"Growth, Distribution and School Policy"

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Growth, Distribution and School Policy

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Abstract

This paper investigates the relationship between growth, income inequality, and educational policies. An endogenous growth model is built in which there are two types of labor, skilled and unskilled, and the quality of the labor force (measured by the fraction of skilled workers) will ultimately determine the economic growth rate. We show that multiple inequality and growth paths may arise. Countries will not necessarily converge to the same economic growth and income distribution. When the proportion of skilled workers is low, the economy grows slow, and the Gini coefficient is high. Low expected growth rate inhibits investments in human capital and the quality of the labor force tomorrow turns out to be low again, keeping the economy in the bad equilibrium.

We then analyze the effects on growth and inequality of two types of government intervention: introduction of public schools and vouchers. Both types can induce the economic agents to invest more in education. The consequence will be an increase in the quality of the labor force, leading to higher growth rates and less inequality.

Finally, we examine the welfare consequences of these interventions and conclude that they may be Pareto improving.
1 Introduction

It is a well-established idea that there is a close link between investments in human capital and economic growth. Empirical evidence shows that measures of educational attainment are important variables to explain future growth rates. In other words, the quality of the labor force is a crucial indicator of a country's prospects of future development. Moreover, the greater the quality of the labor force, the lower is the proportion of unskilled workers, and "greater the scarcity value of unskilled labor." Hence, the income disparity between the bottom and the upper classes is lower leading to less income inequality. According to this reasoning, countries with different labor force quality should grow at different rates and register different income distribution. Clarke (1995) finds that different measures of inequality are negatively and significantly correlated with the growth of per capita GDP using data for 79 countries. However, should one expect all countries to converge to the same economic growth as well as to the same income inequality? This is the first question addressed in this paper.

The empirical evidence suggests that it might not be the case. The classical example is the comparative performance between Latin American and East Asian countries. The latter has grown faster with a more equitable income distribution since the 50's. Moreover, Africa ranks much the same in terms of inequality as Latin America with very poor economic performance. Furthermore, numerous cross-chapter growth regressions indicate that initial inequality is detrimental to long-run growth. About these results, Bénabou (1996) writes: "a one standard deviation decrease in inequality raises the annual growth rate of GDP per capita by 0.5 to 0.8 percentage points (...) implying an income gap of about 25% after 30 years."

If the quality of the labor force is so important as it affects growth and income distribution, policymakers should design policies to encourage investments in education to increase the fraction of skilled workers in the society. These policies might be desirable even if there are no perfect capital markets to finance education. However, in particular, if there is no capital market to finance investment in human capital and there are decreasing returns in these investments, the government ought to equalize the access to educa-

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1 See, for example, Levine and Renelt (1992).
2 The World Development Report (1992) suggests that it is the distribution of human capital and not of physical assets that is very closely related to the distribution of income. Stokey (1996) writes: "while inherited wealth may be a significant contributor to persistence in the upper tail of the income distribution, for the rest its importance is modest and for the bottom half it is apparently of little or no consequence. It is persistence in earnings that has the greater influence, because earnings are the bulk of national income and because earnings are important in all ranges of the income distribution."
3 See the next section for a survey of these cross-section growth regressions.
tion. This seems to have been the motivation of policymakers in East Asian countries when they chose to emphasize primary and secondary education in detriment to tertiary education back in the 50's. As another example, Chile is currently implementing a major educational reform. The numbers of hours that a public school student will stay in school per day will double, reaching 8 hours, the same as current hours in private schools. Two questions come to mind. Can a school policy effectively increase the quality of the labor force, leading to higher growth rates and less inequality? Are investments in public schools and the supply of vouchers to be spent in private schools alternative ways to reach this objective? This paper addresses these two questions.

Finally, we examine if these government interventions are desirable. Members of the current generation will pay the price and the benefits will come only in the future through higher growth rates and income. If individuals are very impatient, the net benefit may be negative.

To investigate the relation between growth, income inequality, and educational policies and to address the above questions, an endogenous growth model is built. A simplified version of the theoretical framework in Stokey (1988) and Young (1991) is used in which the accumulation of knowledge, through economywide learning by doing, is the only ‘engine’ of growth. The idea is that a firm that increases its production learns simultaneously how to improve the quality of the good produced. Growth is measured by the rate of increase of this quality level. To retain the framework of perfect competition, it is assumed that all improvements in the quality of the good produced are unintended by-products of the production process and that they immediately become common knowledge. On the other hand, one could interpret that “the learning and spillovers involve human capital and that each producer benefits from the average level of human capital in the economy (...). Thus, instead of thinking about the accumulated knowledge or experience of other producers, we have to think here about the benefit from interacting (freely) with the average person, who possess the average level of skills and knowledge.” (See Barro and Sala-i-Martin, 1995) Under this interpretation, the economy would grow faster the greater is the quality of the labor force.

Furthermore, it is used a simple overlapping generation model with intergenerational altruism as in Loury (1981). There are two types of agents, skilled (with high level of human capital) and unskilled (with low level of human capital). The offspring’s earning capacity, and therefore his economic status, depends primarily on his parent’s investment in his human capital. The greater this investment, the more likely a child will become an individual with high level of human capital. The quality of the labor force is measured by the fraction of skilled workers.4

4As discussed below, it is assumed that the population is constant over time.
These two types of agents are the only inputs in the production function. As an individual with high level of human capital is more productive than one with low level of human capital, the greater the quality of the labor force the more experience is accumulated in the production process. Hence, faster the economy grows. Moreover, the greater is the proportion of skilled workers greater is the scarcity value of unskilled labor and less unequal is the income distribution.

The focus is on stationary equilibrium that arises from this setup, where the growth rate (measure by the increase in the quality of the good produced) and the fraction of skilled workers are constant. It is shown that multiple growth and inequality paths may arise. The path with higher growth rate is also the one with more egalitarian income distribution, where the latter is measured by the Gini coefficient. The explanation runs as follows. We assume, as in Azariadis and Drazen (1990), a “technological externality with a ‘threshold’ property” that permits returns to scale to rise very rapidly whenever the quality of the labor force reaches a certain point. In such a framework, an economy with a low quality labor force today is expected to grow slowly. Agents with forward-looking behavior realize it and they are discouraged to invest in their child’s human capital as the benefits of these investments are perceived to be low in such stagnant environment, despite the great differences in income of skilled and unskilled workers. In a more technical way, the slower the expected economic growth rate, the greater is the discount imposed by the present generation on future consumption in the intertemporal utility functions that guide consumption and investment in human capital decisions. Hence, low expected growth rate inhibits investments in human capital and the quality of the labor tomorrow turns out to be low again, maintaining the economy in an equilibrium with very unequal income distribution and low growth rates.

Next, it is investigated the effects on growth and income distribution of two different school policies. First, it is introduced public schools and parents are allowed to choose between sending their children to private or public schools. Second, it is supplied vouchers to be spent in private schools. In both schemes the government imposes income taxes to finance the program. Both will affect private investments in human capital in the same way. In one hand, private investments in education will be discouraged because of the negative income effect related to the income tax and the effects of skill quantities on skill prices. In other hand, the prospect of future higher growth rate that accompanies the expectations of an economy with an improved labor force quality stimulates greater investments in human capital. We show that both systems can induce the economic agents to invest more in education. The consequence will be an increase in the quality of the labor force, leading to higher growth rates and less inequality.
Finally, we examine the welfare consequences of these interventions and conclude that they may be Pareto improving. In other words, the utility level of both type of agents may be greater after the government intervenes either through public schools or vouchers.

Azariadis and Drazen (1990) and Becker, et al. (1993) are examples of endogenous growth models with multiple balanced growth path. However, they do not consider redistribution issues as they work with a representative agent framework.

The closest paper related to this one is Glomm and Ravikumar (1992). They develop an endogenous growth model with intergenerational altruism that tries to connect private and public education, income inequality, and economic growth. There are important differences between this paper and theirs: (i) the current generation values the wealth they pass on to their descendants in their model whereas the current generation values the next generation’s utility in my model, (ii) they do not allow a mixed regime with private and public education, (iii) they do not consider a voucher system, and (iv) they do not analyze welfare effects from different educational regimes.

To my knowledge, Chou and Talmain (1996) is the only paper that addresses the welfare consequences of a redistribution policy. They build an endogenous growth model in which the wealth of the households determines their permanent income, which in turn determines their labor supply, and ultimately the growth rate. They show that it is possible for the rich to be better off as a result of a wealth distribution. They experience a loss of wealth today but benefits from increased growth and income in the future. However, the accumulation of human capital is not the ‘engine’ of growth and, consequently, they do not consider the welfare effects of different educational policies as it is done here.

The rest of the paper is organized as follows. In the next chapter, we present the empirical evidence that links growth and income distribution. In chapter 3, the model is presented. The competitive equilibrium is characterized in chapter 4. In the following chapter, the stationary growth path is analyzed. In chapter 6, the effects of government intervention on growth and inequality is presented, as well as it is compared the utility levels of both types of agents in the stationary equilibria with and without government intervention. Next, we illustrate the effects of small changes in the amount of vouchers upon the level of utility of both types of agents and the fraction of skilled workers in the economy, which ultimately determines the economic growth and the income distribution. Chapter 8 concludes.
2 The Empirical Evidence

Several empirical studies have analyzed the link between income distribution and economic growth. They run cross-country growth regressions in which the dependent variable is either the average growth of the per capita income or the average growth rate of GDP over a long period of time. They include, among other explanatory variables, a measure of income distribution. These studies differ one from another mainly because either they use different measures of income distribution or they consider different time periods.

These empirical investigations use a measure of the income distribution variable at the start of the period over which growth is measured. This procedure is employed in order to circumvent potential endogeneity and reverse causation problems.

To my knowledge, Persson and Tabellini (1990) and Alesina and Rodrik (1991) are the first papers that run growth regressions using a measure of income distribution as an explanatory variable. They initially develop a theoretical model to argue that greater inequality slows growth. The theoretical argument in both papers is similar. High levels of income inequality generate greater conflict over distributional issues and encourage government to impose higher taxes to reduce it. The implications of these higher taxes are a reduction on the rate of return on private investments, limiting capital accumulation and reducing the economic growth rate.

Persson and Tabellini (1990) use the average growth rate of per capita real GDP over the period 1960-1985 as the dependent variable. They use the fraction of income received by the third quintile of the distribution (this variable is called 'middle') and the Gini coefficient for the distribution of land ownership as measures of income distribution. Their results led to the conclusion that "a more unequal size distribution of income is bad for growth in democracies." Moreover, they conclude that the effects of distribution on growth are also quantitatively important: "a one-standard deviation change

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5 Among these explanatory variables, two are always incorporated in these regressions: the initial level of per capita income and a variable that indicates the educational attainments of the population. As presented below, Clarke (1995) shows strong empirical support to the link between income distribution and economic growth for different specifications of the growth regression.

