Comparative Advantage, Heterogeneous Firms and Variable Mark-ups
Comparative Advantage, Heterogeneous Firms and Variable Mark-ups

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COMPARATIVE ADVANTAGE, HETEROGENEOUS FIRMS AND VARIABLE MARK-UPS.

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Abstract

We develop a model of comparative advantage with monopolistic competition, that incorporates heterogeneous firms and endogenous mark-ups. We analyse how these features vary across countries with different factor endowments, and across markets of different size. In this model we can obtain trade gains via two channels. First, when we open the economy, most productive firms start to export their product, then, they demand more producing factors and wages rises, thus, those firms that are less productive will be forced to stop to produce. Second channel is via endogenous mark-ups, when we open the economy, the competition gets “tougher”, then, mark-ups falls, thus, those firms that are less productive will stop to produce. We also show that comparative advantage works as a “third channel” of trade gains, because, all trade gains results are magnified in comparative advantage industry of both countries. We also make a numerical exercise to see how endogenous variables of the model vary when trade costs fall.

KEYWORDS: International Trade. Comparative Advantage. Heterogeneous Firms. Variables Mark-ups.
# Contents

1 Introduction 7

2 Closed Economy 9
   2.1 Consumption 10
   2.2 Profit Maximizing Price 11
   2.3 Production 11
   2.4 Cutoff Productivity Levels 12
   2.5 Market Clearing 14

3 Costly Trade 15
   3.1 Cutoff Productivity Levels 17
   3.2 Market Clearing 20

4 Results in the Costly Trade Equilibrium 21

5 Numerical Example 25

6 Conclusion 32

7 Appendix 36

## List of Figures

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Zero-profit and export cut-offs</td>
<td>27</td>
</tr>
<tr>
<td>2</td>
<td>Average productivity</td>
<td>27</td>
</tr>
<tr>
<td>3</td>
<td>Average firm output</td>
<td>28</td>
</tr>
<tr>
<td>4</td>
<td>Differences in comparative advantage</td>
<td>28</td>
</tr>
<tr>
<td>5</td>
<td>Exporting probability</td>
<td>29</td>
</tr>
<tr>
<td>6</td>
<td>Mass of firms (domestic varieties)</td>
<td>29</td>
</tr>
<tr>
<td>7</td>
<td>Mass of entrants</td>
<td>30</td>
</tr>
<tr>
<td>8</td>
<td>Welfare evaluation</td>
<td>30</td>
</tr>
<tr>
<td>9</td>
<td>Weighted average mark-ups</td>
<td>32</td>
</tr>
</tbody>
</table>
1 Introduction

Since Melitz (2003), many authors have used his model of heterogeneous firms as a starting point to their own models. By using Melitz’s model, researchers could obtain results of trade gains by two channels. In one hand, as in Melitz (2003), trade induces competition for scarce labor resources as real wages are bid up by the relatively more productive firms who expand production to serve export markets. Bernard et al. (2007) obtained trade gains by this channel, but their model also focus in comparative advantage, which magnifies trade gains at comparative advantage industry. On the other hand, in Melitz and Ottaviano (2008) and in Rodriguez-Lopez (2011), import competition increases competition in the domestic product market, shifting up residual demand price elasticities for all firms at any given demand level. However, increased factor market competition plays no role in those models.

The differences in the origin of trade gains between those models is in the utility of the consumers. The first group, which follows Melitz (2003), uses CES utility and it does not permit price elasticity to vary. When it does vary we have endogenous mark-ups, what permits trade gains via the second channel, which is done by the second group. The main contribution of this paper is to put these two channels together into a unique framework. Thus, our model permits trade gains via both channels. We analyse international trade in an environment that allows endogenous mark-ups and yet have two industries and comparative advantage effects. We find in one model many results that were before found separately.

We develop a monopolistic competitive model of trade with heterogeneous firms, two industries, and endogenous mark-ups. This environment allows our model to obtain trade gains by both channels already mentioned. Firm heterogeneity is introduced similarly to Melitz (2003), by productivity differences. We introduce endogenous mark-ups using translog expenditure functions in the demand side. This technique was introduced in Feenstra (2003), used in Arkolakis et al. (2010), and in Rodríguez-López (2011). Translog expenditure functions are useful because it permit price elasticity to vary, differently from CES, although, preference still homothetic. We follow basically three papers, Bernard, Redding and Schott (2007), Melitz and Ottaviano (2008) and Rodriguez-Lopez (2011).

As in Melitz and Ottaviano (2008), in our model, market size and trade affect the toughness of competition in a market, which then feeds back into the selection of heterogeneous producers and exporters in that market. The fact that we have two industries also influences this selection. We are able to find similar results to those found in Bernard, Redding and Schott (2007) with the advantage that in our model the mark-ups are free to vary. Although, our model remains tractable.

First we develop a closed-economy version of our model. As in Melitz and Ottaviano (2008), differently from Melitz (2003), the market size induces important changes in the equilibrium distribution of firms and their performance measures. Larger markets require higher productivity cutoffs. Larger markets also has firmsth that are larger and earn higher profits. We then present the open-economy in costly trade.

When the economy moves from autarky to costly trade we show that larger markets still exhibit larger and more productive firms, lower prices, and lower mark-ups. We also

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1 Other international trade models also incorporate heterogeneous firms, Bernard et al. (2003); Helpman et al. (2004); Yeaple (2005).

2 Asplund and Nocke (2006) investigate the effect of market size on the entry and exit rates of heterogeneous firms.
find the same results found in Bernard, Redding and Schott (2007). When countries move simultaneously from autarky to costly trade, firms export opportunities increase. It promotes greater entry from the competitive fringe. However, the mass of domestic producers gets lower in both countries. Most productive firms start to sell at the export market. The proportion of firms exporting will be higher in comparative industry. The productivity cut off necessary to produce increases and the most productive firms start to export, inducing aggregate productivity level in each industry to grow. This result is higher in comparative advantage industry and intensifies its ex ante comparative advantage, which raises trade gains. After opening the market, all firms set smaller mark-ups, an effect that is higher in firms from comparative advantage industry. However, average mark-ups in both industries do not change. These findings contrast with the homogeneous-firm imperfect competition model of Helpman and Krugman (1987), where industry productivity remains constant and, depending on the value of fixed and variable trade costs, either all or no firms export, when there is trade liberalization.

When we take a look into distributional implications, our framework is able to find the famous Stoper-Samuelson result, but also, there is an other effect. In consequence of aggregate productivity growth, average price of the variety is reduced in each industry and thereby elevates real income of both factors. Thus, even if the real wage of the scarce factor falls during opening trade, its decline is less than it would be in a Neoclassical setting. The falling of all mark-ups of those remaining firms reduces the price for each variety, but, once average mark-up is constant, there is no effect on average prices.

As in Bernard, Redding and Schott (2007), our approach also generates predictions about the impact of trade liberalization on job turnover that are different from those obtained in a Neoclassical model. We show that a reduction in trade barriers encourages simultaneous job creation and job destruction in all industries, but gross and net job creation vary with country and industry characteristics.

Differently from Bernard et al. (2007), we have closed functions for all endogenous variables. Using these facility of the model, we constructed a numerical exercise, where we calibrate our model to an symmetric environment and vary trade costs, then we analyse how endogenous variables vary. In this numerical exercise, we calculate a weighted average mark-up, thus it vary when trade costs change. We found that when competition gets “tougher”, weighted average mark-up fall, as average mark-up in Melitz and Ottaviano (2008).

All models using endogenous mark-ups generate the equilibrium property that more productive firms charge higher mark-ups. Bernard et al. (2003) also incorporate firm heterogeneity and endogenous mark-ups in their model. However, the distribution of mark-ups is invariant to country characteristics and to geographic barriers. Melitz and Ottaviano (2008) develop a non-degenerate distribution of mark-ups, that depends on country characteristics and on geographic barriers, but they have only one industry and there are no effects from comparative advantage. They analyse asymmetric trade liberalization scenarios. Rodriguez-Lopez (2011) presents a sticky-wage model of exchange rate pass-through with heterogeneous producers and endogenous mark-ups. Arkolakis, Costinot and Rodriguez-Clare (2010) provide a simple example with Translog Expendi-

---

3Empirical studies strongly confirm these selection effects of trade (only most productive firms export). For example, see Clerides et al. (1998); Bernard and Bradford Jensen (1999); Aw et al. (2000); Pavcnik (2002); and Bernard et al. (2006).

