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Investment, credit and endogenous cycles

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Abstract: In a general equilibrium Ramsey type model with heterogeneous agents we study the conditions for which a credit restriction can be a source of endogenous cycle to credit and capital.

Keywords: business cycles; investment; credit.

JEL Classification : E12, E32

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Investment, credit and endogenous cycles

1. Introduction

In the current literature there are two different approaches to deal with the relationship between money and cycles. The first considers exogenous shocks according to the real business cycle (rbc) theory. These shocks are studied in four different ways²: i) shocks in the financial industry (King and Plosser, 1984); ii) cash injections in the cash in advance restriction (Cooley and Hansen, 1989); iii) cash injection in the financial intermediaries and different timing between agents' decisions and the occurrence of shocks (Christiano, 1994) and iv) monetary shocks in imperfect financial markets (Scheinkman and Weiss, 1986).

The second approach is close to the keynesian tradition that assumes that economic fluctuations arise endogenously from the inherent instability of decentralized decision-making in market economies (Foley, 1992). One of the main differences between both approaches are the tools used to generate cycles. Technically the models in the rbc tradition are developed from micro founded dynamic models to be numerically tested by calibration and have some empirical support (Kydland and Prescott, 1996). In the endogenous business cycle the models are mainly theoretical and use qualitative differential equations theory to solve them. The application of the Poincaré-Bendixson (e.g., Jarsulic, 1988, Gonzalez-Calvet and Sanchez-Choliz, 1994) and the Hopf bifurcation (e.g., Kiefer, 1996) theorems are common tools used to create limit cycles in these models. Just recently have micro founded models emerged in the second tradition (Foley, 1992; Asada and Semmler, 1995).

Our paper presents a general equilibrium dynamic Ramsey type model that can generate endogenous cycle. We assume two different representative agents, borrowers and lenders, as in Mossetti (1990), and financial intermediaries with inside and outside money as in Ohkusa (1993). We investigate under which conditions this model presents a cyclical relationship between investment and credit using the method developed by Feichtinger, Novak and Wirl (1994). The sources of endogenous fluctuations in this model come from a credit restriction in the representative borrower problem. The paper is structured as follows. Next section presents the basic model. Then in section 3 we show how endogenous cycle can be achieved and in section 4 we conclude.

2. The Model

We assume that the representative lender derives utility from consumption (c) and money holdings (m). Her budget constraint is given by the difference between the net revenue of deposits in banks $(i - \frac{\pi}{100})D$, (where D is the real amount of deposits, i is the nominal interest rate paid by deposits and $\frac{\pi}{100}$ is the inflation rate), and consumption and inflationary tax $(\frac{\pi}{100}m)$. The problem of the representative lender is the following:

$$\underline{\underline{Max_{c,m} \int_0^{\infty} U(c,m) e^{-\rho t} dt}} \quad (P1)$$

$$\underline{\underline{\dot{D} + \dot{m} = (i - \pi) D - c - \pi m}}$$

² For a exhaustive survey on rbc and money, see Van Els (1995).

where r is the intertemporal rate of preference. Notice that representative lender allocates in its portfolio inside and outside money.

The representative borrower maximizes a flow of discounted consumption (c') subject to a dynamic budget constraint and to a credit restriction in which part of his loans (sL), if s is defined in the interval $(0, 1]$, finance his investment (dk/dt), adjustment costs $[C(dk/dt, k)]$, and consumption decisions. In the case in which $s > 1$, the credit restriction says that part of his expenditures is financed by loans, so the rest is financed by retained profits. His budget constraint corresponds to the allocation of the production ($f(k)$) and a new loan (dL/dt) in consumption, investment, adjustment costs, and the payment of his debt iL . Inflation represents a new source of income to the borrowers, since the real value of their debt decreases with it. We assume, as Mossetti (1990), that representative borrower does not hold outside money. The problem of the representative borrower is the following:

$$\underline{\underline{Max_{c'} \int_0^{\infty} U(c') e^{-rt} dt}} \quad (P2)$$

$$\underline{\underline{\dot{k} - \dot{L} = f(k) - c' - (i - \pi)L - \delta k - C(\dot{k}, k)}}$$

$$\underline{\underline{sL \geq c' + \dot{k} + C(\dot{k}, k)}}$$

where L is the real amount of loans, δ is the depreciation rate and i is the rate of interest paid for the loan.

