( m_{2,0} \) in period 0. The problem of a representative agent of type 2 is to choose the optimal sequences for consumption \( \{c_{2t}\} \), labor \( \{l_{2t}\} \) and money holding \( \{m_{2t}\} \) such that his/her expected discounted utility is maximized subject to the per period budget constraint, cash-in-advance constraint, and feasibility. Formally, a type 2 agent’s problem is given by:
such that,
\[
m_2 \ t + l + \ m_2 \ trt + C_2t + \ldots + (1 - t_w)w_t h_{2t} + m_{2t} + tr_t
\]
\[
p_t C_{2t} + m_{2t} + M_{2t+1} - M_{2t}
\]
\[
h_{2t} + l_{2t} = 1, \quad t \geq 0
\]

Note that \( h_i \) and \( m_i \) denote particular type \( i = 1, 2 \) agent’s labor choice and money holdings. Let \( \lambda_i \) denote the proportion of type \( i = 1, 2 \) agents such that \( \lambda_1 + \lambda_2 = 1 \). The aggregate per capita variables are \( K_t = \lambda_1 K_1 \) for capital stock, \( M_t = \lambda_1 M_{1t} + \lambda_2 M_{2t} \) for the pre-transfer money supply, \( H_t = \lambda_1 H_{1t} + \lambda_2 H_{2t} \) for total hours worked, and \( X_t = \lambda_1 X_{1t} \) for aggregate investment.

Observe that the budget constraint does not allow either type of agent to purchase or sell state-contingent claims to units of output next-period. Also, consumption goods are only purchased in cash: agents enter period \( t \) with nominal balances \( m_{i,t} \) which are augmented by a lump sum transfer of newly printed money \( M_{i,t+1} - M_{i,t} \). Therefore, the (post-transfer) amount of nominal money, the right hand side of cash-in-advance constraint, is available at \( t \) to finance the purchase of “cash” good \( c_{i,t} \).

The Government

The per period (pre-transfer) money supply evolves according to the growth factor \( g \). Therefore, given the price level \( p_t \), the real inflation tax revenue collected by the government is given by:
\[
\frac{M_{t+1} - M_t}{p_t} = \frac{(g - 1)M_t}{p_t}
\]
The government also collects taxes on labor and capital income, which together with the inflation revenue is refunded in a lump-sum fashion to agents. Thus, the government budget constraint is expressed as:
Moreover the following is assumed to hold for both types of agents:

**Assumption 1** The period utility function \( u(\cdot,\cdot) : \mathbb{R}_+ \to \mathbb{R} \) is bounded, continuously differentiable, strictly concave, strictly increasing in both arguments and satisfies the Inada conditions, i.e. \((u_1/u_2) \to \infty \) as \((c_t/(1-h_t)) \to 0\) and \((u_1/u_2) \to 0 \) as \((c_t/(1-h_t)) \to \infty\). Hence both consumption and leisure are normal goods.

### The Firm

The productive sector of this model economy hires labor \( h_t^f \) and capital services \( k_t^f \) at every \( t \) to produce output \( y_t^f \), according to a constant return to scale technology. Thus, without loss of generality it is assumed that there is only one firm, operating at zero profit using the technology given by:

\[
y_t^f = z_t f(k_t^f, h_t^f)
\]

where,

\( z_t \) : exogenous shock to the technology realized at time \( t \).

Furthermore, the following is assumed:

**Assumption 2** The technology shock described by \( z_t = e^{\omega_t} \), has a random term \( \omega_t \) evolving according to the first order auto regressive process \( \omega_t = \rho \omega_{t-1} + \epsilon_t, 0 < \rho < 1 \) and \( \epsilon_t \) is i.i.d. with mean zero and finite variance \( \sigma^2_\epsilon \).

The term \( \omega_t \) captures the productivity shock which is the source of uncertainty for this economy.

The technology satisfies the following:

**Assumption 3** The constant return to scale production function \( f(\cdot,\cdot) : \mathbb{R}_+^2 \to \mathbb{R}_+^1 \) is twice continuously differentiable, strictly increasing and strictly concave with \(zf(0,h^f) = zf(k^f,0) = 0\) and \( \lim_{k^f \to 0} zf_1(k^f,h^f) = \infty \) for \( h^f > 0 \) and for all \( z \in \mathbb{R}^1, z \neq 0 \).
The per period wage rate $w_t$ and the rate of return on capital $r_t$ are derived from the first order conditions of the firm's profit maximization problem and are given by:

$$w_t = z_t f_2(k_t^I, h_t^I) \quad (5)$$

$$r_t = z_t f_1(k_t^I, h_t^I) \quad (6)$$

Using market clearing conditions $k_t^I = K_t N$ and $h_t^I = H_t N$, denoting the total number of households by $N = \lambda_1 N_1 + \lambda_2 N_2$, the following equilibrium conditions are obtained:

$$w_t = z_t f_2(K_t, H_t) \quad (7)$$

$$r_t = z_t f_1(K_t, H_t) \quad (8)$$

It is also assumed that the aggregate per capita capital stock evolves according to the law of motion given by:

$$K_{t+1} = (1 - \delta) K_t + I_t \quad (9)$$

In order to focus on stationary equilibrium for the households, the monetary variables are transformed\(^4\) in the following fashion:

$$\hat{m}_t = \frac{m_t}{M_t} \quad \text{and,}$$

$$\hat{p}_t = \frac{p_t}{M_{t+1}},$$

such that real money balances becomes:

$$\frac{m_{t+1}}{p_t} = \frac{\hat{m}_{t+1}}{\hat{p}_t}.$$

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\(^4\)This transformation is needed because if the monetary growth factor is positive, i.e. $g > 0$, then the money supply and the price level would grow without limit.
Introducing the transformed monetary variables and the government transfer by type of agent, problem (1) becomes for \( t \geq 0 \):

\[
\max_{x_{1t}, \bar{m}_{1,t+1}} E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left( u(c_{1t}, 1 - h_{1t}) \right) \right\}
\]

such that,

\[
x_{1t} + \frac{\hat{m}_{1,t+1}}{\hat{p}_t} \leq w_t(h_{1t} + t_w(H_{1t} - h_{1t}) + r_t(k_{1t} + t_k(K_{1t} - k_{1t})) + \delta t_k(k_{1t} - K_{1t})
\]

\[
c_{1t} \leq \left( \frac{\hat{m}_{1t} - \hat{M}_{1,t}}{\hat{p}_t} + \hat{M}_{1,t+1} \right) / \hat{p}_t.
\]

Similarly, problem (2) becomes for \( t \geq 0 \):

\[
\max_{m_{2,t+1}} E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left( u(c_{2t}, 1 - h_{2t}) \right) \right\}
\]

such that,

\[
\frac{\hat{m}_{2,t+1}}{\hat{p}_t} \leq w_t(h_{2t} + t_w(H_{2t} - h_{2t})) \geq 0,
\]

\[
c_{2t} \leq \left( \frac{\hat{m}_{2t} - \hat{M}_{2,t}}{\hat{p}_t} + \hat{M}_{2,t+1} \right) / \hat{p}_t.
\]

Solving the respective budget constraints for \( h_{it}, t = 1, 2 \), and aggregating over each type of agent, expressions for \( H_{it}, i = 1, 2 \) are obtained. Aggregating once more over both types of agents \( H_t = \lambda_1 H_1 + (1 - \lambda_1) H_2 \). The following symbolic expression is derived:

\[
H_t = \left( \frac{X_{1t} \hat{p}_t \lambda + 1}{z_t(\lambda_1 K_{1t})^\beta \hat{p}_t} \right)^{1-\theta}
\]

Substituting above expression into \( H_{it} \), the latter is used to find the functions \( h_{1t} = h_{1t}(z, K_1, k_1, \hat{M}_{1t}, \hat{p}_t) \) and \( h_{2t} = h_{2t}(z, K_1, \hat{M}_{2t}, \hat{p}_t) \) that enter the utility function, where \( w_t = z_t f_2(\lambda_1 K_{1t}, H_t) \), \( r_t = z_t f_1(\lambda_1 K_{1t}, H_t) \) from the firm’s profit maximization condition and, \( c_{it}, i = 1, 2 \) are obtained from the cash-in-advance constraint.

To translate the agent’s problem into a dynamic programing framework, let \( K = (K_1, 0) \), \( H = (H_1, H_2) \), \( X = (X_1, 0) \) and \( \bar{M} = (\bar{M}_1, \bar{M}_2) \) be vectors.
describing the entire distribution of the capital stock, hours worked, investment and money balances over type \( i = 1, 2 \) agents. The agents’ optimization problem can now be expressed as the dynamic programing problem below.

For type 1 agents:

\[
v_1(z, K, k_1) = \max \{ r_1(z, K, k_1, \tilde{m}_1, X, X_1, \tilde{M}', \tilde{m}_1', \tilde{p}) + \beta \mathbb{E}[v(z', K', k_1', \tilde{m}_1')/z] \}\]

subject to:
- \( z' = \rho z + \epsilon' \)
- \( k_1' = (1 - \delta)k_1 + x_1 \)
- \( K_1' = (1 - \delta)K_1 + X_1 \)
- \( X_1 = X_1(z, K) \)
- \( H_1 = H_1(z, K) \)
- \( \tilde{M}_1' = \tilde{M}_1(z, K) \)

For type 2 agents:

\[
v_2(z, K) = \max \{ r_2(z, K, \tilde{m}_2, X, \tilde{M}', \tilde{m}_2', \tilde{p}) + \beta \mathbb{E}[v(z', K', \tilde{m}_2')/z] \}\]

subject to:
- \( z' = \rho z + \epsilon' \)
- \( K_1' = (1 - \delta)K_1 + X_1 \)
- \( H_2 = H_2(z, K) \)
- \( \tilde{M}_2' = \tilde{M}_2(z, K) \)

The last three equation indicate that for each type of agent hours worked and money balances are functions of the state of the economy described by the productivity shock \( z \) and the vector \( K \) of distribution of capital stock across the economy.