6 They obtain similar results when they replace 'middle' by the percentage of income received by the top 5% of the population or the Gini coefficient obtained from the distribution of pre-tax income.

7 To test that the political system might affect the link between economic growth and income distribution the authors add to the regressions a dummy variable. It takes a value of 1 for democratic countries and 0 otherwise. This variable enters in the regression alone and interactively with the distributional variables. The equality variable (middle) is significant only when interacted with the dummy variable.
Alesina and Rodrik (1991) use the average growth rate during 1960-85 as the dependent variable. They run regressions with six different distributional indicators. These indicators are the income shares of each of the five quintiles in the population and the income share of the top five percentile. The idea is that “increases in the income shares of the poorer quintiles are associated with increases in growth, while increases in the income shares of the richest quintile and of the top five percent are associated with decreases in growth.” All coefficients related to the distributional variables in the regressions have the expected sign but only the third and highest quintiles and the top five percentile are statistically significant.

The next experiment executed by the authors was to divide the sample in two groups, democratic and non-democratic countries, and to run separate regressions for each one. All coefficients associated with the distributional variables in the latter group have the expected sign but they are not statistically significant. In contrast, the results obtained with the former group are similar to the ones when the full sample is included. About these results, they write: “These results suggest that growth may be particularly sensitive in a democracy to the income shares of the middle class and of the richest quintile. The income share of the poorest individuals may exert little influence. One possible reason is that the middle class is likely to be politically much more active than poorer income groups.”

Clarke (1995) is the most complete empirical investigation of the link between growth and income distribution. The dependent variable is the average growth of per capita GDP over the period 1070-1988. He also uses better measures of income distribution. These measures are: the coefficient of variation, the Theil's index, the Gini coefficient and the ratio of the share of total income earned by the poorest 40 percent of the population to the share of income earned by the richest 20 percent of the population. The author has three objectives in the paper: to show that an initial unequal income distribution is detrimental to growth, to show that this link is important in democratic countries as well as non-democratic ones, and, finally, to show that these results are robust across different specifications for the growth regression.

The coefficients of the distributional variables have the expected sign, that is, a greater initial inequality leads to lower growth, and are statistically significant. According to the author's estimation, a reduction in inequality from one standard deviation above to one standard deviation below the mean can increase the long-term growth rate by up to approximately 2.5%.

The author includes an interaction term between the type of regime, democratic or non-democratic, and inequality in the regressions to check if
inequality is detrimental to growth only in democratic countries, as indicated in other studies mentioned above. He concludes that inequality appears to have a negative impact on both types of regime, with the interaction term being statistically insignificant in many regression and the distributional variable being statistically significant.

Finally, Clarke addresses the issue that the sign and significance of variables in cross country growth regressions are highly sensitive to the inclusion or exclusion of variables that are considered significant in other studies. He follows the procedure proposed in Levine and Renelt (1992) to test the robustness of regression results. Departing from a 'Barro type regression' as the base regression, he introduces variables chosen as proxy for aspects of monetary policy, trade policy, and macroeconomic and social stability. The significance of the distributional variables is robust across different specifications of the growth regression.

Bénabou (1996) conducts a survey of other papers which considers the link between income distribution and economic growth. The main conclusion he draws from this survey is the following: “These regressions, run over a variety of data sets and periods with many different measures of income distribution, deliver a consistent message: initial inequality is detrimental to long-run growth.”

Bénabou performs an additional empirical investigation. He checks if countries are converging to the same level of inequality. In order to test it, he basically run regressions with the rate of change in the Gini coefficient on a constant and the initial value of the Gini. His conclusion is the following: “The general picture which emerges at the end of this empirical exercise is thus a mixed one: general stability in the world distribution of Ginis, within which countries and regions seem to be experiencing some non-negligible amount of relative mobility. The question of whether there is actual convergence or whether countries oscillate around distinct long-run levels of inequality still awaits a definite answer.” He emphasizes that the limitations of the data are the main obstacle to a more definite answer.

3  The Environment

The model is formulated in discrete time, beginning at date $t = 0$. Consider an overlapping generation economy in which agents live for two periods. In the first period of life, each individual is supported by his parent, the head of the family, who decides the family's consumption and the amount invested in child's education in that period. In the beginning of the second period of life, each individual gives birth to another and becomes the head of the family. Each individual works in his second period of life supplying inelastically one unit of time. There is no utility from leisure. The population remains
constant over time. Each generation consists of a continuum of agents with measure one.

It is assumed that only one good is produced in the economy, good $Y$, but it can potentially be supplied in different quality levels, indexed by $z$.\textsuperscript{8} At time $t = 0$, it is assumed that producers have the knowledge to produce good $Y$ with quality $z_0$. In order to simplify the problem, it is also assumed that it is produced and consumed only the good with the highest quality level available at each point in time.\textsuperscript{9} The improvement in the quality level of good $Y$ will depend on the experience accumulated in producing a good with lower quality level. Let $Y(z)$ be the total output produced today of good $Y$ with quality level $z$. Therefore, the newest and highest quality level available next period, $z'$, will be given by:

$$z' = [1 + g(Y(z))]z$$

where:

\textbf{Assumption 1:} $g$ is a function with the following properties: (i) $g(0) = 0$; (ii) $g$ is continuous; and (ii) $g'() \geq 0$.\textsuperscript{9}

The spillover of the experience in producing a less sophisticated good to the production of a more sophisticated one is the only ‘engine of growth’ in this model. This formulation captures the same idea to explain growth as the models developed by Stokey (1988) and Young (1991). The difference is that in those models the knowledge to produce an immediately more sophisticated good is acquired only after the country has accumulated enough experience in producing an immediately less sophisticated good and the productivity in the production of the good of any quality level decreases over time. Here we use a simplified version in which the greater the production of the single good with a certain quality level today higher will be the jump in the quality level of this good tomorrow.

Two types of labor — high level of human capital labor $N_h$, and low level of human capital labor $N_l$ — are used to produce the single quality level of good $Y$ available at each point in time. The single technology, which has constant returns to scale, produces goods that can be consumed, or invested in education. There is no physical capital in this model. As in Stokey (1996), “labor provides two distinct productive services, which we may think of as physical effort (‘brawn’) and mental effort (‘brains’).” We assume that\textsuperscript{8}

\textsuperscript{8}Alternatively, $y$ could be seen as a bundle composed of many different goods consumed always at fixed proportions at all possible quality levels where the quality of all goods are increased at the same time.

\textsuperscript{9}As in Stokey (1991), a particular pricing convention can be imposed to the quality levels in zero supply: they are priced at the lowest price consistent with zero demand by consumers.
workers with high level of human capital can provide both services, while workers with low level of human capital supplies only the former — that is, in order to be able to execute more sophisticated tasks the individual must acquire a higher level of human capital. The aggregate production function is the following:

\[ Y = AN^a_h (N_h + N_l)^{1-a} \]  

(2)

where \( A \) is a positive constant and \( 0 < \alpha < 1 \).\footnote{As only the highest quality level of good \( Y \) will be produced at each point in time the index \( z \) will be omitted from now on.}

On a per capital basis, the technology is:

\[ Y = \bar{y} = Aq^a \]  

(3)

where \( \bar{y} \) is the output per capita and \( q \) is defined as the the fraction of the population with high level of human capital, i.e., \( q = \frac{N_h}{N_h + N_l} \).\footnote{Recall that there is a continuum of agents with measure one. Hence, total output is equal to output per capita.}

Using equation (3) in (1), we get:

\[ z' = [1 + g(Aq^a)]z \Rightarrow z' = \psi(q)z \]  

(4)

where:

\[ \psi(q) = [1 + g(Aq^a)]. \]

From the above equation, one can conclude that the higher the fraction of individuals with high level of human capital \( q \), greater the total production today and, hence, greater the quality of the good tomorrow. Economic growth in this model is measured by \( \psi(q) \), the rate of good \( Y \)'s quality increase. Therefore, \( q \) and economic growth are positively correlated. In other words, \( q \) can be seen as a measure of the quality of the labor force. An economy would grow faster, the greater it is the quality of its labor force. Equation (4) indicates the highest quality level of good \( y \) available next period.

All parents have identical preferences. These preferences manifest inter-generational altruism. Each parent takes as given his level of human capital, high \((h)\) or low \((l)\), earns a wage \( w \) and decide how much to invest in his offspring's education and spend on consumption. If a parent invests \( E \) units of good \( Y \) in his offspring's education, with probability \( \pi(E) \) and \( 1 - \pi(E) \), respectively, his child will acquire \( h \), a high level of human capital, and \( l \), a low level of human capital. Therefore, there are two types of individuals at each date.
Assumption 2: \( \pi \) is a function with the following properties: (i) \( \pi \) is continuous; (ii) \( \pi(0) = 0 \); (iii) \( \pi_E > 0 \); (iv) \( \pi_{EE} < 0 \); and (v) \( 0 \leq \pi < 1. \)

The assumption that the function \( \pi \) is increasing implies that a child is more likely to become an individual with high level of human capital the greater is his parent’s investment in his education. However, a parent can never be sure that his offspring will turn out to be a high type individuals as \( \pi \) is lower than 1. We also assume that the function \( \pi \) is concave. This assumption is made in order to capture the notion that there are decreasing returns on educational investments. There is empirical evidence to support it. Galor and Zeira (1993) cites empirical studies that estimate that yearly social returns to investment in education in developing countries are: 26% for primary education, 17% for secondary education and 13% for higher education. Stokey (1996) cites empirical evidence of the rates of return on investments in child’s human capital for the United States: 17-22% for lower education, 15-16% for high school, 12-13% for college, and 7% for graduate school.

The individual type \( x \)'s actions \( (x \in \{h,l\}) \) can be summarized by the outcome of the following dynamic programming problem: (as only one quality of the good \( Y \) will be produced at any point in time, we set the price of this quality level produced at each time to be one)

\[
V(z, q, x) = \max_{C_x, E_x} u(C_x, z) + \beta \{ \pi(E_x) V(z', q', h) + [1 - \pi(E_x)] V(z', q', l) \} \tag{5}
\]

such that:

\[
C_x + E_x = w_x
\]

where:

(i) \( V(z, q, x) \) is the parent type \( x \)'s utility;
(ii) \( V(z', q', x) \) is the child's utility if he turns out to be type \( x \);
(iii) \( C_x \) is the total consumption of individual type \( x \);
(iv) \( w_x \) is the wage paid to an individual with human capital \( x \);
(v) \( E_x \) is the total expenditures on offspring's human capital of parent type \( x \);
(vi) \( \beta \) is the degree of altruism and \( 0 < \beta < 1 \);
(vii) \( u(C_x, z) \) is the current utility of individual type \( x \) if consumes \( C_x \) units of the good \( y \) with quality \( z \);
(viii) \( u_c(.,.) > 0, u_{cc}(.,.) < 0, \) and \( u_z(.,.) > 0. \)

Implicit in the above set-up is the assumption that there is no market to finance investments in education.\(^{12}\) The same assumption is made in Loury (1981) among others.