4Other works on imperfect competition and comparative advantage include Krugman (1981); Helpman (1984); Markusen and Venables (2000).
ture Functions and Pareto distribution of firm-level productivity, they have an environment close to ours, where mark-ups vary. However, they also have only one industry in their model. In this paper we focus on the effects that variable mark-ups cause in an economy when it moves from closed to open economy. Not only has our model two industries, but we also analyse the effect of endogenous mark-ups in this environment with advantage comparative. Our approach permits us to find closed-solutions for all endogenous variables of the equilibrium.

Zhelobodko et al. (2012) propose a model of monopolistic competition with additive preferences and variable marginal costs. They use the concept of “relative love for variety” (RLV) to provide a full characterization of the free-entry equilibrium. In their work they show that when we use CES utility, as the elasticity of substitution, RLV will also be constant, it is a specific case where prices and mark-ups are not affected by firm entry and market size, but they are interested in those cases where RLV are free to vary. Our model goes in this direction and our results are corroborate by those results from Zhelobodko et al. (2012) when RLV increases.

In Arkolakis et al. (2012), the authors analyse if micro-level data have had a profound influence on research in international trade over the last years. They found that, although models have become more detailed, the amount of welfare gains did not change so much when compared with those obtained by simpler models, like the Armington model. They made some assumptions like CES utility that we do not use in our model, but Arkolakis et al. (2010) shows that the main result holds in a model similar to that used in Rodríguez-López (2011). However, in Arkolakis, Costinot, Donaldson, and Rodríguez-Clare (2012), the authors study the pro-competitive effects of international trade in models that allow variable mark-ups, including models with the continuous translog expenditure function. They find out that gains from trade liberalization are weakly lower than those predicted by the models with constant mark-ups considered in Arkolakis et al. (2012). They argument that “there is incomplete pass-through of changes in marginal costs from firm to consumers”.

Finally, our model could be a more useful benchmark than the existing theory for predicting the pattern of trade. The Neoclassical standard Heckscher-Ohlin-Vanek model presents poor empirical performance, because it does not capture the existence of trading costs, the factor price inequality, and the variation in technology and productivity across countries\(^5\).

The remainder of the paper is structured as follows. Section 2 introduces the model for closed economy. Section 3 expand the model to a costly trade economy. In section 4, we present the results that we find when the economy moves from autarky to costly trade. Section 5 present a numerical exercise, and section 6 concludes the study.

## 2 Closed Economy

In this section we will solve the model for the case that the economy is closed, so there are no exporters in this section. Consider an economy with \(L\) consumers.

\(^5\)See, among others, Bowen et al. (1987); Trefler (1993); Trefler (1995); Davis and Weinstein (2001); Schott (2003), Schott (2004).
2.1 Consumption

The representative consumer’s utility in the upper tier is given by a Cobb-Douglas utility function, in the lower tier the utility is given by the continuous translog expenditure function as in Rodriguez-Lopez (2011). Preferences are defined for a continuum of differentiated goods, in each industry, \( h = \{ x, y \} \), set of goods, \( \Omega_h \). Each set includes the total number of actual, and potential (not yet invented) goods and has measure of \( \tilde{N}_h \). Let \( \Omega'_h \), with measure \( N_h \), be the subset of \( \Omega_h \) that contains the set of goods that are available for purchase in the economy. Utility level is \( U \), and \( P_h \) is the price index for industry \( h \), then the expenditure function of the representative consumer is given by:

\[
\ln E = \ln U + \alpha_x \ln P_x + \alpha_y \ln P_y.
\]

Where \( \alpha_x + \alpha_y = 1 \), and;

\[
\ln P_h = \frac{1}{2\gamma_h N_h} + \frac{1}{N_h} \int_{i \in \Omega'_h} \ln p_{ih} di + \frac{\gamma_h}{2N_h} \int_{i \in \Omega'_h} \int_{j \in \Omega'_h} \ln p_i (\ln p_j - \ln p_i) dj di.
\]

(1)

Where \( \gamma_h > 0 \) indicates the degree of substitutability between goods: with larger values of \( \gamma_h \) implying higher substitutability between goods in that industry (low differentiation). Consumers exhibit “love of variety”: when the set of goods in the economy, \( N_h \), is larger, the expenditure necessary to achieve utility \( U \) is lower.

In this economy we have two kind of agents, skilled workers and unskilled workers, each type of worker will offer one unit of labour. The size of the economy is given by \( L = L_s + L_u \). \( w_k \) is the wage earned by the worker, \( k = \{ s, u \} \). The aggregate income is given by \( I = w_s L_s + w_u L_u \).

The share \( s_{ih} \) of good \( i \) from industry \( h \) in the expenditure of the consumer using Shephard’s lemma is given by;

\[
s_{ih} = \frac{\partial \ln E}{\partial \ln p_{ih}} = \alpha_h \left( \frac{1}{N_h} + \frac{\gamma_h}{N_h} \int_{j \in \Omega'_h} \ln p_{jh} dj - \gamma_h \ln p_{ih} \right).
\]

Note that when \( s_{ih} = 0 \) we have that \( p_{ih} = \hat{p}_h \), where \( \hat{p}_h = \exp \left( \frac{1}{\gamma_h N_h} + \tilde{\ln p}_h \right) \) is the chock-off price, that is the highest price that a firm \( i \) can charge in industry \( h \), and sell anything and, \( \tilde{\ln p}_h = \frac{1}{N_h} \int_{j \in \Omega'_h} \ln p_{jh} dj \) is the average log price. Then,

\[
s_{ih} = \alpha_h \gamma_h \ln \left( \frac{\hat{p}_h}{p_{ih}} \right).
\]

(2)

Using 2 we can write the quantity demanded of a good \( i \) in industry \( h \).

\[
q_{ih} = \gamma_h \ln \left( \frac{\hat{p}_h}{p_{ih}} \right) \frac{\alpha_h I}{p_{ih}}.
\]

Where, \( I_h = \alpha_h I \). From now on, define \( \alpha_x = \alpha \), thus \( \alpha_y = 1 - \alpha \).
2.2 Profit Maximizing Price

Assuming a constant marginal cost for each firm \(i\), each firm will solve the maximization problem:

\[
max_{p_{ih}} p_{ih}q_{ih} - c_{ih}q_{ih}.
\]

The solution is \(p_{ih} = [1 + \ln(\hat{p}_h/p_{ih})]c_{ih}\). Note that,

\[
\frac{p_{ih}}{c_{ih}} = 1 + \ln\left(\frac{\hat{p}_h}{c_{ih}p_{ih}}\right),
\]

\[
\frac{p_{ih}}{c_{ih}}e^{\frac{p_{ih}}{c_{ih}}} = \frac{\hat{p}_h}{c_{ih}},
\]

\[
\frac{p_{ih}}{c_{ih}} = W\left(\frac{\hat{p}_h}{c_{ih}}\right).
\]

Where \(W(z)\) is the Lambert function defined as the inverse of \(x = ze^z\) for \(x > 0\)^6. Thus;

\[
p_{ih} = W\left(\frac{\hat{p}_h}{c_{ih}}\right)c_{ih}.
\]

Using 3 we define the mark-up of firm \(i\) in industry \(h\) as

\[
\mu_{ih} \equiv W\left(\frac{\hat{p}_h}{c_{ih}}\right) - 1.
\]

Then,

\[
p_{ih} = (1 + \mu_{ih})c_{ih},
\]

\[
\ln p_{ih} = \ln \hat{p}_h - \mu_{ih},
\]

\[
s_{ih} = \gamma_h \mu_{ih}.
\]

2.3 Production

Each industry \(h = \{x, y\}\) uses skill and unskilled labour with different intensities. Industry \(x\) is intensive in skilled labour and industry \(y\) in unskilled labour, to that \(\beta_x > \beta_y\). Marginal cost of production of each firm in each industry is

\[
c_h(\varphi) = \frac{(w_s)^{\beta_h}(w_u)^{1-\beta_h}}{\varphi}.
\]