**Definition of Recursive Competitive Equilibrium (RCE)**

According to Hansen and Prescott (1995) a RCE is defined as follows:

**Definition 1** A RCE for the model economy described above is:
(i) A set of decision rules:
   For type 1 agent:
   \[ x_1 = x_1(z, K, k_1) \]
   \[ \hat{m}_1' = \hat{m}_1'(z, K, k_1) \]
   For type 2 agent:
   \[ \hat{m}_2' = \hat{m}_2'(z, K) \]

(ii) A set of aggregate decision rules, for \( i = 1, 2 \):
   \[ I_i = I_i(z, K) \]
   \[ \hat{M}_i = \hat{M}_i(z, K) \]

(iii) A function determining the aggregate price level:
   \[ \hat{p} = P(z, K) \]

(iv) A set of value functions \( v_i(z, K) \), \( i = 1, 2 \) such that:

1. Given the decision rules in (i) and the aggregate price function in (iii), the value function \( v_i \) satisfies the value function (12)/(13) above and, (ii) are the associated decision rules.

2. Given the pricing function in (iii), individual decisions are consistent with aggregate outcomes, i.e. for \( i = 1, 2 \):
   \[ I_i(z, K) = x_1(z, K, K_1) \]
   \[ \hat{M}_i(z, K) = \hat{m}_i(z, K), \ i = 1, 2 \]

where \( \lambda_1 \hat{M}_1(z, K) + (1 - \lambda_1) \hat{M}_2(z, K) = 1 \), because by definition \( \hat{M}_i \) represents type \( i \) agent's aggregate per capita money holdings relative to aggregate per capita money supply.

3 Computational Procedure

In order to numerically solve for the above defined RCE, the method proposed by Hansen and Prescott (1995) is implemented as follows.\(^5\)

Step 1. Computation of steady state values.

Step 2. Assume the functional form \( u_{it} = a \log(c_{it} + (1 - \alpha) \log(1 - h_{it})), \)
\( i = 1, 2 \) for the utility function and a Cobb-Douglas production function
\( f(z_t, K_t, H_t) = z_t K_t^\delta H^{(1-\delta)}, \) the return function of problems (12) and (13) are
approximated numerically by the first three terms of the Taylor series expansion
around the variables' steady state values to obtain the negative semi-definite matrices \( Q_i, \) \( i = 1, 2, \) in order to express the problems in a linear quadratic programing framework, i.e.:

For type 1 agents:

\[
v_1(z, K) = \max\{y_1' Q_1 y_1 + \beta E[v(z', K')/z]\} \tag{14}
\]
subject to:
\( z' = \rho z + \varepsilon' \)
\( k_1' = (1 - \delta)k_1 + x_1 \)
\( K_1' = (1 - \delta)K_1 + X_1 \)
\( X_1 = X_1(z, K) \)
\( H_1 = H_1(z, K) \)
\( M_1' = M_1'(z, K) \)
where \( y_1 = (z, K_1, k_1, \hat{m}_1, X_1, x_1, \hat{p}, \hat{M}_1', \hat{m}_1') \)

For type 2 agents:

\[
v_2(z, K) = \max\{y_2' Q_2 y_2 + \beta E[v(z', K')/z]\} \tag{15}
\]
subject to:
\( z' = \rho z + \varepsilon' \)
\( K_1' = (1 - \delta)K_1 + X_1 \)
\( H_2 = H_2(z, K) \)
\( M_2' = M_2'(z, K) \)
where \( y_2 = (z, K_1, \hat{M}_2, \hat{m}_2, X_1, \hat{p}, \hat{M}_2', \hat{m}_2') \)

Step 3. Successive approximations of the value function are performed simultaneously for both types of agents iterating on the mapping below:
\[ v_{t+1}^{n}(z, K) = \max \{ y(Q, y) + \beta E[v^n(z', K')/z] \} \]  

subject to:

\[ z' = \rho z + \varepsilon' \]

\[ k_1' = (1 - \delta) k_1 + x_1 \]

\[ K_1' = (1 - \delta) K_1 + X_1 \]

\[ X_1 = X_1(z, K) \quad \text{and,} \]

\[ H_i = H_i(z, K) \]

\[ \tilde{M}_i = \tilde{M}'_i(z, K), \quad i = 1, 2 \]

**Step 4** When the above iterations have converged, the associated optimal (linear) decision rules and pricing function are obtained, i.e.

For type 1 agent:

\[ X_1(z, K_1, \tilde{M}_1) = x_1(z, K_1, \tilde{M}_1, \tilde{M}_1) \]

\[ \tilde{M}_1(z, K_1, \tilde{M}_1) = \tilde{m}_1(z, K_1, \tilde{M}_1, \tilde{M}_1) \]

For type 2 agent:

\[ \tilde{M}_2(z, K_1, \tilde{M}_2) = \tilde{m}_2(z, K_1, \tilde{M}_2, \tilde{M}_2) \]

Observing the restriction \( \lambda_1 \tilde{M}_1 + (1 - \lambda_1) \tilde{M}_2 = 1 \) and,

\[ \hat{p} = P(z, K_1) \]

### 4 Results

The parameter values, in a quarterly bases, used to compute the RCE of the standard model are as follows:

**Technology parameters:**

\[ \theta = 0.4908, \quad \delta = 0.0168, \quad \rho = 0.78 \quad \text{and} \quad \sigma_\varepsilon = 0.0016. \]

**Preferences parameters:**

\[ \beta = 0.9835 \quad \text{and} \quad \alpha = 0.5885. \]

**Tax policy parameters:**

\[ g = 1.0403, \quad t_w = 0.2553 \quad \text{and,} \quad t_k = 0.3379. \]

The capital share in output parameter, \( \theta \), has been estimated by Araújo (1997) and those pertaining to the productivity shock were estimated as the
Solow residual using real quarterly GNP and labor input data from Indicadores IBGE (1974-1991), assuming quarterly variation in capital stock to be approximately zero.