\(^{12}\)For a discussion about the absence of this market, see Becker and Tomes (1993).
In addition to his type \((h\) or \(l)\), each individual also takes as given \(z\) and \(q\). These are the two state variables in the model. When the consumer goes to the market to buy good \(Y\) he will get higher utility as greater \(z\) is but he can not affect the quality of the product available. As mentioned above, higher \(q\) implies greater economic growth (and greater utility growth) but each individual can not affect \(q\).

The above formulation implies that each parent can not affect the possible values of his offspring's utility, \(V(z', q', l)\) and \(V(z', q', h)\). In other words, he can not affect the environment where his children will live tomorrow. However, he can influence the type his children will turn out to be through his investment in his child's education, i.e., if his child will have utility \(V(z', q', l)\) or \(V(z', q', h)\).

**Assumption 3:** \(u(C, z) = \frac{(zC)^{1-\sigma}}{1-\sigma}\). ■

In the appendix, where all results are derived or proved, we show:

**Proposition 1:** Under assumption 3, \(V(z, q, x) = z^{1-\sigma}v(q, x)\), where:

\[
v(q, x) = \max_{E_x} \left( \frac{w_x - E_x}{1 - \sigma} \right)^{1-\sigma} + \beta\psi(q)^{1-\sigma} \{ \pi(E_x)v(q', h) + [1 - \pi(E_x)]v(q', l) \}.
\]

\[(6)\]

■

In the above proposition, we reformulated the individual's problem and showed that in deciding which optimal decision to take with respect to consumption and investment he does not take into account the level of \(z\). He considers only the growth rate of \(z\), that is, the economic growth rate. Therefore, \(q\) becomes the only state variable relevant in this model as it determines the economic growth rate.

Let \(I_x(q)\) and \(c_x(q)\) be the optimal policy functions for the problem (6) of the representative individual type \(x\).

**Assumption 4:** \(\beta\psi(1)^{1-\sigma} < 1\). ■

**Proposition 2:** Under assumptions 1-4, there exists a unique value function \(v\) (that is bounded and continuous) satisfying (6). For each \(q\), the maximum in (6) is attained by a unique value \(I_x(q)\), and the policy function \(I\) is continuous. ■

### 4 Competitive Equilibrium

At each time \(t, t \geq 0\), there are perfectly competitive markets for the quality level of good \(Y\) produced at time \(t\) and the two types of labor.
Firms act as price-takers. They, taking as given prices and wage rates, decide how much units of labor of the different skills to hire in order to produce good $Y$. Note that, as in Stokey (1991), “since learning spills over completely (…) to other firms, it is not in the interest of any producer to suffer current losses in order to accelerate learning.”

A representative firm maximizes profits which are given by:

$$AN_h^\alpha (N_h + N_i)^{1-\alpha} - w_h N_h - w_l N_l$$  \hspace{1cm} (7)

As there is perfect competition in all markets, labor is paid its marginal product. Therefore, the first-order conditions for the representative firm’s problem are: (using the fact that $N_h = q$ and $N_i = (1 - q)$)

$$w_l = A q^\alpha (1 - \alpha)$$  \hspace{1cm} (8)

and

$$w_h = A q^\alpha (1 - \alpha) + Aaq^{\alpha-1} = w_l + Aaq^{\alpha-1}$$  \hspace{1cm} (9)

Note that firms earn no profits.

It is obvious that parents would invest in their child’s human capital only if their child can benefit by this investment. The benefit in this model is obtained through a higher level of human capital that will lead to higher wages. The wage difference between an individual with high level of human capital and the one with low level of human capital is the premium for investing in education. Hence, this premium must be positive in order to have positive investment in education in equilibrium. Looking at equation (9), one can see that this premium is positive for any possible value for $q$ (recall that $0 \leq q \leq 1$) and it is decreasing in $q$ (i.e., $\frac{d(W_l - W_i)}{dq} < 0$).

The income share of the individuals with low level of human capital is given by: (using equations (3) and (8))

$$\frac{w_i(1-q)}{Y} = (1-\alpha)(1-q).$$

We can compare the Gini coefficient for two economies with different $q$’s. Let $q_A$ and $q_B$ denote, respectively, the fraction of individuals with low level of human capital in economies $A$ and $B$ such that $q_A < q_B$. Point $A$ in figure 1 indicates that the fraction $(1-q_A)$ of the population in economy $A$, the bottom income class, holds the fraction $(1-\alpha)(1-q_A)$ of the total income. Point $B$ is the correspondent plot for economy $B$. Looking at figure 1, one can see that the Gini coefficient in economies $A$ and $B$ are given, respectively, by the area of the triangles $OAC$ and $OBC$. The fact that $\Delta OAC > \Delta OBC$.

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As it is assumed that only the highest quality of good $Y$ will be produced at each point in time, we normalize this price to be one every period.
indicates that economy $A$ has a greater Gini coefficient than economy $B$, i.e., it has a more unequal income distribution. As discussed above, economy $A$ would also grow at a lower rate in comparison with economy $B$ as $q_A < q_B$. Therefore, if there are two economies with different $q$'s, the one with higher $q$ will grow faster and have a more egalitarian income distribution. The only exception to this rule will occur in the extreme case when $q = 0$. In that case, all individuals will have the low level of human capital, and economic growth rate will be zero. However, this is not an interesting case as it would imply a Gini coefficient equal to zero which is not observed empirically.\textsuperscript{14}

\textsuperscript{14}In this model, note that when $q = 0$ not only the economic growth rate will be zero but total output will also be zero. There would be no income to be distributed.
10. Appendix 2

FIGURE 1

\[ \text{Fraction of Income} \]

\[ (1-\alpha)(1-q_s) \]

\[ (1-\beta)(1-q_b) \]

\[ q_A < q_B \]
Let's now turn to the household's dynamic problem. Each individual, forming expectations about $v(q', h)$ and $v(q', l)$ (or $q'$) and taking as given wages and prices, make decisions about investment in offspring's education and total consumption. It is assumed that they have rational expectations.

The first-order condition for the dynamic problem of the representative household type $x$ is given by:

$$ (w_x - I_x(q))^{-\sigma} = \beta \psi(q)^{1-\sigma} \pi E(I_x(q))[v(q', h) - v(q', l)]. \quad (10) $$

This equation indicates that each individual equates the marginal benefit of consuming one additional unit of the good $Y$ to the marginal benefit of investing one additional unit of the same good $Y$ in his child's human capital.

The law of motion for the state variable $q$ is:

$$ q' = \pi(I_h(q))q + \pi(I_l(q))(1 - q). \quad (11) $$

Note that while there is individual uncertainty about the level of his child's human capital, there is no aggregate uncertainty. If a fraction $q$ of the population decides to invest $E^*$ in his child's education, we know that $\pi(E^*)$ of those will succeed in giving a high level of human capital to their offspring.

**Definition 1:** A recursive competitive equilibrium, given the initial condition $q_0$, $q_0 \in (0, 1)$, is defined to be value functions $v : [0, 1] \times \{x\} \rightarrow \mathbb{R}$, $x = h, l$, policy functions $c_x : [0, 1] \rightarrow \mathbb{R}^+$, and $I_x : [0, 1] \rightarrow \mathbb{R}^+$, an economy-wide law of motion for the fraction of individuals with high level of human capital $Q : [0, 1] \rightarrow [0, 1]$, and factor prices functions $w_x : [0, 1] \rightarrow \mathbb{R}^+$ such that:

(i) $v$ satisfies the household problem (6);

(ii) $c_x$ and $I_x$ are the optimal policy functions for (6);

(iii) the wage functions are given by equations (8) and (9);

(iv) the economy-wide law of motion for $q$ is given by equation (11).

This competitive equilibrium implies an economy-wide law of motion $Z : [0, 1] \times \mathbb{R}_+ \rightarrow \mathbb{R}_+$ for the quality of good $Y$. This law of motion is given by equation (4).

## 5 Stationary Equilibrium

### 5.1 Theoretical Examination

We confine our analysis to a stationary (balanced growth) equilibrium.
Definition 2: A stationary growth path is a competitive equilibrium in which the fraction of individuals with high level of human capital, \( q \), is constant.

This condition pins down the invariance (with time) or growth rate of the remaining variables on a stationary growth path. It follows from (4) that the quality of good \( Y \), the measure of economic growth, will grow at the constant rate \( \psi(q) \). From (8) and (9), we get the implication that, respectively, \( w_t \) and \( w_h \) are time-invariant. Hence, total expenditures in human capital, \( E_z \), and consumption, \( C_z \), are also constant, as can be seen from (10). From equation (3), it follows that the number of units produced of good \( Y \) is time-invariant too.

With the constancy of \( q, w_x, \) and \( E_z \), we can use the result in proposition 1 to infer how \( V \) and \( v \) evolve over time in this stationary equilibrium. Using equation (4), we get:

\[
\frac{V(z', q', x)}{V(z, q, x)} = \frac{(z')^{1-\sigma}v(q', x)}{(z)^{1-\sigma}v(q, x)} = (z')^{1-\sigma} = \psi(q)^{1-\sigma}.
\]

Therefore, the household’s utility \( V \) grows at a constant rate given by \( \psi(q)^{1-\sigma} \) and \( v_x \) is a constant, where \( v_x \equiv v(q, x) \). The growth in the utility \( V \) reflects the increase in the quality of the good available to the household. The quality of consumption, investment in education, and wages is increasing over time in this stationary equilibrium, only the quantity is constant. In that sense we can say that both types of agents are becoming wealthier over time.

There are seven equations that characterize this stationary equilibrium with seven unknowns \((E_t, E_h, q, w_h, w_t, v_t, v_h)\):

- the two first-order condition for the firms’ problem:

\[
w_t = Aq^\alpha(1 - \alpha), \tag{12}
\]

and

\[
w_h = w_t + A\alpha q^{\alpha-1}. \tag{13}
\]

- the first-order condition for each type of agent:

\[
(w_t - E_t)^{-\sigma} = \beta\psi(q)^{1-\sigma}\pi_E(E_t)[v_h - v_t], \tag{14}
\]

and
\[(w_h - E_h)^{-\sigma} = \beta \psi(q)^{1-\sigma} \pi(E_h)[v_h - v_l]. \quad (15)\]

- one equation for each \(v:\)

\[v_h = \frac{(w_h - E_h)^{1-\sigma}}{1 - \sigma} + \beta \psi(q)^{1-\sigma} \{\pi(E_h)v_h + [1 - \pi(E_h)]v_l\}, \quad (16)\]

and

\[v_l = \frac{(w_l - E_l)^{1-\sigma}}{1 - \sigma} + \beta \psi(q)^{1-\sigma} \{\pi(E_l)v_h + [1 - \pi(E_l)]v_l\}. \quad (17)\]

- the law of motion for \(q\) (imposing the condition \(q = q'):\)

\[q = \pi(E_h)q + \pi(E_l)(1 - q). \quad (18)\]

We now turn to the task of proving that there is at least one solution to this system of equations. We start by choosing a functional form for the probability function \(\pi\).