We can rewrite the problem (P2) by making : (1) $\dot{k} = I$. This transformation makes the borrower problem looks like a firm's problem restricted by the availability of credit. Using (1) in the second restriction of (P2) yields: (2) $c' = sL - I - C(I, k)$,

since the inequality in this restriction reduces to an equality due to the insatiability of consumers. Defining the adjustment costs as an increasing function of the ratio of investment to capital yields ³: (3) $C(I, k) = \alpha I k^{-1}$. Inserting (1), (2) and (3) in problem (P2) yields:

$$\underline{\underline{Max_I \int_0^{\infty} U(sL - I(1 + \alpha k^{-1})) e^{-rt} dt}} \quad (P3)$$

$$\underline{\underline{\dot{L} = (s + i - \pi) L + \delta k - f(k)}}$$

$$\underline{\underline{\dot{k} = I}}$$

To close the model we have two equilibrium conditions for the financial intermediaries. They lend deposits $(1 - \frac{\theta}{\underline{\quad}}) D$, where $\frac{\theta}{\underline{\quad}}$ is the preparation rate, plus non-stochastic cash injection $Z(m)$ from the monetary authority⁴, which is the way outside money enters into the economy⁵:

$$L = Z(m) + (1 - \frac{\theta}{\underline{\quad}}) D \quad (4)$$

We assume that all deposits and loans clear up:

$$L = D \quad (5)$$

Solving problems (P1) and (P3), where we assume a CRRA utility function, in

the case of the representative borrower we have⁶: $\underline{\underline{U(\bullet) = \frac{(sL - I(1 + \alpha k^{-1}))^{1-\sigma}}{1-\sigma}}}$.

Taking the equilibrium conditions for the financial sector, we obtain a system of eight

³ For the case of desinvestment, we can assume that eq. (3) is a module function, which is a weakly convex function (Das, 1991).

⁴ The function $Z(m)$ can be associated to the seignorage from money.

⁵ See Christiano (1994) and Ohkusa (1993).

⁶ Despite the use of a linear adjustment cost function, the CRRA yields the necessary conditions for an optimum as in Lucas (1967).

equations to eight unknowns: I, L, D, i, k, q, c, m , where q is the costate variable associated with k in the problem (P3):

$$I = 0 \quad (6)$$

$$i = r + \frac{\pi}{\sigma} \quad (7)$$

$$L = D \quad (8)$$

$$L = Z(m) + (1 - \frac{\theta}{\sigma}) D \quad (9)$$

$$c + im = (i - \frac{\pi}{\sigma})(D + m) \quad (10)$$

$$\underline{U_m} = i \underline{U_c} \quad (11)$$

$$f(k) = L(s + i - \frac{\pi}{\sigma}) + \frac{\delta}{\sigma} k \quad (12)$$

$$\underline{L} = (1 + \alpha k^{-1})^{\frac{1}{\sigma}} q^{\frac{-1}{\sigma}} s^{-1} \quad (13)$$

This system is block recursive: equation (6) determines optimal I and equation (7) determines optimal i . Equation (7) is the Fisher equation, obtained from the first order conditions in the problem of representative lender. From equations (8), (9), (10) and (11) the optimal values of L, D, m and c are simultaneously determined. This allows us to determine the optimal value of k by equation (12) With the optimal k we obtain the optimal q from equation (13). Notice that credit and money market are the first to adjust in this model, followed by the goods market.

3. The Cycle

The endogenous cycle arises from the analysis of problem (P3). Following Feichtinger, Novak and Wirl (1994) it is enough to show that the signals of :

$$\det J = \begin{vmatrix} \frac{\partial \dot{L}}{\partial L} & \frac{\partial \dot{L}}{\partial k} & \frac{\partial \dot{L}}{\partial \lambda} & \frac{\partial \dot{L}}{\partial q} \\ \frac{\partial \dot{k}}{\partial L} & \frac{\partial \dot{k}}{\partial k} & \frac{\partial \dot{k}}{\partial \lambda} & \frac{\partial \dot{k}}{\partial q} \\ \frac{\partial \dot{\lambda}}{\partial L} & \frac{\partial \dot{\lambda}}{\partial k} & \frac{\partial \dot{\lambda}}{\partial \lambda} & \frac{\partial \dot{\lambda}}{\partial q} \\ \frac{\partial \dot{q}}{\partial L} & \frac{\partial \dot{q}}{\partial k} & \frac{\partial \dot{q}}{\partial \lambda} & \frac{\partial \dot{q}}{\partial q} \end{vmatrix} \quad (\text{A.1})$$

$$\Omega = \begin{vmatrix} \frac{\partial \dot{L}}{\partial L} & \frac{\partial \dot{L}}{\partial \lambda} \\ \frac{\partial \dot{\lambda}}{\partial L} & \frac{\partial \dot{\lambda}}{\partial \lambda} \end{vmatrix} + \begin{vmatrix} \frac{\partial \dot{k}}{\partial k} & \frac{\partial \dot{k}}{\partial q} \\ \frac{\partial \dot{q}}{\partial k} & \frac{\partial \dot{q}}{\partial q} \end{vmatrix} + 2 \begin{vmatrix} \frac{\partial \dot{L}}{\partial k} & \frac{\partial \dot{L}}{\partial q} \\ \frac{\partial \dot{\lambda}}{\partial k} & \frac{\partial \dot{\lambda}}{\partial q} \end{vmatrix} \quad (\text{A.2})$$

and a condition to determine the value of the bifucation parameter:

$$\det J = (\Omega / 2)^2 + r^2 (\Omega / 2) \quad (\text{A.3})$$

are positive when calculated with the optimal solutions of L , k , λ and q in order that the matrix J has a pair of purely imaginary eigenvalues . The value of λ is determined from the first order conditions to problem (P3), where λ is the costate variable associated with the state variable L . The novelty here is in the use of the solutions to the overall system instead of the partial equilibrium solutions to problem (P3)⁷.