Preference parameters were also been estimated by Araújo (1997) and here are adjusted to quarterly bases. Among the tax policy parameters, the tax rate on labor income as well as capital income, \( t_w \) and \( t_k \), were obtained as the average of the progressive tax rates based on data from Secretaria da Receita Federal (1995). The money supply growth factor \( g \) was computed as the quarterly average growth of \( M_1 \) based on data from Boletim do Banco Central do Brasil, taking 1980 as the base year.

### 4.1 Steady State Analysis

The steady state variable values of the standard model (S.M.) economy with heterogeneous agents as well as the simulated model with different tax policies are reported in Table ?? . All policies are revenue neutral. When the inflation tax is eliminated the foregone revenue is replaced by either of the following tax regimes.

- **Policy 1 (P.1.)** refers to a tax policy in which the relation between the two income tax rates \( t_w \) and \( t_k \) were kept constant. Eliminating the inflation tax implied an increase in both tax rates, on labor income to \( t_w^1 = 0.2899 \) and \( t_k^1 = 0.3837 \).

- With **Policy 2 (P.2.)**, the inflation tax was substituted by an increase only in the labor income tax rate, the latter increases in this case 23.97% to \( t_w^2 = 0.3165 \).

- **Policy 3 (P.3.)** replaces the inflation tax in a revenue neutral way increasing only the capital income tax rate to \( t_k^3 = 0.4435 \).

- **Policy 4 (P.4.)** institutes an uniform tax rate over all sources of income. This policy implies an increase in the tax rate on labor income of 27.03% and an decrease in the tax rate on capital income of 4.02%, leading to an uniform tax rate on income of \( t_u = 0.3243 \).

Table 2 shows the aggregate per capita steady state variable values, corresponding to the homogeneous agent models for the standard as well as the simulated economies with alternative tax policies.

The standard model (S.M.) values provide a bench mark that the four alternative policies can be compared with in the analysis that follows.
Table 1: Steady State Variable Values - per (type) capita

<table>
<thead>
<tr>
<th>Var.</th>
<th>$K_1$</th>
<th>$X_1$</th>
<th>$H_1$</th>
<th>$H_2$</th>
<th>$M_1$</th>
<th>$M_2$</th>
<th>$C_1$</th>
<th>$C_2$</th>
<th>$\bar{p}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>S.M.</td>
<td>193.3058</td>
<td>3.2475</td>
<td>0.0520</td>
<td>0.5017</td>
<td>1.5522</td>
<td>0.8160</td>
<td>5.1804</td>
<td>2.7229</td>
<td>0.2996</td>
</tr>
<tr>
<td>P.1.</td>
<td>175.3948</td>
<td>2.9466</td>
<td>0.0406</td>
<td>0.4997</td>
<td>1.5598</td>
<td>0.8134</td>
<td>4.9859</td>
<td>2.6000</td>
<td>0.3128</td>
</tr>
<tr>
<td>P.2.</td>
<td>187.8765</td>
<td>3.1563</td>
<td>0.0430</td>
<td>0.4902</td>
<td>1.5395</td>
<td>0.8202</td>
<td>4.9936</td>
<td>2.6602</td>
<td>0.3083</td>
</tr>
<tr>
<td>P.3.</td>
<td>157.7802</td>
<td>2.6507</td>
<td>0.0366</td>
<td>0.5116</td>
<td>1.5868</td>
<td>0.8044</td>
<td>4.9353</td>
<td>2.5020</td>
<td>0.3215</td>
</tr>
<tr>
<td>P.4.</td>
<td>191.3821</td>
<td>3.2152</td>
<td>0.0435</td>
<td>0.4873</td>
<td>1.5335</td>
<td>0.8221</td>
<td>4.9912</td>
<td>2.6756</td>
<td>0.3568</td>
</tr>
</tbody>
</table>

Table 2: Steady State Variable Values (aggregate per capita)

<table>
<thead>
<tr>
<th>Var.</th>
<th>$K$</th>
<th>$X$</th>
<th>$H$</th>
<th>$C$</th>
<th>$Y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>S.M.</td>
<td>48.3264</td>
<td>0.8119</td>
<td>0.3893</td>
<td>3.3373</td>
<td>4.1492</td>
</tr>
<tr>
<td>P.1.</td>
<td>43.8487</td>
<td>0.7367</td>
<td>0.3849</td>
<td>3.1964</td>
<td>3.9331</td>
</tr>
<tr>
<td>P.2.</td>
<td>46.9691</td>
<td>0.7891</td>
<td>0.3784</td>
<td>3.2436</td>
<td>4.0326</td>
</tr>
<tr>
<td>P.3.</td>
<td>39.4451</td>
<td>0.6627</td>
<td>0.3828</td>
<td>3.1103</td>
<td>3.7730</td>
</tr>
<tr>
<td>P.4.</td>
<td>47.8455</td>
<td>0.8038</td>
<td>0.3763</td>
<td>3.2545</td>
<td>4.0583</td>
</tr>
</tbody>
</table>