In this economy, production will follow Melitz (2003) and Bernard, Redding and Schott (2007). Each firm will invest a sunk cost \(f_E(w_s)^{\beta_h}(w_u)^{1-\beta_h}\), to draw a productivity \(\varphi\) from a distribution \(g(\varphi)\) to \(\varphi \in [\varphi, +\infty)\) seeking to produce in industry \(h\), so that production and investment to enter the industry imply factors in the same proportion, given wages. Price in the domestic market is then given by:

\[\text{Lambert function has the properties of } W_z > 0, W_{zz} < 0, W(0) = 0 \text{ and } W(e) = 1.\]
\[ p_h(\varphi) = (1 + \mu_h(\varphi)) \frac{(w_s)^{\beta_h}(w_u)^{1-\beta_h}}{\varphi}. \]  \hspace{1cm} (8)

The mark-up is:

\[ \mu_h(\varphi) = \mathcal{W} \left( \frac{\hat{p}_h}{(w_s)^{\beta_h}(w_u)^{1-\beta_h}} \right) - 1. \]  \hspace{1cm} (9)

Output, \( y_h(\varphi) \), revenue, \( r_h(\varphi) \), and the profit, \( \pi_h(\varphi) \), of each firm can be calculated substituting optimum price (5) in the demand (2).

\[ y_h(\varphi) = \frac{\gamma_h I_h}{c_h(\varphi)} \left( \frac{\mu_h(\varphi)}{1 + \mu_h(\varphi)} \right), \]

\[ r_h(\varphi) = p_h(\varphi)y_h(\varphi) = \gamma_h I_h \mu_h(\varphi), \]

\[ \pi_h(\varphi) = \gamma_h I_h \frac{\mu_h(\varphi)^2}{1 + \mu_h(\varphi)}. \]

More productive firms set lower prices and earn higher revenues and profits and set higher mark-ups.

### 2.4 Cutoff Productivity Levels

The cutoff \( \varphi_h \) determines which firms will produce in each industry. Firms only produce if their profit is non-negative, that is, if \( \varphi \geq \varphi_h \) where \( \varphi_h = \inf\{\varphi : \mu_h(\varphi) > 0\} \). Thus;

\[ \varphi_h = \frac{(w_s)^{\beta_h}(w_u)^{1-\beta_h}}{\hat{p}_h}. \]

The cutoff productivity is higher the lower is the check-off price and the higher are input costs.

To proceed further we adopt the Pareto distribution for productivities, as in Melitz and Ottaviano (2008) with density function for \( \varphi \in [\underline{\varphi}, \infty) \) and \( k \geq 1 \).

\[ g(\varphi) = \frac{k \varphi^k}{\varphi^{k+1}}, \]

and distribution function,

\[ G(\varphi) = 1 - \left( \frac{\varphi}{\underline{\varphi}} \right)^k. \]

Thus, the distribution of those firms that draw a productivity \( \varphi \) high enough to be active in industry \( h \) is

\[ g(\varphi | \varphi > \varphi_h) = \begin{cases} \frac{k \varphi^k}{\varphi^{k+1}}; & \text{if } \varphi > \varphi_h \\ 0; & \text{otherwise} \end{cases} \]

The average productivity in each industry increases linearly with the cutoff and decreases with the parameter \( k \) of the distribution.
\[
\bar{\varphi}_h(\varphi_h) = \int_{\varphi_h}^{\infty} \frac{k\varphi_h^k}{\varphi^{k+1}} d\varphi = \frac{k}{k-1}\varphi_h.
\]

Note that, we can write the mark-ups of the firms as a function of their productivity and the cutoff;

\[
\mu_h(\varphi, \varphi_h) = \mathcal{W}\left(\frac{\varphi}{\varphi_h}\right) - 1.
\]

Firms with higher productivities set higher mark-ups, but they are lower in industries with higher cutoffs.

Since the distribution of \(\varphi/\varphi_h\) is invariant under a Pareto distribution for productivity, the average mark-up depend only on the parameter \(k\) of the distribution and so it is the same for both industries.

\[
\int_{\varphi_h}^{\infty} \mu_h(\varphi, \varphi_h) \frac{k\varphi_h^k}{\varphi^{k+1}} d\varphi = k \int_{1}^{\infty} \frac{\mathcal{W}(xe) - 1}{x^{k+1}} dx = \tilde{\mu}(k).
\]

As in Melitz (2003), in every period each firm has a positive probability \(\delta\) to have a bad shock and “die”. The value of the firm with productivity \(\varphi\) is

\[
v_h(\varphi) = \max\{0, \Sigma(1 - \delta)\pi_h(\varphi)\} = \max\{0, \frac{\pi_h(\varphi)}{\delta}\}
\]

With free entry, the expected value of the firm should equal the sunk cost incurred to draw a productivity from \(g(\varphi)\). The ex-ante expected profit of the firm \(\pi^e_h\) is given by:

\[
\pi^e_h = \int_{\varphi_h}^{\infty} \pi_h(\varphi)g(\varphi)d\varphi = \frac{\psi_h I_h}{\varphi_h^k}.
\]

Where, \(\psi_h = \gamma_h \bar{\chi}(k) \varphi_h^k\), and \(\chi(k) = k \int_{1}^{\infty} \frac{\mathcal{W}(xe) - 1)^2}{\mathcal{W}(xe)x^{k+1}} dx\), is a constant depending only on parameter \(k\).

The free entry condition is

\[
\frac{\pi^e_h}{\delta} = \frac{\psi_h I_h}{\varphi_h^k} = \frac{\psi_h I_h}{\varphi_h^k} = \delta \frac{f_E(w_s)^{\beta_h}(w_u)^{1-\beta_h}}{\varphi_h^k}.
\]

Thus,

\[
\frac{\psi_h I_h}{\varphi_h^k} = \delta \frac{f_E(w_s)^{\beta_h}(w_u)^{1-\beta_h}}{\varphi_h^k}.
\]

(10)

Using the FECs we can determine the cutoff productivity levels as functions only of parameters and wages;

\[
\varphi_h = \left[\frac{\psi_h I_h}{\delta f_E(w_s)^{\beta_h}(w_u)^{1-\beta_h}}\right]^\frac{1}{k}.
\]

(11)

In the closed economy, larger market size, as measured by \(I_h\), and a higher degree of substitutability between the goods, \(\gamma_h\), implies a higher cutoff productivity. Because in this environment the competition is “tougher”. Higher probability of “dying”, \(\delta\), or higher investment factor costs imply a smaller cutoff to increase expected profitability.
With the cutoffs determined and the definition for chock-off prices we can determine the mass of producing firms in each industry.

**Proposition 2.1.** The mass of available goods in each industry depends only on the substitutability among varieties;

\[ N_h = \frac{1}{\gamma_h (\ln p_h - \ln \hat{p}h)} = \frac{1}{\gamma_h \hat{\mu}(k)}. \]  

(12)

*Proof.* See Appendix. 

The mass of available goods is larger in the industry with lower \( \gamma_h \). Number of varieties is higher when they are less substitutable.

Given \( N_h \), Let \( N_{ph} \) denote the measure of the pool of existing firms in industry \( h \), that is the mass of firms that pay the sunk cost to draw a productivity. These firms in the pool can be producing or not, depending on their productivity \( \varphi \) and the productivity cutoff, \( \varphi_h \). Since \( 1 - G(\varphi_h) \) is the fraction of investors that became producing firms;

\[ N_h = (1 - G(\varphi_h)) N_{ph} = \left( \frac{\varphi}{\varphi_h} \right)^k N_{ph}. \]

In steady-state, we have that the following relation is valid:

\[ N_{ph:t+1} = (1 - \delta) N_{ph:t} + N_{Eh:t+1}. \]

Where \( N_{Eh} \) is the mass of entrant firms in industry \( h \). In steady-state, we must have \( N_{ph:t+1} = N_{ph:t} = N_{ph} \). Thus\(^7\),

\[ \delta N_{ph} = N_{Eh}. \]

In conclusion, the mass of firms that “die” every period in each industry is equal the mass of entrants firms in the same industry.