**Assumption 5:** \(\pi(E) = k \left(\frac{E}{A}\right)^{1-\theta}, \) where \(0 < k, \theta < 1. \) \(\blacksquare\)

This above functional form for \(\pi\) satisfies the properties in assumption 2. In particular, the parameter \(A\) enters in this functional form to guarantee that \(\pi(.) < 1,\) for any possible \(E.\) The maximum units of good \(Y\) that can be produced at any point in time is equal to \(A.\) This is the total output when \(q = 1.\) We have that \(\pi(A) < 1,\) as \(k < 1.\) Hence, there will always be two types of agents in the economy and no parent can assure through investment that his child will turn out to be the high type individual.

One implication of this model’s set-up is that it is increasingly more difficult for an individual to become high type. As mentioned above, the quality of the investments in education increases over time in the stationary equilibrium. One could expect that an increase in the quality of these investments would augment the likelihood that one’s child becomes high type and, eventually, all individuals in the economy would turn out to be high type ones. However, a next generation parent must invest the same units of good \(Y\) that his parent invested in him, but with a higher quality, to give to his offspring the same likelihood to become a high type individual that he had. Implicitly it is assumed that the high (low) type individual tomorrow is more qualified than the high (low) type individual today. In order to simplify the notation we just call it ‘high’ and ‘low’ types all periods.
In the following proposition, we prove existence of stationary growth path to a particular case in which $\sigma = \theta$ in order to get an analytical solution. This assumption implies that both types of agents spend the same fraction of their income investing in their child’s education. To work out this particular case is interesting because it is possible to understand which assumption drives the result of multiple balanced growth path, discussed below.\(^{15}\)

**Proposition 3:** With assumptions 1, and 3-5, if $\sigma = \theta$, then there exists at least one stationary growth path.\(^{\square}\)

In order to capture the phenomenon of multiple balanced growth, we borrow the idea of “technological externality with a ‘threshold’ property” used in Azariadis and Drazen (1990). This technological feature “permits returns to scale to rise very rapidly whenever economic state variables, such as the quality of the labor, take on values in a relatively narrow ‘critical mass’ range.”\(^{16}\) We then assume that the learning by doing technology exhibits increasing returns when the fraction of skilled workers in the society passes a certain level, say, $q_L$. These returns are decreasing when $q$ goes beyond a certain level, say, $q_H$.

**Assumption 6:** The function $g$ has the following properties: (i) $g' \equiv 0$ for any $q$ such that $0 < q < q_L$, (ii) $g' > 0$ and $g'' > 0$ for any $q$ such that $q_L < q < q_H$, and (iii) $g' > 0$ and $g'' < 0$ for any $q$ such that $q_H < q < 1$; where $0 < q_L < q_H < 1$.\(^{\square}\)

The proof of proposition 3 in the appendix shows that the system of seven equations that characterize the stationary equilibrium can be reduced to a single equation, equation (25), with the unknown variable $q$. The left-hand side ($LHS$) of this equation is equal to $\frac{1}{\beta(q)}$, where the denominator is the discount factor applied to the marginal benefit of investment. The right-hand side ($RHS$) is the ratio of the current value of the marginal benefit of investment in education to the marginal benefit of consumption. In proposition 3, it is shown that this ratio declines with $q$, as drawn in figure 2. Using assumption 6, the $LHS$ has the shape depicted in figure 2.\(^{17}\)

\(^{15}\)In a later version of this paper, it will be computed solutions with different values for the parameters.

\(^{16}\)About this technological feature, Azariadis and Drazen writes: “Among the types of externalities we consider are spillovers from the stocks of different types of capital (to capture in a primitive way the notion of “infrastructure”) as well as the labor-augmenting outcomes of externalities arising in the process of creating human capital.”

\(^{17}\)Recall that $\psi(q) = [1 + g(Aq^\alpha)]$. 

19
Looking at figure 2, we can see that there are multiple values for $q$ that satisfies the equation that characterizes the stationary equilibria (equation (25)). In other words, it indicates that there are multiple balanced growth path. Each path is characterized by a different value for $q$: $q = q_A$ (equilibrium A), $q = q_B$ (equilibrium B), and $q = q_C$ (equilibrium C), where $q_A < q_B < q_C$.

Using the discussion above, we can compare the economic growth rate and income distribution in these different equilibria. The economy in equilibrium $C$, with the highest $q$, will grow faster with less inequality in comparison with economies in equilibrium $A$ and $B$. The same is true for economy in $B$ in comparison with one in $A$. Therefore, one should not necessarily expect all countries to converge to the same economic growth as well as to the same income inequality.

The explanation for an economy to stay in a 'bad' equilibrium such as $A$ runs as follows. An economy with a low quality labor force today is expected to grow slow. Agents with forward-looking behavior realize it and they are discouraged to invest in their child's human capital as the benefits of these investments are perceived to be low in such stagnant environment. This is true in despite of the great differences in income of skilled and unskilled workers (great income inequality). The slower the expected economic growth rate, the greater is the discount imposed by the present generation on future consumption, which is given by $\beta \psi(q)^{1-\sigma}$. Hence, low expected growth rate inhibits investments in human capital and the quality of the labor tomorrow, $q$, turns out to be low again, maintaining the economy in an equilibrium with very unequal income distribution and low growth rates.

Note that the assumption of absence of markets to finance investments in education does not drive the above result of multiple balanced growth path. The crucial assumption to obtain this result is the 'threshold property'. If this last assumption is dropped, equation (25) would be satisfied by only one value of $q$. That is, there would be only one stationary equilibrium.

5.2 Numerical Example

In order to show in proposition 3 that there exists at least one stationary growth path, we made the simplifying assumption that $0 < \sigma = \theta < 1$. As mentioned above, this assumption implied that both types of agents invest the same fraction of their income in their offspring’s education. The objective in this chapter is to use more commonly accepted parameter values in numerical examples and to make the above points more concrete. In particular, we want to consider examples in which the coefficient of relative

\[18\text{Use equation (4) to compare the economic growth in these different equilibria. Look at figure 1 to compare the income distribution in different economies.}\]
risk aversion, \( \sigma \), is greater than one.

Values have to be chosen to six parameters in the model. There are two parameters related to the individuals' preferences: the discount factor, \( \beta \), and the coefficient of relative risk aversion, \( \sigma \). There are two parameters associated with the production function: the multiplicative term, \( A \), and \( \alpha \). Finally, using the same probability function specified in assumption, there are the parameters \( k \) and \( \theta \), which have to be between 0 and 1. In addition, a functional form for the learning by doing technology has to be specified.

The preference parameter, \( \sigma = 1.5 \), is within the range of values commonly used in real business cycle models.

We consider one period in this model to be equivalent to 30 years. Assuming an annual interest rate to be around 3% per year, we set the discount factor \( \beta = 0.4 \).

We follow Stokey (1996a) and consider a skilled worker to be one with at least a high school education. As pointed out by Stokey (1996a), "In 1985, 20% of the adult population in the U.S. had completed college, 13% had 1-3 years of college, 38% had completed high school, and 29% had not completed high school." Hence, we look for a stationary equilibrium in which \( q \) is approximately equal to .71.

Stokey (1996a) points out that "in the U.S. over the period 1963-1989, the wages of college graduates, college dropouts, and high school dropouts relative to high school graduates were 1.4, 1.15, and 0.75." Weighting by the population proportions mentioned above, she calculates the ratio of the skilled wage to the unskilled wage in the U.S. economy over this period to be 1.5. Thus, in the calculations below, we look for a stationary equilibrium in which \( \frac{w_s}{w_u} \) is approximately equal to 1.5.

In the theoretical framework, the learning by doing technology is the 'engine' of growth. Economic growth over the 30 years period is equal to \( \psi(q) = [1 + g(Aq^{\sigma})].^{19} \) The quality of labor force, \( q \), will ultimately determine the growth rate. We specify the following functional form for the function \( g \):

\[
g(x) = \frac{bx}{Aq^{\sigma-1}}.\]

This functional form satisfies the properties of the \( g \) function in assumption 1. We set the parameter \( b \) equal to 1.24. It implies that the steady state GDP per capita growth over the 30 years period with the fraction of skilled workers in the economy equal to .71 \( (q = .71) \) fits the actual data for the U.S. The average GDP per capita growth of the U.S. economy over the period 1960-1990 was 2.1%.

The remaining parameters, \( \alpha, \theta, k, \) and \( A \) are chosen such that the model's stationary growth path roughly fits the facts mentioned above. Therefore, the parameters values used in the computation are:

\[
\alpha = .3, \ \theta = .95, \ k = .93, \text{ and } A = 100.
\]

\(^{19}\text{See equation (4).}\)
As pointed out above, seven non-linear equations with seven unknowns characterize the stationary equilibrium. We reduce it to a system with five equations and five unknowns \((q, v_l, v_h, E_t, E_h)\). The computer method used in the numerical calculations in this paper is Newton's Method for Multivariate Equations. The calculated steady state values are:

\[
q = 0.7103, \quad v_l = -0.3408, \quad v_h = -0.2875, \\
E_t = 0.2682, \quad E_h = 0.5641.
\]

Consequently, the ratio \(\frac{E_h}{E_t}\) is equal to 1.60.

In the theoretical examination in the first part of this chapter, we showed that there is a possibility of multiple stationary equilibria. The path with higher economic growth rate would also be the one with a more egalitarian income distribution. We also showed that the crucial assumption to obtain this result is the imposition of a 'threshold' property in the technology with externality as used in Azariadis and Drazen (1990). The simplifying assumption \(0 < \sigma = \theta < 1\) was imposed in the theoretical analysis. Our objective now is to give a numerical example, using more accepted values for the parameters, to illustrate the possibility of multiple equilibria.

In order to capture the 'threshold' property in the learning by doing technology, we assume the following: \(\beta \psi(q)^{1-\sigma} = \beta\) if \(0 < q \leq 0.3\) and \(\beta \psi(q)^{1-\sigma} \approx 1\) for \(0.3 < q < 1\). This assumption implies that there would be no economic growth rate if there is any stationary equilibrium in which the fraction \(q\) is lower than 0.3. It also implies that the economic growth rate would be very high if there is any stationary equilibrium in which the fraction \(q\) is greater than 0.3.