From the first order conditions of problem (P3) we have:

$$I = [sL - (1 + \alpha k^{-1})^{\frac{1}{\sigma}} q^{\frac{-1}{\sigma}}] (1 + \alpha k^{-1})^{-1} \quad (14)$$

$$\dot{\lambda} = \lambda (r - s - i + \pi) - [sL - I(1 + \alpha k^{-1})]^{-\sigma} s \quad (15)$$

$$\dot{q} = rq - \lambda (\delta - f_k) - [sL - I(1 + \alpha k^{-1})]^{-\sigma} (I \alpha k^{-2}) \quad (16)$$

⁷ This is possible with a special equivalence condition. It is enough to show that the general optimal solution to q (k or L) is equal to the partial optimal solution to q (k or L). If one of these equalities is achieved, the others are easily obtained. The proof is available from the authors upon request.

from (1) and (14) in the steady-state we obtain equations (6) and (10). In the steady-state (15) or (16) determines the optimal value of λ .

Consider the following two inequalities:

$$\text{I) } \underline{(f_k - \delta)^2 (sL)^{-\sigma-1} s^2 I_q > sr(I_k(s+r) + (f_k - \delta)I_L)}$$

$$\text{II) } \underline{I_k r > s(s+r)}$$

where $\underline{I_x}$ denotes the partial derivative of I in relation to $x = q, L, k$.

If inequalities I) and II) calculated with the optimal solutions of the model (6)-(13) are preserved and the bifurcation parameter $\underline{\alpha}$ calculated from (A.3) is positive for the same optimal solutions then there is a limit cycle, by the Hopf bifurcation theorem, between loans $(L)^8$ and capital (k) in the economy described by problems (P1), (P3) and the equilibrium conditions (4) and (5).

We can see this result noticing that the inequality I) yields $\det J > 0$, and inequality II) yields $\underline{\Omega} > 0$. And if by (A.3) the bifurcation parameter is positive, these three conditions are necessary such that the matrix J possesses a pair of purely imaginary eigenvalues. These fulfill the conditions for the existence of a limit cycle by the Hopf bifurcation theorem (Feichtinger, Novak and Wirl (1994)). This result holds true for specific values of parameters, using a Cobb-Douglas production function:

$\underline{f(k) = A k^\beta}$, and assuming: $A = 0.25$, $Z = 5$, $r = 0.02$, $s = 0.015$, $\underline{\theta} = 0.5$, $\underline{\sigma} = 0.6$, $\underline{\delta} = 0.02$, $\underline{\beta} = 0.7$, we have positive signals for $\det J$ and $\underline{\Omega}$ and $\alpha = 1.5$. If $\underline{\sigma} = 1.1$ and $s = 0.01$ and keeping the other parameters values we obtain positive signals for $\det J$ and $\underline{\Omega}$ and $\underline{\alpha} = 1.5$.

⁸ As $L = D$ in the general equilibrium, then the cycle is between credit and investment.

The cyclical behavior has two sources in this model. The first results from the positive externality due to L in the objective functional of problem (P3), which allows k to differ from the optimal k from the modified golden rule. The modified golden rule can be obtained by solving problem (P2) after dropping the adjustment cost and the credit restrictions from it.

The second source of cycles is related to the penalization of changes in the investment. The main mechanism to guarantee this penalty is given by the impact of k on I . When the impact is positive this implies that the penalty is high. We can see this by inequality II), that demands a positive effect of k on I to be verified. Notice that this effect is only possible given the dependence of the adjustment costs on k . It is easy to see that an increase in k decreases adjustment costs and it stimulates the investment.

It is important to notice that both sources of cycles in our model come from the credit restriction in the problem of the representative borrower.

4. Conclusions

Our general equilibrium set up with heterogeneous representative agents in the Ramsey framework gives us the mechanics of the endogenous cycle between capital and credit. In the problem of the representative borrower the presence of a positive externality given by the existence of L in the objective functional, and a penalty for changes in the investment associated with capital are the main features of our model which suffice to generate an endogenous cycle for specific values of the parameters. These two features come from the inclusion of a credit restriction in the borrower problem. The cycle is still consistent with optimal choices from the representative

lender and from the financial intermediaries, and it shows how in a general equilibrium set up it is possible to have a cyclical pattern between capital and credit.

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