### 2.5 Market Clearing

In this section we will use the marketing clearing conditions to determine the wage vector \([w_s, w_u]\).

Revenue that a firm with productivity \( \varphi \in [\varphi_h, \infty] \) in the industry \( h = \{x, y\} \) earns in the domestic market is

\[ r_h(\varphi) = p_h(\varphi)y_h(\varphi) = \gamma_h I_h \left( W \left( \frac{\varphi}{\varphi_h} e \right) - 1 \right). \]

Let \( R_h \) be the total revenue of industry \( h = \{x, y\} \),

\[ R_h = N_h \int_{\varphi_h}^{\infty} \gamma_h I_h \left( W \left( \frac{\varphi}{\varphi_h} e \right) - 1 \right) \frac{\varphi^k e^k}{\varphi^{k+1}} d\varphi. \]

Using a change of variables, \( x = \varphi / \varphi_h \), and the previous result for \( N_h \) and average mark-up \( \bar{\mu}(k) \), we have that,

\(^7\)So that \( N_{ph} \) could be interpreted as the mass of entrants before they are hit by the exogenous chock \( \delta \).
\[ R_h = \gamma_h N_h I_h \mu(k) = I_h. \]  

(13)

The total revenue of industry \( h \) is equal the total expending on varieties from that industry. FEC is used to show that all profit made in each industry will be spent paying labour employed in the entry technology for the same sector.

**Proposition 2.2.** Expenditures on entry investment employment are equal to profits in each sector.

\[ N_{Eh} f_E(w_s)^{\beta_h} (w_u)^{1-\beta_h} = \Pi_h. \]

*Proof.* See Appendix.

Market clearing of labour market require that \( L_k = L_{kx} + L_{ky} \). Where \( L_{kh} = L_{kh}^D + L_{kh}^E \) and superscripts refer to workers employed in production and entry investment respectively. We can determine relative wages using labour demand and labour market clearing conditions.

**Proposition 2.3.** Equilibrium wages defined only on comparative advantage parameters.

\[ \frac{w_u}{w_s} = \frac{1 - (\alpha \beta_x + (1 - \alpha) \beta_y)}{\alpha \beta_x + (1 - \alpha) \beta_y} \frac{L_x}{L_u}. \]

*Proof.* See Appendix.

With wages determined, all the equilibrium for the closed economy can be described: Because average mark-up are constant and equal for both industries, the equilibrium relative wages reflect directly relative demand and supply facts, independently of intra industries competition.

### 3 Costly Trade

We extend the model for two countries, “Home Country” and “Foreign Country”, that will be represented by an asterisk. Firms in each of the two industries, \( h = \{x, y\} \), can sell in markets \( r = \{D, X\} \), domestic market and export market. We adopt the standard Heckscher-Ohlin assumption that countries are identical in terms of preferences and technologies, but differ in terms of endowments. The Home Country is the skilled labour abundant country and the Foreign Country is the skilled labour scarce country, as described by their relative endowments \( \frac{L_S}{L_U} > \frac{L_S^*}{L_U^*} \). Factors of production can move between industries within countries, but not across countries.

Costs to export are modelled as ice-berg costs. It is necessary to ship \( \tau_h > 1 \) units of the good for one unit to be delivered in the other country for consumption, we allow for different ice-berg costs to each industry \( h \). As in Bernard et al. (2007) we show how these trade costs interact with comparative advantage to determine responses to trade liberalization that vary across firms, industries, and countries. Market size, factor intensity and factor abundance also play an important role in shaping within-industry reallocations of resources from less to more productive firms. But differently from Bernard et al. (2007), the equilibria in our model also have mark-ups heterogeneity.

In open economy, producing firms can sell in two different markets. They sell output \( y_{Dh}(\varphi) \) in the domestic market by price \( p_{Dh}(\varphi) \), and, if the firm has a productivity high
enough, it sells output \( y_{Xh}(\varphi) \) to the Foreign Country by price \( p_{Xh}(\varphi) \). Firms in Foreign Country are in the same environment. In consequence of transportation costs, only the more productive firms will make profitable sales in the export market. That will define two different cutoffs productivities, one for firms selling exclusively in the domestic market market, and the other for exporters.

Since the markets are segmented\(^8\) and firms produce under constant marginal costs, they independently maximize profits earned from domestic and export sales. Using the results from before and taking account the transportation cost, \( \tau_h \), we have the following mark-ups for domestic and export sales:

\[
\mu_{Dh}(\varphi) \equiv \mathcal{W} \left( \frac{\hat{p}_h}{\left(\hat{w}_s(\hat{w}_u)^{1-\beta_h}\right)^{\varphi}} \right) - 1 = \mathcal{W} \left( \frac{\varphi}{\varphi_{Dh}} \right)^e - 1,
\]

\[
\mu_{Xh}(\varphi) \equiv \mathcal{W} \left( \frac{\hat{p}_h^*}{\left(\tau_h\hat{w}_s(\hat{w}_u)^{1-\beta_h}\right)^{\varphi}} \right) - 1 = \mathcal{W} \left( \frac{\varphi}{\varphi_{Xh}} \right)^e - 1.
\]

Other domestic and export firm variables can be written as functions of the respective cutoffs and mark-ups:

\[
p_{Dh}(\varphi) = (1 + \mu_{Dh}(\varphi))^\left(\frac{\hat{w}_s(\hat{w}_u)^{1-\beta_h}}{\varphi}\right),
\]

\[
p_{Xh}(\varphi) = (1 + \mu_{Xh}(\varphi))^\left(\frac{\hat{w}_s(\hat{w}_u)^{1-\beta_h}}{\varphi}\right),
\]

\[
y_{Dh} = \frac{\gamma_h I_{h}}{c_h(\varphi)} \left( \frac{\mu_{Dh}(\varphi)}{1 + \mu_{Dh}(\varphi)} \right),
\]

\[
y_{Xh} = \frac{\gamma_h I_{h}^*}{\tau_h c_h(\varphi)} \left( \frac{\mu_{Xh}(\varphi)}{1 + \mu_{Xh}(\varphi)} \right),
\]

\[
r_{Dh}(\varphi) = p_{Dh}(\varphi)y_{Dh}(\varphi) = \gamma_h I_{h} \mu_{Dh}(\varphi),
\]

\[
r_{Xh}(\varphi) = p_{Xh}(\varphi)y_{Xh}(\varphi) = \gamma_h I_{h}^* \mu_{Xh}(\varphi),
\]

\[
\pi_{Dh}(\varphi) = \gamma_h I_{h} \left( \frac{\mu_{Dh}(\varphi)^2}{1 + \mu_{Dh}(\varphi)} \right),
\]

\[
\pi_{Xh}(\varphi) = \gamma_h I_{h}^* \left( \frac{\mu_{Xh}(\varphi)^2}{1 + \mu_{Xh}(\varphi)} \right).
\]

\(^8\)We show ahead that this holds in equilibrium.
3.1 Cutoff Productivity Levels

To determine the cutoff productivity levels we use the fact that a firm will sell to a determinate market only if it draws a productivity high enough to turn in positive profits, that is: \( \varphi_{rh} = \inf \{ \varphi : \mu_{rh}(\varphi) > 0 \} \). Using the definitions for the demand check-off price in each market, \( \hat{p}_h \) and \( \hat{p}_h^* \),

\[
\varphi_{Dh} = \frac{(w_s)^{\beta_h}(w_u)^{1-\beta_h}}{\hat{p}_h^*},
\]

\[
\varphi_{Xh} = \frac{\tau_h (w_s)^{\beta_h}(w_u)^{1-\beta_h}}{\hat{p}_h^*},
\]

\[
\varphi_{Dh}^* = \frac{(w_s^*)^{\beta_h}(w_u^*)^{1-\beta_h}}{\hat{p}_h^*},
\]

\[
\varphi_{Xh}^* = \frac{\tau_h (w_s^*)^{\beta_h}(w_u^*)^{1-\beta_h}}{\hat{p}_h^*}.
\]

The cutoffs productivities for an industry in a market is the ratio between the factor basket cost in the country and the industry check-off price in the market.