The parameters' values used in the computation are:

\[
\beta = 0.4, \quad \sigma = 1.5, \quad A = 100, \quad k = 0.93, \quad \theta = 0.7 \text{ and } \alpha = 0.9.
\]

There are two sets of steady state values that satisfy the five equations that characterize the stationary equilibrium. The two steady state values for \(q\) are 0.2998 and 0.5711. It indicates that there are two stationary growth paths with different economic growth rates.

20 All the calculations in this paper to obtain the steady state values used the same technique.
21 Next section, when we calculate the effects of government intervention on growth, income distribution and welfare, we use the stationary growth path associated with these steady state values as the benchmark case.
22 Recall that the term \(\beta \psi(q)^{1-\sigma}\) is the "extended discount factor" which includes the discount factor \(\beta\), and the economic growth rate to the power \((1 - \sigma)\), \(\psi(q)^{1-\sigma}\). In our numerical example, we make sure that \(\beta \psi(1)^{1-\sigma} < 1\), as specified in assumption 4.
23 The wages' steady-state values are: \(w_l = 63.1725\) and \(w_h = 101.2894\).
24 The other steady state values are: (i) when \(q = 0.2998\): \(E_t = 0.0916, E_h = 45.3609, \quad v_l = -0.6311, \quad v_h = -1.7525, \quad w_l = 3.3815\) and \(w_h = 104.9044\); (ii) when \(q = 0.5711\): \(E_t = 0.10388, E_h = 66.4126, \quad v_l = -57.1484, \quad v_h = -58.4719, \quad w_l = 6.04\) and \(w_h = 101.2257\).
paths, with different growth rates and income distribution. As expected by the theory, the 'bad' equilibrium, with \( q = .2998 \), has lower economic growth rate and more unequal income distribution in comparison with the other equilibrium. As pointed out in the theoretical discussion, economic agents would not have great incentives to invest in their child's human capital when the expected economic growth rate is low. In the 'bad' equilibrium, the low and high type agents spend, respectively, 2.7% and 43% of their total income in their child's education. The correspondent numbers for the equilibrium with growth, when \( q = .5711 \), are 16% and 65%.

6 Government intervention

There are two inefficiencies in this economy that justifies government intervention. First, there is no market to finance investment in education. With decreasing returns on this investment (\( \pi_{EE} < 0 \)) and same ability for all members of the young generation (same \( \pi \)), the quality of the labor force (\( q \)) could be improved if the access to education becomes more equal. This equalization would occur with complete capital markets. Second, there are the external effects associated with the learning by doing technology. As mentioned above, each firm does not have any incentive to suffer any loss to accelerate learning because its piece of knowledge spills over instantly across the whole economy. In this model, the learning process can only be accelerated through an improvement in the quality of the labor force. Therefore, a country's educational policy will be a crucial element to determine its economic path.

An efficient government intervention should focus on two aspects: equalizing the access to education and improving the quality of the labor force. Intervention along these lines is one of the main explanations for the extraordinary economic performance with a relatively equitable income distribution of the East Asian countries. Since the sixties, the government of these countries have focused their educational expenditures on primary and secondary education, where the capital market imperfections and information problems are more severe. Public expenditures on tertiary education increased only in the last two decades after universal and near universal enrollment rates had been achieved at the primary and secondary levels and a shift in the structure of production toward more sophisticated and skill-intensive

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\[ ^{25} \text{It seems realistic to assume that someone's education depends in great length on his parent's income.} \]

\[ ^{26} \text{From equation (3), one can see that total output is only a function of } q. \text{ Total production can only be increased through an increase in } q. \text{ Note that subsidizing production to accelerate the learning process is not an alternative policy for the government in this set-up.} \]
products had occurred.27

In this chapter, we introduce two different educational policies with the
objective to reduce the inefficiencies mentioned above. First, the government
supplies vouchers that can only be used to finance education. Second, it
offers public schools as an alternative to private schools. We then analyze the
effects of this intervention on economic growth rate and income inequality.

6.1 Public Schools
Government decides total expenditures in education per pupil who studies in
public school, $E_g$. It imposes the same income tax rate ($\tau$) to all individuals
to finance it.

The parent must opt for sending his child to public or private school. He
pays taxes independently of his choice. If he chooses the first alternative,
then his child will receive $E_g$ units of investment in his education. If he picks
the second alternative, he must decide how much to invest in his offspring’s
education.28

The individual type $x$’s problem with the option of public schools can be
summarized by the following dynamic problem:

$$v(q,x) = \max\{PU_x, PR_x\}$$

where:

- $$PU_x = u(C_x) + \beta \psi(q)^{1-\sigma} \{\pi(E_g)v(q',h) + [1 - \pi(E_g)]v(q',l)\}$$
  such that:
  $$C_x = w_x (1 - \tau).$$
  $PU_x$ is the individual type $x$’s utility if he chooses to send his child to
  public school. The parent will consume his net income and $E_x = 0$.

- $$PR_x = \max_{C_x, E_x} u(C_x) + \beta \psi(q)^{1-\sigma} \{\pi(E_x)v(q',h) + [1 - \pi(E_x)]v(q',l)\}$$
  such that:
  $$C_x + E_x = w_x (1 - \tau).$$

28We assume in this paper that all families live in the same community. As an example
of a paper in which individuals decide in which community they want to live, see Fernandez
and Rogerson (1996).
$PR_x$ is the individual type $x$'s utility if he chooses to send his child to private school. His problem will be exactly equal to the environment without government intervention. The only difference is that he will pay taxes to finance public schools expenditures even sending his kid to private school.

In the above formulation, we limit the investments in education in someone who goes to public school by $E_g$, an amount chosen by the government. One can imagine that a parent might want to invest additional amounts in tutoring and private classes to complement his child education. In this case, the parent's problem if he sends his child to public school would become:

$$PU_x = \max_{C_x, E_x} u(C_x) + \beta \psi(q)^{1-\sigma} \{\pi(E_g + E_x)v(q', h) + [1 - \pi(E_g + E_x)]v(q', l)}$$

such that:

$$C_x + E_x = w_x(1 - \tau).$$

$E_x$ would be the additional investment. It is obvious that if faced with this alternative, the option private school ($PR$) would be always inferior and $\max(PR, PU) = PU$.

However, a student is required to spend many hours per day in a school, private or public. He can 'waste' significant hours of his day if he studies in a low quality school. In particular, if the quality of the public school is very low, additional expenditures in education may not guarantee a better education in comparison with a private school. Moreover, the option of a public school can turn out to be more expensive. The additional expenditures to give a public school student the same probability of becoming an individual with high lever of human capital as a private school student may be enormous. Hence, a parent may prefer to send his child to private school and ignore the option of 'free' education. These are good explanations for the existence of private schools when public ones are available. To capture this possibility, we maintain from now on the earlier formulation in which the alternative to invest in education beyond $E_g$ is not possible.

**Definition 3:** A recursive competitive equilibrium with public schools, given the initial condition $q_0$, $q_0 \in (0,1]$, and $\{E_g\}_{t=0}^\infty$, is defined to be value functions $v : [0,1] \times \{x\} \rightarrow R$, policy functions $c_x : [0,1] \rightarrow R_+$, and $I_x : [0,1] \rightarrow R_+$, an economy-wide law of motion for the fraction of individuals with high level of human capital $Q : [0,1] \rightarrow [0,1]$, and factor prices functions $w_x : [0,1] \rightarrow R_+$ such that: ($x \in \{h, l\}$)

(i) $v$ satisfies the household problem (19);

(ii) $c_x$ and $I_x$ are the optimal policy functions for (19);
(iii) the wage functions are given by equations (8) and (9);
(iv) the economy-wide law of motion for $q$ is given by:

$$q' = \pi(I_h(q))(q - \lambda_h) + \pi(I_l(q))(1 - q - \lambda_l) + \pi(E_g)\lambda,$$

where $\lambda_x$ is the fraction of the young generation whose parents are type $x$ who goes to study in public schools, and $\lambda = \lambda_h + \lambda_l$.
(v) the government budget is in equilibrium:

$$\lambda E_g = [w_l(1 - q) + w_h q]r = yr.$$

The definition of stationary equilibrium with public schools is exactly equal to the one without it. Again, we confine our analysis to balanced growth equilibrium.

Note that if the government expenditures in education per pupil ($E_g$) is low enough, all parents will send their children to private school, i.e., $\lambda = 0$. Such equilibrium will be exactly equal to the one in chapter 4, without government intervention.

**Proposition 4:** With the introduction of public schools, there is an $E_g$ such that the fraction of skilled workers, $q$, is greater than the one in the equilibrium without public schools.

The above proposition says that, by introducing public school, the government can induce the whole society to invest a greater amount in education. It will lead to an improvement in the quality of the labor force (greater $q$) and, ultimately, to a better economic performance and less inequality. This will happen in spite of the distortion caused by the income tax. Private investments in education will be discouraged because of the negative wealth effect implied by the income tax. It will also be discouraged by the reduction in the differences in income between low and high types caused by the increase in the quality of the labor force.

**Proposition 5:** With the introduction of public schools, there is an $E_g$ such that the fraction of skilled workers, $q$, is lower than the one in the equilibrium without public schools.

Proposition 5 shows that the introduction of public schools will not necessarily move the economy to another stationary equilibrium with higher growth rate and less income inequality. It can actually move the economy to the opposite direction. The intuition behind this result runs as follows. The government introduces public schools and attracts only students from low-income families in a way that they would receive the same level of education as they had without the option of public schools. However, high-income
families would be discouraged to invest in education due to the negative wealth effect related to the income tax. The result is that the whole society would invest less in education. The consequence is a reduction in the quality of the labor force ($q$), lower economic growth and more inequality.

6.2 Numerical Example with Public Schools

To make the above points more concrete, we provide a few examples. The objective here is to calculate the steady state values of the utility levels of both types of agents as well as the fraction of workers with the high level of human capital in the stationary equilibria with different levels of government expenditures in public school. The next step is to compare them with the correspondent values in the equilibrium without public schools. Finally, the numerical examples are expected to support the theoretical results obtained in propositions 4 and 5.

The first equilibrium computed in chapter 5.2, without public schools, is used here as a benchmark. The steady state values of this equilibrium is reproduced in the first column of table 1. The same calibration specified in this benchmark case is used throughout this chapter.

The first experiment is to set government’s expenditures per pupil in public school to be 10% greater than the amount the low type agent spends in their child’s education in the benchmark case ($E_g = 1.1E_i^\text{public}$). The new steady state equilibrium is reported in the second column of table 1. The fraction of workers with high level of human capital ($q$) increases .08%, from .7103 to .7109. Consequently, the annual GDP per capita growth increases slightly from 2.1279% to 2.1299%. Moreover, both type of agents are better off as a result of this intervention. The utility levels of the high and low type agents augments from -.2875 and -.3408 to, respectively, -.2874 to -.3401. It indicates that this policy is Pareto improving.