4.1.1 Welfare Analysis

Based on the results presented in the tables above, an aggregate per capita welfare measure is constructed as the weighted sum of the heterogeneous agents' utilities, i.e. $WF = \lambda_1 u_1 + (1 - \lambda_1) u_2$. Table 3 presents in column (1) the level of per capita aggregate output and columns (2) to (5) introduce the aggregate per capita utility and welfare measures in terms of the aggregate per capita output, where $WF^h$ refers to the welfare measure corresponding to the homogeneous agents model.

Measured in terms of per capita aggregate output, the welfare measure improves if the inflation tax is replaced in a revenue neutral way by any of the alternative tax policies except for policy 3 which increases the tax rate on capital income only. It is also interesting to observe that in the model with homogeneous agents, policy 3 also leads to an improvement in welfare in terms of output. Moreover, with policies 2 and 4 the heterogeneous agents
Table 3: Utility per Type of Agent and Aggregate Welfare

<table>
<thead>
<tr>
<th>Var.</th>
<th>$\frac{u^1}{Y}$</th>
<th>$\frac{u^2}{Y}$</th>
<th>$\frac{W^F}{Y}$</th>
<th>$\frac{W^{FE}}{Y}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>S.M.</td>
<td>0.2280</td>
<td>0.0730</td>
<td>0.1118</td>
<td>0.1220</td>
</tr>
<tr>
<td>P1.</td>
<td>0.2360</td>
<td>0.0705</td>
<td>0.1119</td>
<td>0.1230</td>
</tr>
<tr>
<td>P2.</td>
<td>0.2302</td>
<td>0.0740</td>
<td>0.1131</td>
<td>0.1232</td>
</tr>
<tr>
<td>P3.</td>
<td>0.2449</td>
<td>0.0649</td>
<td>0.1099</td>
<td>0.1226</td>
</tr>
<tr>
<td>P4.</td>
<td>0.2286</td>
<td>0.0750</td>
<td>0.1134</td>
<td>0.1134</td>
</tr>
</tbody>
</table>

The model displays a greater improvement in terms of aggregate welfare than the homogeneous agents model. Conversely, the former model shows a smaller improvement in welfare with policy 1.

In aggregate per capita bases, policy 4 shows the best welfare improvement of 1.43%. Policy 2 leads to the second best improvement of 1.16% followed by policy 1. Implementing policy 3 leads to a deterioration in the welfare measure. In the homogeneous agents model, the best policy in terms of welfare improvement is still policy 4 but the increase in welfare is only 1.07%, 25% less than the improvement in the heterogeneous agents model. Surprisingly, all other policies, including policy 3, show an improvement in welfare.

Therefore, the steady state results show that the heterogeneous agents model leads to qualitative as well as quantitatively different results than the ones obtained by the corresponding homogeneous agents model. Strikingly, policy 3 which replaces revenue from the inflation tax solely by increasing the tax rate on capital income leads to the best outcome in terms of utility-output ratio for type 1 agents (capital owners) while the same policy represents the worst choice for type 2 agents (employers). For these agents, policy 4 and policy 2, even though they imply an increase in the tax rate applied to their only source of income (labor), lead to an increase in their utility-output ratio by 2.74% and 1.37% respectively. This is due to the fact that both policies imply a lower labor ($H_2$) choice and aggregate per capita output as shown.
in Tables 1 and 2.

4.1.2 Income Distribution Analysis

The interesting feature of using an heterogeneous agents model is permits the analysis of distributive questions. The table below reports the (per capita - per type) aggregate income in columns (1) and (2), the aggregate per capita income in column (3) and the relative participation of each type of agent in aggregate income in columns (4) and (5), for the standard as well as the simulated policy models.

<table>
<thead>
<tr>
<th>Var.</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>S.M.</td>
<td>8.4279</td>
<td>2.7229</td>
<td>4.1492</td>
<td>0.5078</td>
<td>0.4922</td>
</tr>
<tr>
<td>P.1.</td>
<td>7.9325</td>
<td>2.6000</td>
<td>3.9331</td>
<td>0.5042</td>
<td>0.4958</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(-0.71%)</td>
<td>(+0.73%)</td>
</tr>
<tr>
<td>P.2.</td>
<td>8.1499</td>
<td>2.6602</td>
<td>4.0326</td>
<td>0.5052</td>
<td>0.4948</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(-0.51%)</td>
<td>(+0.53%)</td>
</tr>
<tr>
<td>P.3.</td>
<td>7.5860</td>
<td>2.5020</td>
<td>3.7730</td>
<td>0.5027</td>
<td>0.4973</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(-1.00%)</td>
<td>(+1.04%)</td>
</tr>
<tr>
<td>P.4.</td>
<td>8.2064</td>
<td>2.6756</td>
<td>4.0583</td>
<td>0.5055</td>
<td>0.4945</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(-0.45%)</td>
<td>(+0.47%)</td>
</tr>
</tbody>
</table>

From the last two columns (4) and (5) it is clear that the elimination of the inflation tax in a revenue neutral way by either of the considered income tax schemes improves the distribution of income between the two types of agents, characterizing the inflation tax as a regressive distributive mechanism.