Under costly trade, there are four different cutoffs: each country has one cutoff to produce for domestic market and one cutoff to produce for foreign market.

These equations imply direct relations between the cutoffs productivity levels: for national and foreign firms competing in each country;

\[
\varphi_{Xh}^* = \frac{\tau_h \zeta_h}{\varphi_{Dh}},
\]

\[
\varphi_{Xh} = \frac{1}{\tau_h \varphi_{Dh}^*},
\]

where \( \zeta_h = \left[ \frac{(w_s^*)^{\beta_h}(w_u^*)^{1-\beta_h}}{(w_s)^{\beta_h}(w_u)^{1-\beta_h}} \right] \) is the relative price of foreign factor basket. And between the cutoff productivity level for each country’s firms competing in domestic and export markets;

\[
\varphi_{Xh} = \Lambda_h \varphi_{Dh},
\]

\[
\varphi_{Xh}^* = \Lambda_h^* \varphi_{Dh}.
\]

Where, \( \Lambda_h = \tau_h \hat{p}_h / \hat{p}_h^* > 1 \) and \( \Lambda_h^* = \tau_h \hat{p}_h / \hat{p}_h > 1 \).

Where the inequalities ensure that there is selection of firms into domestic only and exporters in both industries.

Note that equations (16) and (17) establish a relation between the export cutoff productivity level and the domestic cutoff productivity level in different country markets. It says that trade barriers make trade harder for exports to break even relative to domestic producers. Equations (18) and (19) ensure that the cutoff to export is higher than the cutoff to domestically production, then we will have separation in the market, only most productivity firms will export. We can show that non-arbitrage conditions over prices are verified (Claim 3.1).

\[9\] Otherwise the least productive producing firm in each country could also export.
The average productivity level is the same linear function of the cutoff that we have already found for the closed economy;

\[ \bar{\varphi}_{rh}(\varphi_{rh}) = \int_{\varphi_{rh}}^{\infty} \varphi \frac{k_{rh}}{\varphi_{rh+1}} d\varphi = \frac{k}{k-1} \varphi_{rh}. \]

The average mark-up for exports is the same constant as the one for domestic sales, \( \tilde{\mu}(k) \).

The FEC still need \( \frac{\pi^e}{\delta} = f_E(w_s)\beta_h(w_u)^{1-\beta_h} \), but now expected profits include export profits;

\[ \pi^e_h = \pi^e_d + \pi^e_x. \]

Where,

\[ \pi^e_d = \frac{\psi_h I_h}{\varphi^k_{Dh}}, \quad \pi^e_x = \frac{\psi_h I_h^*}{\varphi^k_{Xh}}. \]

As before, \( \psi_h = \gamma_h \bar{\chi}(k) \varphi^k \), and \( \bar{\chi}(k) = k \int_1^{\infty} \frac{W(xe)^{-1}}{W(xe)x^{k+1}} dx \), is a constant depending only on parameter \( k \).

The FECs are:

\[ \frac{1}{\delta} \left[ \frac{\psi_h I_h}{\varphi^k_{Dh}} + \frac{\psi_h I_h^*}{\varphi^k_{Xh}} \right] = f_E(w_s)\beta_h(w_u)^{1-\beta_h}, \]

\[ \frac{1}{\delta} \left[ \frac{\psi_h I_h^*}{\varphi^k_{Dh}} + \frac{\psi_h I_h^*}{\varphi^k_{Xh}} \right] = f_E(w^*_s)\beta_h(w^*_u)^{1-\beta_h}. \]

Using (16), (17), (20) and (21) we can determine the cutoffs \([\varphi_{Dh}, \varphi_{Xh}, \varphi^*_{Dh}, \varphi^*_{Xh}]\) as functions of wages.

As result, we have that:

\[ \varphi_{Dh} = \left[ \frac{\psi_h I_h(1 - \tau_{rh}^2)}{\delta f_E w_s^\beta u_a^1 - \beta_h (\zeta_{rh}^{k+1} - \tau_{rh})} \right]^{\frac{1}{\delta}} \frac{1}{\tau_{rh}}, \]

\[ \varphi_{Xh} = \left[ \frac{\psi_h I_h^*(1 - \tau_{rh}^2)}{\delta f_E w_s^\beta u^a_1 - \beta_h (1 - \tau_{rh} \zeta_{rh}^{k+1})} \right]^{\frac{1}{\delta}} \frac{1}{\tau_{rh}}, \]

\[ \varphi^*_{Dh} = \left[ \frac{\psi_h I_h^*(1 - \tau_{rh}^2)}{\delta f_E w_s^\beta u_a^1 - \beta_h (1 - \tau_{rh} \zeta_{rh}^{k+1})} \right]^{\frac{1}{\delta}} \frac{1}{\tau_{rh}}, \]

\[ \varphi^*_{Xh} = \left[ \frac{\psi_h I_h(1 - \tau_{rh}^2)}{\delta f_E w_s^\beta u_a^1 - \beta_h (\zeta_{rh}^{k+1} - \tau_{rh})} \right]^{\frac{1}{\delta}} \zeta_{rh}. \]
Claim 3.1. There is no opportunity of arbitrage in the economy.

1. \(p_{Xh}(\varphi) / \tau_h < p_{Dh}(\varphi)\). There is no profitable export resale by a third party of a good produced and sold in a country.

2. \(p_{Dh}(\varphi) / \tau_h < p_{Xh}(\varphi)\). There is no profitable resale of a good exported to a country, back in its origin country.

Proof. See Appendix.

Under free trade average prices and average \(\ln\) of the prices would be trivially equal for domestic and imported goods. Under costly trade, even though not trivial, but it is still true.

Proposition 3.2. The average prices and \(\ln\) average prices of domestic and imported goods are equal.

\[
\tilde{p}_h = \tilde{p}_{Dh} = \tilde{p}_{Xh} \quad \text{and} \quad \tilde{\ln} p_h = \tilde{\ln} p_{Dh} = \tilde{\ln} p_{Xh}.
\]

Proof. See Appendix.

Having determined average prices we can use the definition of the choke-off price and determine the mass of available goods for consumption in each country and industry.

Proposition 3.3. The mass of available goods in each industry depends only on the substitutability among varieties:

\[
N_h = \frac{1}{\gamma_h (\ln \tilde{p}_h - \ln \tilde{p}_h)} = \frac{1}{\gamma_h \mu(k)} = N^*_h. \quad (20)
\]

Proof. See Appendix.

Under costly trade only the most productive firms will export, so the mass of firms that export is different and smaller than the mass of for the domestic market. We have that,

\[
N_h = N_{Dh} + N^*_{Xh} \quad \text{and} \quad N^*_h = N^*_{Dh} + N^*_{Xh}. \quad (21)
\]

We also have that:

\[
N_{rh} = (1 - G(\varphi_{rh})) N_{ph} = \left(\frac{\varphi}{\varphi_{rh}}\right)^k N_{ph}. \quad (22)
\]

Then, note that, using (16), (17), (22), (23) and (24) we can determine \(N_{ph}\).

\[
N_{ph} = \frac{N_h (\tau^2_h \varphi^{2k}_{Dh} - \varphi^{2k}_{Xh})}{\varphi^{2k}} \quad \text{and} \quad N^*_{ph} = \frac{N_h (\tau^{2k}_h \varphi^{*2k}_{Dh} - \varphi^{*2k}_{Xh})}{\varphi^{2k}} \quad (23)
\]

The steady-state condition is the same,

\[
\delta N_{ph} = N_{Eh}.
\]
3.2 Market Clearing

As in the closed economy, we will use the marketing clearing conditions to, once more, determine the wage vector \([w_s, w_u, w_s^*, w_u^*]\).