In the second experiment, government’s expenditures per student in public school corresponds to 20% greater than the amount spent in education by the low type agent in the benchmark case ($E_g = 1.2E_i^\text{public}$). The results are reported in the third column of table 1. As in the first experiment, the fraction $q$ jumps. It goes from .7103 to .7118. The new annual GDP growth rate would be 2.1313%, greater than the rate 2.1279% without public schools. In addition, as in the first experiment, this greater increase in the quality of public schools is still Pareto improving in comparison with the benchmark case. The utility levels of the high and low type agents increases from -.2875 and -.3408 to, respectively, -.2874 to -.3400.

It is interesting to note that the equilibria resulted from the intervention in the first and second experiment are welfare comparable. The equilibrium obtained in the second experiment is Pareto improving in comparison with the equilibrium in the first experiment. The low type agent’s utility in-
creases from \(-.3401\) to \(-.3400\), whereas the high type agent’s utility remains unaltered at \(-.2874\).

The above experiments confirm, as expected, the theoretical result obtained in proposition 4: the introduction of public schools can move the economy to an equilibrium with higher growth rates and lower inequality. As discussed in the above in the theoretical examination, a greater \(q\) implies a lower Gini coefficient, i.e., less inequality.

Finally, we introduce public schools in which the government spends per student only 80\% of the amount spent in education by the low type agent in the equilibrium without the option of public schools. The results are reported in the last column of table 1. There is a reduction in \(q\). It drops from \(.7103\) to \(.7079\) in comparison with the benchmark case. This numerical example supports the theoretical result obtained in proposition 5. Low type agents would prefer, in this case, to take advantage of the ‘free’ public school system, invest less in their child’s human capital and benefit from more consumption. Their utility level would increase from \(-.3408\) to \(-.3406\). In contrast, high type agents would be worst off with a reduction in utility from \(-.2875\) to \(-.2876\).

Table 1

<table>
<thead>
<tr>
<th>(E_g)</th>
<th>(E_g = 0)</th>
<th>(E_g = (1.1)E^{ss}_I)</th>
<th>(E_g = (1.2)E^{ss}_I)</th>
<th>(E_g = (0.8)E^{ss}_I)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(q)</td>
<td>.7103</td>
<td>.7109</td>
<td>.7118</td>
<td>.7079</td>
</tr>
<tr>
<td>(v_l)</td>
<td>-.3408</td>
<td>-.3401</td>
<td>-.3400</td>
<td>-.3406</td>
</tr>
<tr>
<td>(v_h)</td>
<td>-.2875</td>
<td>-.2874</td>
<td>-.2874</td>
<td>-.2876</td>
</tr>
<tr>
<td>(\tau)</td>
<td>0</td>
<td>.0009</td>
<td>.001</td>
<td>.0007</td>
</tr>
</tbody>
</table>

6.3 Vouchers

Government decides to transfer resources in the form of vouchers (\(T\)) that can be used exclusively to finance education. The recipients of these vouchers can be restricted to a specific group in the society or not. Again, it imposes the same income tax rate (\(\tau\)) to all individuals to finance it.

A parent receives vouchers or not and decide how much to invest in his child’s education. He is not restricted to invest only the vouchers in his offspring’s education but he is obliged to invest at least that amount.\(^{29}\)

\(^{29}\)As it is assumed that public and private schools have the same probability function \(\pi\), it does not matter if we allow parents to use vouchers in public schools and private schools or only private schools. For simplicity we consider only the latter case.
The individual type $x$'s problem with vouchers can be summarized by the following dynamic problem:

$$v(q, x) = \max_{c_x, E_x} u(c_x) + \beta \psi(q)^{1-\sigma} \{(\pi(\mu_x T + E_x) v(q', h) + [1 - \pi(\mu_x T + E_x)] v(q', l))\}$$

such that:

$$C_x + E_x = w_x(1 - \tau)$$

and

$$E_x \geq \mu_x T.$$  

$\mu_x$ is an indicator function:

$$\begin{cases} 0, & \text{if the government does not supply vouchers to families whose parents are type } x; \\
1, & \text{if the government does supply vouchers to families whose parents are type } x. 
\end{cases}$$

**Definition 4:** A recursive competitive equilibrium with vouchers, given the initial condition $q_0$, $q_0 \in [0, 1]$, $\{T\}_{t=0}^\infty$, and $\{\mu_x\}_{t=0}^\infty$, is defined to be value functions $v : [0, 1] \times \{x\} \to R$, policy functions $c_x : [0, 1] \to R_+$, and $I_x : [0, 1] \to R_+$, an economy-wide law of motion for the fraction of individuals with high level of human capital $Q : [0, 1] \to [0, 1]$, and factor prices functions $w_x : [0, 1] \to R_+$ such that: ($x \in \{h, l\}$)

(i) $v$ satisfies the household problem (20);

(ii) $c_x$ and $I_x$ are the optimal policy functions for (20);

(iii) the wage functions are given by equations (8) and (9);

(iv) the economy-wide law of motion for $q$ is given by:

$$q' = \pi(I_h(q) + \mu_h T)(q) + \pi(I_l(q) + \mu_l T)(1 - q)$$

(v) the government budget is in equilibrium:

$$\mu_h T + \mu_l T = (qw_h + (1 - q)w_l)\tau = y\tau.$$  

The definition of stationary equilibrium with vouchers is exactly equal to the one without it. Again, we confine our analysis to balanced growth equilibrium. In order to prove the following proposition, we assume that $\mu_l = \mu_h = 1$.  

---

30In the next section, we consider the case in which $\mu_h = 0$ and $\mu_l = 1$. Only families from low-income background receive vouchers.
Proposition 6: With the introduction of vouchers, there is a $T$ such that the fraction of skilled workers, $q$, is greater than the one in the equilibrium without vouchers.

The results obtained with public schools are replicated with vouchers. It indicates that both types of intervention are equivalent, at least theoretically. As in the public school case, the government through vouchers can force the whole society to invest a greater amount in education. It will also lead to an improvement in the quality of the labor force (greater $q$) and, ultimately, to a better economic performance and less inequality.

6.4 Numerical Example with Vouchers

The analysis in this section is similar to the one performed in section 6.2. The objective here is to calculate the steady state values of the utility levels of both types of agents as well as the fraction of workers with the high level of human capital in the stationary equilibria with different levels of government expenditures in vouchers. Another objective is to compare them with the correspondent values in the equilibrium without vouchers. Finally, the numerical examples are expected to support the theoretical result obtained in proposition 6.

The results obtained in the benchmark case, without vouchers, is reproduced in the first column of table 2. As in the numerical example with public schools, the same calibration specified in this benchmark case is used throughout this chapter.

Two experiments are performed in this chapter. In the first one, the government distributes vouchers to all families with the face value 10% greater than the amount the low type agents spend in their child’s education in the benchmark case $[(1.1)E_l]$. In the second experiment the amount is 20% greater $[(1.1)E_l]^{20}$.

The results are reported, respectively, in the second and last columns of table 2.

In the first experiment, the fraction $q$ moves from .7103 to .7112. Thus, the annual GDP per capita growth rate increases from 2.1279% to 2.13%. As in the similar experiment with public schools, this introduction of the vouchers system is also Pareto improving. The utility levels of the high and low type of agents augments from $-.2875$ and $-.3408$ to, respectively, $-.2871$ and $-.3400$.

Similar qualitative results are obtained in the second experiment. The fraction $q$ moves from .7103 to .7120, representing an increase in the annual GDP growth rate from 2.1279% to 2.1318%. The utility level of both types

31 In the next section we analyse the effects of vouchers being distributed only to families with low income background.
of agents augments. It goes from \(-.2875\) to \(-.2870\) for the high type agents and from \(-.3408\) to \(-.3398\) for the low type agents.

The above experiments confirm, as expected, the theoretical result obtained in proposition 6: the introduction of a vouchers scheme can move the economy to an equilibrium with higher growth rates and lower inequality. As pointed out above, a greater \(q\) implies less inequality.

Table 2

<table>
<thead>
<tr>
<th>(T = 0)</th>
<th>(T = (1.1)E_t^{ss})</th>
<th>(T = (1.2)E_t^{ss})</th>
</tr>
</thead>
<tbody>
<tr>
<td>(q)</td>
<td>.7103</td>
<td>.7112</td>
</tr>
<tr>
<td>(v_l)</td>
<td>-.3408</td>
<td>-.3400</td>
</tr>
<tr>
<td>(v_h)</td>
<td>-.2875</td>
<td>-.2871</td>
</tr>
<tr>
<td>(\tau)</td>
<td>0</td>
<td>.0009</td>
</tr>
</tbody>
</table>

7 Welfare analysis

The analysis in the previous chapter indicates that government intervention through vouchers or public schools can improve economic performance and reduce inequality. Moreover, we were able to compare both type of agents' utility levels in different stationary equilibria characterized by different degrees of government intervention. In this chapter, we are again interested in comparing the welfare of both types of agents when the government intervenes, but we investigate it in a different way.

To analyze the welfare effects of changes in educational policies, we proceed in the following way. We consider equilibria in which the variables follow paths that are close to the values they would have in a stationary equilibrium. The stationary equilibrium considered is the benchmark case explored in the previous chapter. We then contemplate small disturbances in the variable under the government control. These disturbances are small enough such that these equilibria exist. The paths of \(v_l\) and \(v_h\) moving away from the stationary equilibrium and back indicate how the utility of both types of agents evolve when government alters its policy. The path of \(q\) shows the behavior of economic growth rate and income inequality.

Because these disturbances have to be small enough such that the variables follow paths close to the stationary equilibrium, we make the analysis only in the case in which the variable under the government control is the amount of vouchers. When the government introduces public schools, the necessary perturbation in the total government expenditures in order to make it attractive to at least the children from low income families is not
small enough. Therefore, we consider a stationary equilibrium in which the amount of vouchers is zero and change this amount slightly.

As a result of this perturbation, economic agents are expected to invest more in education than they would otherwise choose to. The costs of this intervention can be measured by a reduction in total consumption today. The benefits will come in the future through higher growth rates and income. The net benefit may be negative.

Several factors affect the net result. First, it depends on which rate economic agents value consumption tomorrow vis-à-vis consumption today. The greater is the discount factor ($\beta$), the more importance individuals give to the utility of future generations. Thus, the greater is $\beta$, the more likely households are to give up consumption today and allow the government intervention in order to increase consumption tomorrow. Second, the net benefit is more likely to be positive the more efficient is the government intervention in terms of increasing the quality of the labor force, as lower taxes would have to be imposed today. In our formulation, this efficiency is measured by the probability function $\pi$. Third, the benefits in the future are greater the more important is the external spillover, measured by $\psi$. This is the case because, given a certain level of government intervention, the increase in income tomorrow is positively related with the magnitude of the externality effect.