The above table shows that from the perspective of equity, policy 3 brings the best distributive outcome at steady state, leading to an increase of 1.04% in the participation of type 2 agents in aggregate income. Policy 1 which keeps a constant ratio between the two income tax rates relative to the standard model economy presents the second best distributive result.

Cross-analyzing Table 3 and Table 4 it is also evident that in a political game of the above four policy schemes for eliminating the inflation tax in
a revenue neutral fashion, both types of agents would choose either policy 2 or 4 as they are the Nash equilibrium of such a game. Furthermore, if perfect coordination between the agents is assumed, either policy could be implemented, leading to an improvement in the welfare-output ratio for both types of agents as well as in the equity of income distribution.

4.2 Transition Path

The optimal investment decision rule as well as the corresponding pricing function, as functions of the state variables $z$ and $K$, were derived for the homogeneous agents model solving the analog to problem (14) observing that in equilibrium $\dot{M} = 1$. This is introduced in Table below along with the corresponding optimal rules as well as the pricing function for the heterogeneous agents models, the solution to problems (14) and (15).  

Aggregating over both types of agents, $K_t = \lambda_1 K_{1t} + (1 - \lambda_1) K_{2t}, L_t = \lambda_1 L_{1t} + (1 - \lambda_1) L_{2t}, C_t = \lambda_1 C_{1t} + (1 - \lambda_1) C_{2t}$, the main statistics of the obtained sequences are reported in Table 6 in aggregated per capita terms, together with the transition path implied by the homogeneous agents model over a period of 100 quarters.

As suggested by the Table ??, the introduction of two different types of agents into the model economy increases the variance of the sequence for all considered variables even though the mean values are very close to each other. The representative agent’s model leads to a faster approximation to the corresponding steady state values but also generates a per capita welfare path that is consistently higher than the welfare sequence implied by the heterogeneous agents model. This result suggest that the used heterogeneous agents model captures a consistently lower welfare-output path due to the inflation tax.

Computing, within the heterogeneous agent framework, the capital stock (per type 1 capita) path, the optimal labor choice sequence, the consumption path and the utility path by type of agent over a period of 25 years, it can be seen, as expected, that the consumption and utility values for type 1 agents who have two sources of income are higher than for type 2 agents, whereas the optimal labor choice is consistently higher for type 2 agents, for labor is

6The optimal rules and the pricing function induce the respective steady state variable values up to five decimal places.
the only source of income for them. The sequences corresponding to type 1 agents follow the endogenously determined capital stock cycle.

Furthermore, assuming that either one of the two Nash equilibrium outcomes generated by policy 2 and policy 4, that increases only the tax rate on labor income in a revenue neutral way or a tax scheme which introduces an uniform tax rate across different income sources respectively, is to be implemented as an alternative to the regressive inflation tax, the corresponding

<table>
<thead>
<tr>
<th>Table 5: Optimal Decision Rules</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Homogeneous Agents Model</strong></td>
</tr>
<tr>
<td>Optimal Investment Decision Rule</td>
</tr>
<tr>
<td>( X_t = a_0^h + a_1^h z_t + a_2^h K_t )</td>
</tr>
<tr>
<td>( a_0^h \quad a_1^h \quad a_2^h )</td>
</tr>
<tr>
<td>29.2945 (-0.0010) (-0.5894)</td>
</tr>
<tr>
<td>Pricing Function</td>
</tr>
<tr>
<td>( \hat{p}_t = c_0^h + c_1^h z_t + c_2^h K_t )</td>
</tr>
<tr>
<td>( c_0^h \quad c_1^h \quad c_2^h )</td>
</tr>
<tr>
<td>(-16.2796) (-6.1162e-04) (0.3431)</td>
</tr>
<tr>
<td><strong>Heterogeneous Agents Model</strong></td>
</tr>
<tr>
<td>Optimal Investment Decision Rule</td>
</tr>
<tr>
<td>( X_t = a_0 + a_1 z_t + a_2 K_t )</td>
</tr>
<tr>
<td>( a_0 \quad a_1 \quad a_2 )</td>
</tr>
<tr>
<td>49.4318 (1.6172) (-0.2473)</td>
</tr>
<tr>
<td>Optimal Money Holding - Type 1 RA</td>
</tr>
<tr>
<td>( \hat{M}<em>{1t} = b</em>{10} + b_{11} z_t + b_{12} K_t )</td>
</tr>
<tr>
<td>( b_{10} \quad b_{11} \quad b_{12} )</td>
</tr>
<tr>
<td>554.4837 (-2.2344) (-2.8488)</td>
</tr>
<tr>
<td>Optimal Money Holding - Type 2 RA</td>
</tr>
<tr>
<td>( \hat{M}<em>{2t} = b</em>{20} + b_{21} z_t + b_{22} K_t )</td>
</tr>
<tr>
<td>( b_{20} \quad b_{21} \quad b_{22} )</td>
</tr>
<tr>
<td>(-183.4920) (0.7448) (0.9496)</td>
</tr>
<tr>
<td>Pricing Function</td>
</tr>
<tr>
<td>( \hat{p}_t = c_0 + c_1 z_t + c_2 K_t )</td>
</tr>
<tr>
<td>( c_0 \quad c_1 \quad c_2 )</td>
</tr>
<tr>
<td>(24.6439) (-0.1947) (-0.1249)</td>
</tr>
</tbody>
</table>
transition path can be analyzed using the optimal rules and the pricing functions depicted in the following Table 7:\textsuperscript{7}