In costly trade we have that revenue earned by a firm \(i\), from industry \(h\), in market \(r\), is given by;

\[
r_{Dh}(\varphi) = \gamma_h I_h \left( W\left( \frac{\varphi}{\varphi_{Dh}} e \right) - 1 \right),
\]

\[
r_{Xh}(\varphi) = \gamma_h I^*_h \left( W\left( \frac{\varphi}{\varphi_{Xh}} e \right) - 1 \right).
\]

Thus, in costly trade, we have that total revenue of industry \(h\) in market \(r\), is given by;

\[
R_{Dh} = \gamma_h N_{Dh} I_h \tilde{\mu}(k) = \frac{N_{Dh}}{N_h} I_h,
\]

\[
R_{Xh} = \gamma_h N_{Xh} I^*_h \tilde{\mu}(k) = \frac{N_{Xh}}{N_h} I^*_h.
\]

Note that, in costly trade, we have trade, then, revenue in an industry \(h\) in domestic market is given by a percentage of the income of the Domestic country expended in that industry, and the revenue of this industry with exportation is given by a percentage of income in Foreign country, also, expended in that industry. The total revenue of an industry \(h\) from each country is determined summing these both revenues.

\[
R_h = R_{Dh} + R_{Xh},
\]

\[
R^*_h = R^*_{Dh} + R^*_{Xh}.
\]

Using labour market conditions,

\[
L_k = L_{kx} + L_{ky} \quad \text{and} \quad L^*_k = L^*_{kx} + L^*_{ky},
\]

\[
L_{kh} = L^p_{kh} + L^p_{kh} + L^E_{kh}.
\]

we can show that total profit industry \(h\) equals pays total investment in that industry and that total income expended in this industry equals total revenue of the industry. Then, market clearing condition can determine the wage vector and close the model.

**Proposition 3.4.** Expenditures on entry investment employment are equal to profits in each sector.

\[
N_{Eh} f_E(w_s)^{\beta_h} (w_u)^{1-\beta_h} = \Pi_h.
\]

**Proof.** See Appendix. \(\square\)

Proposition 3.4 shows that investment is funded by the profit of the industry and that each industry equals revenue to labour costs. Also using labour market clearing conditions, we can determinate labour demand in function of wages, exogenous parameters and endowments. Defining \(w_s = 1\) then, we can determinate the wage vector \([1, w_u, w_s^*, w_u^*].\)
Proposition 3.5. There exists a unique costly trade equilibrium referenced by the equilibrium vector, \( \{ \varphi_{DX}, \varphi_{Dx}^*, \varphi_{Dy}, \varphi_{Dy}^*, \varphi_{Xx}, \varphi_{Xy}, \varphi_{Xy}^*, \varphi_{Xx}^*, p_{DX}(\varphi), p_{DX}^*(\varphi), p_{Dy}(\varphi), p_{Dy}^*(\varphi), p_{Xx}(\varphi), p_{Xx}^*(\varphi), p_{Xy}(\varphi), p_{Xy}^*(\varphi), \mu_{DX}(\varphi), \mu_{DX}^*(\varphi), \mu_{Dy}(\varphi), \mu_{Dy}^*(\varphi), \mu_{Xx}(\varphi), \mu_{Xx}^*(\varphi), \mu_{Xy}(\varphi), \mu_{Xy}^*(\varphi), R_x, R_y, R_x^*, R_y^*, w_s, w_u, w_s^*, w_u^* \} \).

Proof. See Appendix.

4 Results in the Costly Trade Equilibrium

Although our model is a complex combination of multiple factors, multiple countries, country asymmetry, firm heterogeneity, variable mark-ups and trade costs, we were able to find closed form solutions for all key endogenous variables, differently from Bernard et al. (2007). In this section we derive several analytical results concerning the effects of opening a closed economy to costly trade. Even though our model is significantly more complex than Bernard et al. (2007) and Melitz and Ottaviano (2008), most of the provide proves follow these papers.

Proposition 4.1. The opening of costly trade increases the steady-state zero-profit cutoff (ZPC) cut-off and average industry productivity in both industries.

1. Other things equal, the increase in the steady-state ZPC and average industry productivity is greater in a country’s comparative industry: \( \Delta \varphi_{DX} > \Delta \varphi_{Dy} \) and \( \Delta \varphi_{Dy}^* > \Delta \varphi_{DX}^* \).

2. Other things equal, the exporting productivity cut-off is closer to the ZPC in a country’s comparative industry: \( \varphi_{Xx}/\varphi_{DX} < \varphi_{Xy}/\varphi_{Dy} \) and \( \varphi_{Xy}^*/\varphi_{Dy}^* < \varphi_{Xx}/\varphi_{DX}^* \).

Proof. See Appendix.

When trade is costly, only a subset of productivity firms will export, there is selection of firms into domestic only and exporters in both industries. The profit of the most productivity firms rises, thus, the expect profit of entering firms rises in both industries, because there is a positive probability of the firm drawing a productivity high enough to export. This induces more firms to entry. In addition, there is a new market where firms sell and there are other firms (from Foreign Country) that sell in domestic market, thus, competition increases. In this model mark-ups are variable, then, all mark-ups fall when the economy move from autarky to costly trade. Moreover, profit of those firms that produce only for domestic market will also fall and firms with lower productivities will exit the industry. The zero-profit cutoff productivity level, \( \varphi_{Dh} \) rises and also rises average productivities, \( \tilde{\varphi}_{Dh} \), in both industries.

Profits in exporter market are larger relative to profits in domestic market in comparative industries, thus, ex-post profit of exporter industries rises more in the comparative advantage industry. Consequentially, those effects described before are magnified in the comparative advantage industry.

Firms stop producing because a “tougher” competition and because an increasing of real wages. Opening costly trade leads to an increase in labour demand at exporters. The increase in labour demand bids up factor prices of non-exporters, then, lower productivity firms exit the industry and it increases the cutoff productivity in both industries.
The increase of labour demand at exporters is higher in comparative advantage industry, then relative price of the factor abundant and used intensively in this industry rises more, furthermore, profits on domestic market falls more in the comparative advantage industry. Then, zero profit cutoff productivity and average cutoff rises more in this industry.

In our model we have this important result via these two channels. Finally, we conclude that when we move from autarky to costly economy, it is more difficult to firms with low productivity to survive in the comparative advantage industry.

**Proposition 4.2.** The opening of costly trade increases steady-state average firm output in both industries, and other things equal the largest increase occurs in the comparative advantage industry.

*Proof.* See Appendix.

This is the same result of Bernard, Redding and Schott (2007), when trade costs falls, the environment becomes more competitive, then, domestic production falls. However, most productive firms sell at the exporter market, those firms rise their production, it rises more than enough to compensate the fall in domestic production, thus, average firm output will be higher than in autarky.

The average profit rises when the economy is opened, but the value of the sunk entry cost remains unchanged, thus, production cutoff rises, then, average output must rises, so average profit also rises. This increase in average output is higher in comparative advantage industry, because cutoff rises more in this industry.

**Proposition 4.3.** The opening of costly trade magnifies ex-ante cross-country differences in comparative advantage by inducing endogenous Ricardian productivity differences at the industry level that are positively correlated with Heckscher–Ohlin-based comparative advantage.

*Proof.* See Appendix.

Under costly trade, the environment is more competitive, then, there are more intensive selection of high productive firms in comparative advantage industry. In this model, this effect occurs via two channels, as it was explained in proposition 4.1. As a result, it gives rise for endogenous Ricardian technology differences at industry level that are no neutral across sectors. Also, average productivity level increases more in comparative industry, thus, Heckscher–Ohlin-based comparative advantage is magnified.

\[ \frac{\varphi_{D_x}}{\varphi_{D_y}} > 1. \]

Price index in industry \( h \) is

\[
\ln P_h = \frac{1}{2\gamma_h N_h} + \ln \bar{p}_h + \frac{\gamma_h}{2N_h} \int_e^{\infty} \int_e^{\infty} \ln p_{rh}(\varphi)(\ln p_{rh}(\varphi') - \ln p_{rh}(\varphi))d\varphi'd\varphi.
\]

Solving this:
\[
\ln P_h = \frac{\bar{\mu}(k)}{2} + 2 \left[ \ln \left( \frac{(w_s)_{\alpha h}(w_u)_{1-\alpha h}}{\varphi_{Dh}} \right) - \bar{\mu}(k) \right] + \frac{\gamma h}{2} \left[ k \int_1^\infty \frac{W(xe)}{x^{k+1}} dx \right]^2 - k \int_1^\infty \frac{W(xe)^2}{x^{k+1}} dx].
\]

Using \( \ln P_h \):

**Proposition 4.4.** The opening of costly trade has three sets of effects on the real income of skilled and unskilled workers:

1. The relative nominal reward of the abundant factor rises and the relative nominal reward of the scarce factor falls.

2. The rise in the zero production cutoff reduces average variety prices in both industries and so reduces consumer price indices.