In order to characterize this type of equilibria, we consider a linearization of the system of equilibrium conditions in the competitive equilibrium. This linearization is around the stationary values of the variables that represent an equilibrium in the absence of disturbances, the benchmark case explored in the previous chapter. $^{32}$

The equations that characterize an equilibrium in which vouchers are supplied only to low income families are the following: (again, note that the subscript $t$ indicates time) $^{33}$

$$w_{l,t} = Aq_t^\alpha (1 - \alpha),$$

$$w_{h,t} = w_{l,t} + Aq_t^{\alpha-1},$$

$$[w_{h,t}(1 - \tau_t) - E_{h,t}]^{-\sigma} = \beta \psi(q_t)1^{1-\sigma} \pi_E(E_{h,t})[v_{h,t+1} - v_{l,t+1}],$$

$$[w_{l,t}(1 - \tau_t) - E_{l,t}]^{-\sigma} = \beta \psi(q_t)1^{1-\sigma} \pi_E(E_{l,t} + T_t)[v_{h,t+1} - v_{l,t+1}],$$

$^{32}$Woodford (1996) uses the same methodology in a different problem.

$^{33}$Recall that in the benchmark case $T = 0$. 

33
The linearized version of these equations are given respectively by:

\[ \tilde{v}_{h,t} = a_2 \tilde{q}_t \]
\[ \tilde{v}_{l,t} = a_1 \tilde{q}_t \]

\[ m_1 \tilde{w}_{h,t} + m_2 \tilde{\tau}_t + m_3 \tilde{E}_{h,t} = m_4 \tilde{q}_t + m_5 \tilde{v}_{h,t+1} + m_6 \tilde{v}_{l,t+1} \]
\[ \sigma_1 \tilde{w}_{l,t} + \sigma_2 \tilde{\tau}_t + \sigma_3 \tilde{E}_{l,t} = \sigma_4 \tilde{q}_t + \sigma_5 \tilde{v}_{h,t+1} + \sigma_6 \tilde{v}_{l,t+1} + \sigma_7 \tilde{T}_t \]
\[ \tilde{v}_{h,t} = n_1 \tilde{w}_{h,t} + n_2 \tilde{\tau}_t + n_3 \tilde{E}_{h,t} + n_4 \tilde{q}_t + n_5 \tilde{v}_{h,t+1} + n_6 \tilde{v}_{l,t+1} \]
\[ \tilde{v}_{l,t} = n_1 \tilde{w}_{l,t} + n_2 \tilde{\tau}_t + n_3 \tilde{E}_{l,t} + n_4 \tilde{q}_t + n_5 \tilde{v}_{h,t+1} + n_6 \tilde{v}_{l,t+1} + n_7 \tilde{T}_t \]
\[ \tilde{q}_{t+1} = s_1 \tilde{E}_{h,t} + s_2 \tilde{q}_t + s_3 \tilde{E}_{l,t} + s_4 \tilde{T}_t \]
\[ \tilde{\tau}_t = a_3 T_t + a_4 \tilde{q}_t + \]

where:

\[ a_1 = Aq^{a-1}(1 - \alpha) + A\alpha(\alpha - 1)q^{a-2}, \quad a_2 = A(1 - \alpha)aq^{a-1}, \]
\[ a_3 = \frac{1 - q}{Aq^a}, \quad a_4 = -\left(\frac{T + \tau Aq^{a-1}}{Aq^a}\right), \quad m_1 = \frac{-\sigma w_h(1 - \tau)}{w_h(1 - \tau) - E_h}, \]
\[
m_2 = \frac{\sigma w_h \tau}{w_h (1 - \tau) - E_h}, \quad m_3 = \frac{\sigma E_h}{w_h (1 - \tau) - E_h} - \frac{E_h \pi_E (E_h)}{\pi_E},
\]

\[
m_4 = \frac{(1 - \sigma) q \psi q}{\psi}, \quad m_5 = \frac{v_h}{v_h - v_l}, \quad m_6 = \frac{-v_l}{v_h - v_l},
\]

\[
o_1 = \frac{1}{w_l (1 - \tau) - E_l} - \frac{1}{E_l - E_l} - \sigma, \quad o_2 = -w_l[w_l (1 - \tau) - E_l]^{-\sigma},
\]

\[
o_3 = \frac{\beta \psi (q)^{1 - \sigma}}{v_h - v_l} \left[ \pi_E (E_l + T) - \frac{w_l (1 - \tau) - E_l}{w_h (1 - \tau) - E_h} \right] - \sigma,
\]

\[
o_4 = \frac{\beta (1 - \sigma) \psi}{\psi} \left[ \pi_E (E_l + T) v_h + (1 - \pi(E_l + T)) v_l \right],
\]

\[
o_5 = \frac{\beta \psi (q)^{1 - \sigma}}{v_h - v_l} \pi_T (E_l + T), \quad o_6 = \beta \psi (q)^{1 - \sigma} \left[ 1 - \pi(E_l + T) \right],
\]

\[
o_7 = \beta \psi (q)^{1 - \sigma} \left[ v_h - v_l \right] \pi_T (E_l + T),
\]

\[
n_1 = \frac{[w_h (1 - \tau) - E_h]^{-\sigma} (1 - \tau) w_h}{v_h}, \quad n_2 = \frac{-[w_h (1 - \tau) - E_h]^{-\sigma} \tau w_h}{v_h},
\]

\[
n_3 = \frac{-[w_h (1 - \tau) - E_h]^{-\sigma} E_h + \beta \psi (q)^{1 - \sigma} [v_h - v_l]}{\psi^\sigma v_h} \pi_E (E_h) E_h,
\]

\[
n_4 = \frac{(1 - \sigma) q \psi}{v_h} \left[ \pi (E_h) v_h + (1 - \pi (E_h)) v_l \right], \quad n_5 = \beta \psi^{1 - \sigma} \pi (E_h),
\]

\[
n_6 = \frac{\beta \psi^{1 - \sigma} (1 - \pi (E_h)) v_l}{v_h},
\]

\[
i_1 = \frac{-\sigma (1 - \tau)}{[w_l (1 - \tau) - E_l]}, \quad i_2 = \frac{\sigma w_l}{[w_l (1 - \tau) - E_l]},
\]

\[
i_3 = \frac{\sigma}{[w_l (1 - \tau) - E_l]} - \frac{\pi_E (E_l + T)}{\pi_E (E_l + T)} \pi_E (E_l + T), \quad i_4 = \frac{-\pi_E (E_l + T)}{\pi_E (E_l + T)},
\]

\[
i_5 = \frac{(1 - \sigma) \psi}{\psi}, \quad i_6 = \frac{1}{v_h - v_l}, \quad i_7 = \frac{-1}{v_h - v_l},
\]

\[
s_1 = \pi_E (E_h) q, \quad s_2 = \left[ \pi (E_h) - \pi (E_l + T) \right],
\]

\[
s_3 = (1 - q) \pi_E (E_l + T), \quad s_4 = (1 - q) \pi_T (E_l + T),
\]

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and \( \hat{w}_{i,t}, \hat{w}_{h,t}, \hat{q}_t, \hat{\tau}_t, \hat{E}_{h,t}, \hat{E}_{l,t}, \hat{v}_{h,t}, \hat{v}_{l,t}, \) and \( \hat{T}_t \) denote deviations of \( w_{i,t}, w_{h,t}, w_{l,t}, q_t, \tau_t, E_{h,t}, E_{l,t}, v_{h,t}, v_{l,t}, \) and \( T_t \) respectively from their stationary values. Note that their stationary values are denoted respectively by \( w_i, w_h, w_l, q, \tau, E_h, E_l, v_h, v_l, \) and \( T \) (i.e., without the time subscript).

Eliminating \( \hat{w}_{i,t}, \hat{w}_{h,t}, \hat{E}_{h,t}, \hat{E}_{l,t} \) and \( \hat{\tau}_t, \) we obtain a system of three difference equation system that can be written in vector form as:

\[
\begin{bmatrix}
\hat{v}_{h,t+1} \\
\hat{v}_{l,t+1} \\
\hat{q}_{t+1}
\end{bmatrix} = R \begin{bmatrix}
\hat{v}_{h,t} \\
\hat{v}_{l,t} \\
\hat{q}_t
\end{bmatrix} + S\hat{T}_t
\]

where:

\[
R = \begin{bmatrix}
(a_0 + o_3i_6) & (a_0 + o_3i_7) & 0 \\
-(s_1m_5 + s_3i_6) & -(s_1m_6 + s_3i_7) & 1 \\
(n_5 + n_3m_5) & (n_6 + n_3m_6) & 0
\end{bmatrix}^{-1} \begin{bmatrix}
0 & 1 & -r_4 \\
0 & 0 & r_5 \\
1 & 0 & r_6
\end{bmatrix},
\]

\[
S = \begin{bmatrix}
(a_0 + o_3i_6) & (a_0 + o_3i_7) & 0 \\
-(s_1m_5 + s_3i_6) & -(s_1m_6 + s_3i_7) & 1 \\
(n_5 + n_3m_5) & (n_6 + n_3m_6) & 0
\end{bmatrix}^{-1} \begin{bmatrix}
r_7 \\
r_8 \\
r_9
\end{bmatrix},
\]

and

\[
\begin{align*}
r_4 &= a_1a_2 + a_2a_4 + a_4 + \frac{a_3}{i_3}(i_5 - i_1a_2 - i_3a_4), \\
r_5 &= s_1\left(m_4 - m_1a_1 - m_2a_4\right) + s_2 + s_3\left(i_5 - i_1a_2 - i_3a_4\right), \\
r_6 &= n_1a_1 + n_2a_4 + n_4 + \frac{n_3}{m_3}(m_4 - m_1a_1 - m_2a_4), \\
r_7 &= a_2a_3 + a_7 - a_3(i_2a_3 + i_4), \\
r_8 &= s_4 - s_1m_2a_3 - s_3(i_2a_3 + i_4), \\
r_9 &= n_2a_3 - n_3m_2a_3.
\end{align*}
\]

The effects of a disturbance to the value of \( \hat{T} \) may usefully be illustrated by a numerical example. We use the same calibration and the steady state values of the benchmark case used in the previous chapter. In figures 3 through 5, impulse response functions are reported for an increase in the amount of vouchers from 0 to 0.02682. This last number corresponds to 20% of the amount spent in education by the low type agent in the stationary equilibrium without vouchers. One can observe that the high type agent’s utility (figure 3), the low type agent’s utility (figure 4), and the fraction of individuals with high level of human capital (figure 5) increase as a result of the government intervention. The conclusion is straightforward: a more equal access to education can lead to a higher growth rate, lower inequality and be Pareto improving.

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8 Conclusion

Market forces would not necessarily drive economies out of an equilibrium with very low economic growth rates and very unequal income distribution. Considerable luck might be needed in the magnitude of shocks to give a sufficiently push to investments in human capital to free an economy from a poverty trap with inequality. An alternative would be to canalize resources from the developed world to finance education in poor countries.