\begin{table}[h]
\centering
\begin{tabular}{|l|c|c|c|c|}
\hline
\textbf{Var.} & \textbf{Mean(Hom.)} & \textbf{Std(Hom.)} & \textbf{Mean(Het.)} & \textbf{Std(Het.)} \\
\hline
Capital Stock & 48.3262 & 0.0037 & 48.3267 & 0.0089 \\
Labor & 0.3889 & 0.0052 & 0.3888 & 0.0096 \\
Consumption & 3.3396 & 0.0188 & 3.3388 & 0.0557 \\
Welfare & 0.5070 & 0.0026 & 0.4620 & 0.0099 \\
\hline
\end{tabular}
\caption{Transition Path Statistics}
\end{table}

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|}
\hline
\multicolumn{4}{|c|}{Table 7: Heterogeneous Agents Model} \\
\hline
\multicolumn{4}{|c|}{Optimal Investment Decision Rule} \\
\hline
& $a_0$ & $a_1$ & $a_2$ \\
\hline
P.2 & 31.9709 & 0.1752 & -0.1543 \\
P.4 & 61.9053 & 0.1394 & -0.3074 \\
\hline
\multicolumn{4}{|c|}{Optimal Money Holding Rule} \\
\hline
Ag.1: $\hat{M}_{1t} = b_{10} + b_{11} z_t + b_{12} K_t$ & & & \\
Ag.2: $\hat{M}_{2t} = b_{20} + b_{21} z_t + b_{22} K_t$ & & & \\
\hline
& $b_{10}$ & $b_{11}$ & $b_{12}$ \\
\hline
P.2 & -57.0703 & -0.0452 & 0.3122 \\
P.4 & -40.4263 & -0.0854 & 0.2197 \\
\hline
& $b_{20}$ & $b_{21}$ & $b_{22}$ \\
\hline
P.2 & 20.3568 & 0.0151 & -0.1041 \\
P.4 & 14.8028 & 0.0285 & -0.0732 \\
\hline
\multicolumn{4}{|c|}{Pricing Function} \\
\hline
& $c_0 + c_1 z_t + c_2 K_t$ & & & \\
\hline
& $c_0$ & $c_1$ & $c_2$ \\
\hline
P.2 & -281.7900 & -0.4603 & 1.5040 \\
P.4 & -192.2403 & -0.4323 & 1.0083 \\
\hline
\end{tabular}
\caption{Heterogeneous Agents Model}
\end{table}

\textsuperscript{7}The above rules induce the respective steady state variable values up to five decimal places.
Transition Path of Alternative Tax Policies, $t=1,\ldots,100$.

Analysing the transition paths for aggregate welfare in terms of aggregate output, the revenue path, the optimal labor sequence and the aggregate per capital stock path, respectively, for the basic model, tax policy 2 and tax policy 4 for an interval of 100 quarters, it can also be seen that policy 2 generates paths for aggregate per capita revenue, labor and capital stock consistently lower than the ones implied by the standard model and by policy 4. In terms of aggregate per capita welfare-output sequence this policy Pareto dominates the others only after $t = 18$.

If for equity criteria the per capita-per type income participation in terms of aggregate per capita income is considered, the implementation of policy 4 presents a more equitable distribution of income. The corresponding mean participation is 0.5071 for type 1 agents and 0.4929 for type 2 agents and its standard deviation is 0.0127 over the same interval.

Therefore, the above results suggest that if what matters for the model economy is an equitable distribution of income between type 1 and type 2 agents, policy 4 which institutes an uniform tax rate over both capital and labor income, would be the most desirable policy choice, although the implementation of this policy leads to a lower welfare-output ratio than the alternative policy 2 after period $t = 18$.

Policy Changes After $t=10$.

Assume now that the economy starts as the standard model economy with an inflation tax and from period $t = 10$ one of the alternative NE tax policy scheme is implemented. In this case, as is expected, policy 4 (the uniform tax rate regime) will bring about a higher level of revenue, capital stock and labor choice after $t = 10$ than policy 2 (which increases only the tax rate on labor income). In terms of the welfare-output ratio, this result is true only up to $t = 20$ along the first 10 periods of policy implementation after which policy 4 results in a consistently lower level of aggregate welfare relative to aggregate output.

The sequences of income distribution show a strong adjustment during the first 10 periods after implementing policy 4. On the other side if policy 2 is to be implemented then this adjustment process is much more accentuated and lasts 10 periods longer than the former. Furthermore, the reported path corresponding to policy 4 has a mean value of 0.5121 for type 1 agents'
income participation (0.4879 for type 2) with a standard deviation of 0.0439 which represent a better outcome from an equity standpoint compared to a mean of 0.5575 for type 1 agent's income participation with a higher standard deviation of 0.1079 associated with the path generated with policy 2.