3. The rise in industry productivity cutoff reduces the mass of firms producing domestically, then, it rises consumer price indices. However, the opportunity to import foreign varieties rises the available mass of goods in the economy, and then, it reduces consumer price indices. These two effects combined does not have any effect on consumer price indices.

**Proof.** See Appendix.

The first effect in opening the economy to costly trade is the famous Stolper-Samuelson Theorem. Relative nominal reward of the abundant factor rises and the relative nominal reward of the scarce factor falls. Since the production of comparative advantage industry good increases, relative demand for the country’s abundant factor also increases.

Another effect is the reduction of consumer price indices in both industries. It happens because opening costly trade imply in an increase in the zero profit cutoff, this means that average log prices falls, then, \( \ln P_h \) falls. It is important to note that, although, our model permit mark-ups to vary, average mark-up do not change when we pass from autarky to an open economy, neither \( N_h \). Thus, we have welfare gains via efficiency increase of the firms.

When trade costs falls the cutoff productivity level increases, then, lower productive firms exit industry, it increases consumer indices, because there are less available goods in economy. However, the opportunity to import brings new goods to the economy, and it reduces consumer prices indices. The final effect is ambiguous in (Bernard et al., 2007). In our model, proposition 3.3 says that \( N_h \) is constant in both industries, thus, the final effect is that \( N_h \) remain constant and there is no real effect in consumer prices indices.

Finally, differently from neoclassical models, real wage increases for both factors. This means that, at least, the fall in real wage of scarce factor will be smaller here than in Heckscher–Ohlin model; Bernard, Redding and Schott (2007) also find this result.

**Proposition 4.5.**

1. The opening of costly trade results in net job creation in the comparative advantage industry and net job destruction in comparative disadvantage industry.

2. The opening of costly trade results in simultaneous gross job destruction in both industries, so that gross job changes exceed net job changes, and both industries experience excess job reallocation.
As in Heckscher–Ohlin model, under costly trade, there is net job creation in the comparative advantage industry and net job destruction in comparative disadvantage industry. The magnitude of these effects differs as a result of endogenous changes in productivity cutoff levels, and in average industry productivity that shape the extent of the reallocation of factors across industries.

The second part of proposition 4.5 is consequence of the approach that is used here. The opening of costly trade rises the productivity level cutoff in both industries, thus, firms that remain in the market and produce only to domestic market will produce less than in closed economy, this implies in gross job destruction in both industries. However, firms with productivity high enough to export will produce more, thus, they experiment gross job creation. Therefore, some firms will have gains from reduction in trade costs, and other not.

Proposition 4.6. The opening of costly trade reduces $\hat{p}_h$ in both industries, this effect is higher in the comparative advantage industry.

Proof. See Appendix.

When we open the economy, mark-ups of all firms became smaller because productivity level cutoff gets higher in both industries, thus, in this environment, the highest price that a firm can charge for its good, the chock-off price, is lower when the economy moves from closed to an open economy. Firms that can’t set a mark-up such that $\mu_{Dh}(\varphi) > 0$ stop to produce.

Proposition 4.7. The opening of costly trade leads to a larger increase in steady-state creative destruction of firms in comparative advantage industry than in comparative disadvantage industry.

Proof. See Appendix.

Each period, a mass of firms receives a bad shock $\delta$ and “dies”, these firms exit the pool of firms that paid the sunk cost. To replace these firms, a mass of entrants firms, $N_{Eh}$, pays the sunk cost, in steady-state equilibrium, $\delta N_{ph} = N_{Eh}$. The costly trade equilibrium displays steady-state creative destruction, it corresponds to the steady-state probability of firm failure. In our model, it varies across countries and industries with comparative advantage, and it is given by:

$$\Psi_h = \frac{\delta(G(\varphi_{Dh}) + 1)}{1 + \delta}.$$  

Note that, higher $\varphi_{Dh}$ means higher $\Psi_h$. From proposition 4.1, $\varphi_{Dh}$ is higher in comparative advantage industry, and, consequentially, the increase of steady-state creative destruction at this industries is also higher. This implication of the model may explain why workers in general report greater perceived job insecurity as countries liberalize\(^\text{10}\).

Proposition 4.8. The opening of costly trade reduces mark-ups in all surviving firms of the market; this effect is higher to firms in comparative advantage industry. However, average mark-ups does not change in both industries.

\(^{10}\)See Scheve and Slaughter (2004).
Proof. See Appendix.

Opening costly trade rises competition, thus, productivity cutoff level rises in both firms, then, those firms that have productivity high enough to stay in market producing will charge a lower price, with lower mark-ups. This effect is higher in comparative advantage industry, because productivity cutoff level rises more in this industry.

Proposition 4.9. In the opening of costly trade, we have that:

\[
\frac{N_{Dx}}{N_{Xx}} < \frac{N_{Dy}}{N_{Xy}} \text{ and } \frac{N_{Dx}^*}{N_{Xx}^*} > \frac{N_{Dy}^*}{N_{Xy}^*}
\]

Proof. See Appendix.

When we pass to costly trade and the cutoff productivity level rises in both industries and countries, it is more difficult to produce in both markets, then, the mass of firms producing domestically fall in both industries, only the most productive firms keep producing, and the even most productive firms export, thus, \(N_{Xh}^*\), is now positive.

Note that, by proposition 4.1, the productivity cutoff in comparative advantage industry is closer to its exporting cutoff than productivity cutoff in comparative disadvantage industry is from its exporting cutoff, thus, it will be easier to firms in comparative advantage industry to export. Firms in comparative advantage industry are more productive. As result, we have that a higher proportion of firms export in comparative advantage industry than in comparative disadvantage industry.

When trade costs falls, firms that export will reach higher profits, although less productive firms stop to produce, more firms will be wondering to get in the industry, then, the mass of entrants firms, \(N_{Eh}\), rises. In comparative advantage industry this effect is higher, because, profits in this industry are higher and probability to draw a productivity high enough to sell in the export market is also higher. However, in disadvantage comparative industry, the lower probability to draw a productivity high enough to sell in the export market can make the net job destruction larger than the gross job creation in this industry, as we will have in the numerical example in next section.

The effect in the pool, \(N_{ph}\), of each firm is similar to the effect an entrants firms, we have that, in steady-state, \(N_{Eh} = \delta N_{ph}\).

5 Numerical Example

In this section, we calibrate our model. It provides a visual representation of the equilibria described in the previous sections and reinforce the intuition behind them. It also allows us to examine the evolution of endogenous variables when trade costs rise.

Once our model provides closed form solutions to all endogenous variables, we set exogenous parameters and compute equilibrium. We used Bernard et al. (2007) calibration, thus, we focus on comparative advantage effects. However, we show some results that are consequence of our framework that permits mark-ups to vary.

Following Bernard et al. (2007), we assume that all industry parameters except for factor intensity (\(\beta_h\)), are the same across industries. In particular, we set \(\gamma_x = \gamma_y = 1\), \(f_E = 2\), \(\tau_x = \tau_y = \tau\). We also consider symmetric differences in country factor

\footnote{Bernard et al. (2007) model doesn’t have this parameter, thus we set it equal to Rodríguez-López (2011)}
endowments ($L_s = L_u^* = 1200$ and $L_u = L_s^* = 1000$) and symmetric differences in industry factor intensities ($\beta_x = 0.6$ and $\beta_y = 0.4$). The share of each industry in consumer expenditure is assumed to be equal one half ($\alpha = 0.5$). We set the Pareto shape parameter $k = 3.4$ and the minimum value for productivity $\varphi = 0.2$. Finally, we set death parameter $\delta = 0.025$.