We obtained the result that the introduction of public schools or voucher system may reduce inequality and increase the economic growth. Members of the current generation would finance the government expenditures necessary to implement the educational policy. Economic agents would be induced to invest more in education with government intervention than they would otherwise choose.

The cost of this intervention could be measured by the reduction in total consumption today and, consequently, in current utility. The benefits would come only in the future through higher growth rate and income. The low income families benefit the most from the government intervention, either through vouchers or public schools. In our model, if high-income parents send their child to public school, then low-income parents do too, whereas the opposite is not true. In the case of vouchers, the advocated policy is to give it to low-income families, in particular if there are no capital markets to finance education. High-income families would be taxed to finance the educational program and would not have a direct benefit today. Nonetheless, we have shown that the implementation of these policies may be Pareto improving. In other words, both types of agents, with high and low level of human capital, may have their utility levels increased as a result of the introduction of vouchers or public schools.

However, the extension of these programs beyond a certain point may not be Pareto optimal anymore. In other words, it may benefit only the low income families and reduce the high income families' utility level, while still increasing the quality of the labor force and, ultimately, the economic growth rate. This possibility may explain why enough resources have not been devoted to raise the education of all citizens to decent levels in some developing countries. The introduction of a political economy framework in the above model would imply that the median voter would choose not to make the access to education more egalitarian if the fraction of skilled workers is greater than .5. In other words, he would prefer not to extend the government intervention.\textsuperscript{34} However, even if this fraction is lower than .5, \textsuperscript{34}See Alesina and Rodrik (1992) and Persson and Tabelini (1992) as examples of papers which consider the link between income distribution and economic growth in a political economy framework.
the extension of these policies would not necessarily be adopted. This would be the case if the political system is "biased against the poor due to a wealth-restricted voting franchise, unequal lobbying power, vote-buying or simply the fact that poor and less educated individuals have lower participation rates in elections." (See Bénabou, 1996, p.11)

Using the framework developed in this paper, one can interpret government expenditures in tertiary education in the following way. It would transfer resources to those individuals who receive a minimum amount of private investment in education, say, \( E \). This minimum amount would correspond to the costs of primary and secondary education. High-income families would be more likely to benefit from such government intervention as they could pay the price \( E \). The final outcome could be an overall reduction in the quality of the labor force because the intervention would make the access to the education more unequal. Nonetheless, high-income families could be made better off; they could have their utility level increased. If they had sufficient political power they could force the government to implement such policy instead of the ones discussed in chapter 6. It might explain the emphasis on tertiary education in detriment to primary and secondary education in Latin American countries in the last three decades. The result was lower economic growth and greater inequality.

The theoretical analysis indicates that public schools and vouchers system are alternative ways to obtain the same objective of increasing the quality of the labor force. It does not indicate, however, which alternative is the most efficient. Policymakers have, so far, given preference to the use of public schools. The vouchers system is rarely adopted. An evaluation of the most appropriate and efficient way to intervene is of fundamental importance. As more efficient this government intervention, the more likely it is to be Pareto improving.

It would be more realistic to incorporate a labor-leisure decision in the household's problem. With this additional feature, the imposition of the income tax to finance the school policy would not only inhibit private investment in human capital but it would also decrease the labor supply. As a result, the quality of the labor force would increase but the number of hours spent in the production process would be reduced. Therefore, the incorporation of this additional feature in the model would weaken the efficacy of the government intervention.\(^{35}\)

---

\(^{35}\)See Chou and Talmain (1996) as an example of an endogenous growth model in which the labor supply determines the economic growth rate.
9 Appendix

9.1 Proposition 1

Under assumption 3, \( V(z, q, x) = z^{1-\sigma} v(q, x) \), where:

\[
v(q, x) = \max_{E_x} \left( \frac{(w_z - E_z)^{1-\sigma}}{1-\sigma} + \beta \psi(q)^{1-\sigma} \{ \pi(E_z)v(q', h) + [1 - \pi(E_x)]v(q', l) \} \right).
\]

**Proof.** Using assumption 3, and the individual's budget constraint in the individual's dynamic problem, we have:

\[
V(z, q, x) = \max_{E_x} \left( z^{1-\sigma} \frac{(w_z - E_z)^{1-\sigma}}{1-\sigma} + \beta \psi(q)^{1-\sigma} \{ \pi(E_z)v(q', h) + [1 - \pi(E_x)]v(q', l) \} \right)
\]

Using equation (4) in the above equation we get:

\[
V(z, q, x) = \left( z^{1-\sigma} \right) \left( \max_{E_x} \left( \frac{(w_z - E_z)^{1-\sigma}}{1-\sigma} + \beta \psi(q)^{1-\sigma} \{ \pi(E_z)v(q', h) + [1 - \pi(E_x)]v(q', l) \} \right) \right)
\]

9.2 Proposition 2

Under assumptions 1-4, there exists a unique value function \( v \) (that is bounded and continuous) satisfying (6). For each \( q \), the maximum in (6) is attained by a unique value \( I_x(q) \), and the policy function \( I \) is continuous.

**Proof.** The proof of this proposition is a straightforward application of theorem 4.6 in Stokey, Lucas and Prescott (1989).

9.3 Proposition 3

With assumptions 1, 3-5, if \( \sigma = \theta \), then there exists at least one balanced-growth equilibrium.

**Proof.** In order to prove the existence of a stationary equilibrium, one has to show that the system of six equations and six unknowns has a solution.

Dividing equation (15) by (14), we get:

\[
\frac{\pi_E(E_h)}{(w_h - E_h)^{-\sigma}} = \frac{\pi_E(E_l)}{(w_l - E_l)^{-\sigma}}.
\]

Using assumption 5 and imposing the restriction \( \sigma = \theta \) in the above equation, we obtain:
The imposition of the restriction \( \sigma = \theta \) implies that both types of individuals spend the same fraction of their income in their child’s education.

Using this result in equations (14) (or equivalently in equation (15)) and (18) and again imposing the restriction \( \sigma = \theta \), we obtain, respectively:

\[
\left( \frac{r}{1 - r} \right)^\sigma = k_1(1 - \sigma) \beta \psi(q)^{1-\sigma}(v_h - v_l),
\]

and

\[
r = \left( \frac{1}{k_1(w_h^{1-\sigma} + w_l^{1-\sigma} \frac{1-q}{q})} \right)^{\frac{1}{1-\sigma}},
\]

where: \( k_1 = \frac{k}{A^{1-\tau}} \).

Subtracting (16) by (17), we get:

\[
v_h - v_l = \frac{(1 - r)^{1-\sigma}(w_h^{1-\sigma} - w_l^{1-\sigma})}{(1 - \sigma)(1 - \beta \psi(q)^{1-\sigma} k_1 r^{1-\sigma}(w_h^{1-\sigma} - w_l^{1-\sigma}))}.
\]

Using (23) in (21), we obtain:

\[
\frac{1}{\beta \psi(q)^{1-\sigma}} = \frac{k_1}{r^2} (w_h^{1-\sigma} - w_l^{1-\sigma}).
\]

Using equations (12), (13), and (22) in (24) reduce the seven equations system into one equation with only one unknown, \( q \):

\[
\frac{1}{\beta \psi(q)^{1-\sigma}} = \frac{k_1}{r(q)^{\beta}} [w_h(q)^{1-\sigma} - w_l(q)^{1-\sigma}].
\]

The right-hand side (\( RHS \)) of equation (25) is the ratio of the current value of the marginal benefit of investment in education to the marginal benefit of consumption. It can be easily proved that: (i) \( RHS \) is continuous in \( q \), (ii) \( \lim_{q \to 0} RHS(q) = +\infty \), and (iii) \( \frac{dRHS}{dq} < 0 \).

The denominator of the left-hand side (\( LHS \)) of (25) is equal to the discount factor applied to the marginal benefit of investment. Using assumptions 1 and 4, we have: (i) \( LHS \) is continuous in \( q \), (ii) \( LHS(q = 0) = \frac{1}{\beta} \), and (iii) \( LHS(q = 1) > 1 \).

In order to complete the proof, we just have to show that \( RHS(q = 1) < 1 \). It is easy to check that: \( RHS(q = 1) = k_1 \beta \left[ 1 - (1 - \alpha)^{1-\sigma} \right] < 1 \).
9.4 Proposition 4

With the introduction of public schools, there is an $E_g$ such that the fraction of skilled workers, $q$, is greater than the one in the equilibrium without public schools.

Proof. Let $E_h^*$ and $E_l^*$ and $q^*$ be, respectively, the investment in education of parent type $h$ and $l$, and the fraction of the population with high level of human capital in a stationary equilibrium without public schools. Set $E_g$ to be such that: (i) $E_g \leq q^*E_h^* + (1 - q^*)E_l^*$ and (ii) $\pi(E_g) > q^*\pi(E_h^*) + (1 - q^*)\pi(E_l^*) = q^*$. (i) guarantees that it is feasible for the government to impose such level of education in public school. Note that all low type agents will choose to send their child to public school as $E_g > E_l^*$ and $\tau > 0$. There are two possible types of stationary equilibrium. In the first one, high type agents also send their children to public school. In this case, $q_{new} = \pi(E_g) > q^*$, where $q_{new}$ is the fraction of the population with high level of human capital in the stationary equilibrium with public school. In the second one, high type agents send their children to private school because they prefer to spend beyond $E_g$ in their child’s education. In this case, $q_{new} > \pi(E_g) > q^*$.  

9.5 Proposition 5

With the introduction of public schools, there is an $E_g$ such that the fraction of skilled workers, $q$, is lower than the one in the equilibrium without public schools.

Proof. Let $E_h^*$ and $E_l^*$ and $q^*$ be, respectively, the investment in education of parent type $h$ and $l$, and the fraction of the population with high level of human capital in a stationary equilibrium without public schools. Set $E_g = E_l^*$. Note that this level of public education is feasible as to be such that: (i) $E_l^* < q^*E_h^* + (1 - q^*)E_l^*$. Note that all low type agents will choose to send their child to public school as $E_g > E_l^*$ and $\tau > 0$. There are two possible types of stationary equilibrium. In the first one, high type agents also send their children to public school. In this case, $q_{new} = \pi(E_g) < q^*$, where $q_{new}$ is the fraction of the population with high level of human capital in the stationary equilibrium with public school. In the second one, high type agents send their children to private school. In this case, they will invest less in education in comparison with an environment without public school as $\tau > 0$. Hence, $q_{new} < q^*E_h^* + (1 - q^*)E_l^* = q^*$.  

9.6 Proposition 6

With the introduction of vouchers, there is a $T$ such that the fraction of skilled workers, $q$, is greater than the one in the equilibrium without vouchers.
Proof. The proof is analogous to the one in proposition 4. ■
References