**Temporary Policy Changes**

Another exercise is performed based on a common fact that so called "stabilization policies" are often temporary. Then, the natural step is to analyze the transition path implied by these unexpected temporary policy changes, for it is assumed to be a "surprise" from the agent's point of view.

As should be expected, the transition paths over an interval of 50 periods (12.5 years) in which either policy 2 or policy 4 are implemented temporarily for 10 quarters (2.5 years) show that the revenue path follows a combination of labor and capital sequences with policy 4 leading to a higher level of these variables over the time period 10 to 30. Moreover, the labor path implied by both policies moves in opposite direction to each other. Policy 2 levies a heavier burden on labor income and has a negative impact on the decision to work (hence on the associated revenue path). The aggregate welfare-output ratio presents a different pattern with policy 4 reflecting a faster adjustment to the temporary change in policy than the alternative policy 2 regime.

In order to analyze the impact on income distribution caused by such a temporary policy change compared to a permanent elimination of the inflation tax, Table 8 reports the main comparative statistics of the respective sequences taking into consideration their first 50 periods.

Comparing the temporary vs. the permanent policy implementation, the results strongly suggest that cutting down the inflation tax only for 10 quarters (2.5 years), with either policy 2 or policy 4, leads to a worse distribution of income over the considered 12.5 years, both in terms of the mean value as well as of the standard deviation of the sequences. If a permanent policy change is to be implemented, policy 4 appears to be more desirable from a distributive point of view leading to a mean income participation of 0.5230 for type 1 agents (0.4770 for type 2 agents) compared to 0.6038 for type 1 agents (0.3962 for type 2) brought about by a permanent change to policy 2.

Nevertheless, cross-comparing the above results, in terms of the mean value of income participation and its standard deviation, even the temporary policy 4 leads to a more equitable distribution than the permanent
Table 8: Permanent vs. Temporary Policy Changes

<table>
<thead>
<tr>
<th></th>
<th>Policy 2</th>
<th>Policy 4</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Permanent (Mean, St. Dev.)</td>
<td>Temporary (Mean, St. Dev.)</td>
</tr>
<tr>
<td></td>
<td>Mean 0.6038, 0.1383</td>
<td>Mean 0.5230, 0.0604</td>
</tr>
<tr>
<td></td>
<td>Mean 0.3962, 0.1383</td>
<td>Mean 0.4770, 0.0604</td>
</tr>
</tbody>
</table>

Implementation of policy 2 for the considered time span.

5 Conclusion

Summarizing the main findings, when comparing the inflation tax policy relative to the other alternative tax policy schemes, the steady state analysis has clearly shown the relevance of using a heterogeneous agents model within a RCE approach. In terms of the aggregate welfare-output ratio, any simulated alternative tax policy to the inflationary model economy, using a RA's model, results in a mild improvement of up to 1.07%. With a heterogeneous agents model, this welfare enhancing effect attains an improvement of up to 1.43% except with policy 3.

It is also shown that from an equity point of view, all alternative revenue neutral tax regimes are desirable compared to the income distribution outcome of the inflation tax scheme. Income equality improves most with policy 3 which increases only the tax rate on capital income. Hence, the trade-off between equity and efficiency becomes apparent: the policy that can improve the income distribution the most is Pareto dominated by the other alternative policies.

By type of agents, their utility in terms of aggregate per capita output
increases consistently only through policy 2 or policy 4 which constitute the
two NE used to examine the dynamics of the transition path.

The transition path analysis over 100 time periods shows that the se­
quences of the aggregate per capita variables implied by the optimal decision
rules and the price function associated with the two (Nash) RCE describe a
different pattern depending on the timing and duration of the implemented
policy.

If from a common arbitrary initial capital stock and productivity shock
the obtained sequences are compared, policy 2 leads to a consistently higher
(Pareto superior) level of aggregate welfare-output ratio after period t=18.
If the alternative tax policies are implemented permanently from t=10 on,
policy 4 leads to a higher welfare-output ratio only from period t=10 to t=20,
from which its path associated with policy 2 Pareto dominates the others.

In the case of a temporary decrease of the inflation tax, only from t=10
to t=20, the aggregate per capita welfare in terms of aggregate output shows
an immediate improvement which dominates policy 2's path over more than
half of the temporary implementation period. The transitory nature of either
policy leads to a sharp deterioration of approximately 0.75% of the welfare
measure which can be recovered only after the following 10 periods.

Moreover, contrasting the distributive impact of temporary vs. perma­
nent policy changes over the interval of 50 periods, the former has a negative
impact on the income distribution for either policy relative to their perma­
nent application.

This study constitutes an example of the relevance of heterogeneous
agents models within a RCE framework to analyze both welfare and dis­
tributive impacts of government revenue funding through an inflation tax.
Furthermore, the dynamic nature of the RCE along with the linear quadratic
algorithm allows one to derive explicitly the associated optimal decision rules.
This is crucial for analyzing the transition path movements towards the
steady state implied by the different policy regimes.

References

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Autor: Bugarin, Mirta Noemi Sataka.
Título: Welfare cost of inflation with heterogeneous...