Our exercise in this numerical example is to vary $\tau$ and see how it modifies the equilibrium. We permit $\tau$ to vary from 1.2 to 1.7. Doing this, we see what happens to the economy (analysing Domestic Country) when trade costs rises and the economy approximates to the closed economy. We analyse endogenous variables such as cutoffs, average productivity, average output, exporting probability, mass of domestic and entrants firms. We also, evaluate if comparative advantage effects are magnified when the economy is liberalized. We use the logarithmic of indirect utility to analyse welfare gains. Finally, we calculate average mark-ups weighted by the fraction of output of each firm, which allows us to study the effect that variable mark-ups cause in our model.

First graphic shows ZPC and export cutoff. The ZPC rises when trade costs fall, only more productive firms stays in the market, and, as we have shown in proposition 4.1, this effect is higher in comparative advantage industry. Export cutoffs are always higher than ZPC and, as proposition 4.1 says, the distance between the cutoffs of comparative advantage industry is smaller than the distance between the comparative disadvantage industry. We also can see that rising trade costs, our economy converges to the closed economy.

The second graphic shows that, when trade costs are smaller, only more productive firms stay in the market, competition is tougher, and, as consequence, average productivity grows. Again, this effect is higher in comparative advantage industry. The third graphic present what happens with average firm output. We can see that, even with higher trade costs, average firm output is higher in commerce and it gets higher when trade costs fall. We also can view that comparative advantage firm always produce more, which is explained by the advantage comparative itself and because the average productivity in this industry is also higher, as we have seen in the last graphic.

In the next graphic, $\varphi_{Dx}/\varphi_{Dx}^* > \varphi_{Dy}/\varphi_{Dy}^*$ is always true. This happens because Domestic country has comparative advantage in industry $x$. The graphic also shows that this difference becomes higher when trade costs fall, which means that liberalization magnifies comparative advantages in this economy, just as Bernard et al. (2007). The exporting probability rises when trade costs fall. Firms in comparative advantage industry have higher probability to export than firms in comparative disadvantage industry.

The next two graphics are about the mass of domestic firms and the mass of entrant firms respectively. The mass of firms producing domestically falls when the economy is opened, ZPC rises and less firms are able to continue producing. However, the mass of firms in comparative advantage industry is higher than in disadvantage industry. Although ZPC is higher in comparative advantage industry, when we open the economy, the probability of exporting is also higher in this industry. Hence, the expected profit ex-ante in this industry rises more than in comparative disadvantage industry and as consequence, the pool in this industry rises, while in the comparative disadvantage industry, it falls. Thus, the mass of domestic firms is higher in the comparative advantage industry. Analogously, in comparative advantage industry, when trade costs fall, the mass of entrants firms rises, and in comparative disadvantage industry falls.

To analyse welfare gains, we used the logarithmic of the indirect utility of the representative consumer. In the following graphic, liberalization imply in welfare gains, note
Figure 1: Zero-profit and export cut-offs

Figure 2: Average productivity
Figure 3: Average firm output

Figure 4: Differences in comparative advantage
Figure 5: Exporting probability

Figure 6: Mass of firms (domestic varieties)
that, when trade costs are lower, indirect utility is higher. We obtain trade gains as Melitz (2003), most productive firms export and demand more labour, then, wages get higher, and, as consequence, less productive firms stop to produce. We also have trade gains like in Melitz and Ottaviano (2008), mark-ups are free to vary, then when the economy moves from autarky to an open economy, competition rise, then, firms set low mark-ups, thus, less productive firms stop to produce. Furthermore, trade gains are magnified in comparative advantage industry, just like in Bernard et al. (2007). Our model allow trade gains that all these previous models permit, but at one unique framework.
Average mark-up is always constant, in intention to analyse the effect of variable mark-ups, we calculate a weighted average mark-up. It is weighted by the proportion of output of each firm.

\[
\hat{\mu}(\varphi_{rh}) = \int_{\varphi_{rh}}^{\infty} \mu(\varphi, \varphi_{Dh}) \frac{y_{rh}(\varphi)}{Y_{rh}} d\varphi.
\]

Where, \( Y_{rh} \) is the total output of industry \( h \) at market \( r \).

\[
Y_{rh} = N_{rh} \int_{\varphi_{rh}}^{\infty} y_{rh} g(\varphi | \varphi > \varphi_{rh}) d\varphi = \frac{N_{rh} \gamma_{h} I_{h} \varphi_{rh}}{w_{n}^{\beta_{h}} w_{u}^{1-\beta_{h}} \eta_{1}}
\]

Where \( \eta_{1} = k \int_{1}^{\infty} \left( W(xe) - 1 \right) / \left( W(xe) x^{k} \right) dx \) is constant depending on parameter \( k \). Calculating \( \hat{\mu}(\varphi_{rh}) \):

\[
\hat{\mu}(\varphi_{rh}) = \frac{\eta_{2}}{N_{rh} \eta_{1}}.
\]

Where \( \eta_{2} = k \int_{1}^{\infty} \left( W(xe) - 1 \right)^{2} / \left( W(xe) x^{k} \right) dx \).

The last graphic shows the weighted average mark up for both industries, in domestic and export market. Firms in the exporter market set higher mark-ups. In this market, mark-ups fall when trade costs fall. It happens because when trade costs fall, more firms are able to export, thus the competition gets tougher and firms set lower mark-ups. Note that firms in comparative disadvantage industry set higher weighted mark-ups than firms in comparative advantage industry in this market. Competition explains it, there are more firms exporting in the comparative advantage industry, thus, competition is tougher. In the comparative disadvantage industry, only few industries are productive enough to sell to the export market. In the domestic market, the weighted average mark-up is almost constant. There are more firms in comparative advantage industry, thus, mark-ups in this industry are lower. Analysing comparative disadvantage industry, we can see that weighted average mark-up rises when trade costs falls, as we have seen before, the domestic mass of firms fall when trade costs fall, thus, weighted average mark-up rise.
We constructed this weighted average mark-up with the proposal to analyse the effects described previously, and to show a result that there isn’t present neither in Bernard et al. (2007) or Melitz and Ottaviano (2008).

6 Conclusion

Our main objective in this paper was to formulate a model that was capable to jointly provide trade gains via two different channels. On one hand, liberalization should shift up labour demand, thus, those firms there were less productive wouldn’t be able to produce after liberalization and would stop to produce. On the other hand, liberalization should make competition “tougher”, thus, mark-ups would get lower and firms that were less productive would stop to produce.

We developed a model of comparative advantage that incorporates heterogeneous firms and endogenous mark-ups that respond to the toughness of competition in a market. In such environment, we accomplished our main objective. We presented several results from Bernard et al. (2007) and from Melitz and Ottaviano (2008).

Market size influences firms in a specific industry: larger market size exhibits firms with larger profits and larger producing cutoff. Tougher competition result in lower mark-ups. However, average mark-up is constant. This is a weakness of our specification. As in Melitz and Ottaviano (2008), it would be better if tougher competition result in lower average mark-ups. Because of this, our model has a constant measure $N_h$ of available goods in each sector, this means that when trade costs fall, the economy present import substitution, although, consumers utility has “love of variety”.

Trade liberalization raises average industry productivity and average firm output in all sectors, because less productive firms stop producing and most productive firms export. This effect is higher in comparative advantage industry, as Bernard et al. (2007) say, it provides a new source of welfare gains from trade.
Endogenous mark-ups permit mark-ups to vary when the economy moves from autarky to an opened economy and they influence each firm individually, they affect profits, and, as a consequence, if the firm will keep producing or not. However average mark-up do not change, so endogenous mark-ups does not present any results in the industry level. Thus, some results remain identical to those found by Bernard et al. (2007). Trade results in gross job creation and gross job destruction in both industries, and the magnitude of these gross job flows varies across countries and industries with comparative advantage.

Taking account welfare gains, our model, as in Bernard et al. (2007), has distinct implications for the distribution of income across factors. In our model it is possible to the real wage of scarce factor also rises with trade liberalization, or, at least, declines less than in Neoclassical models and in the predicted by the Stolper-Samuelson Theorem.

Differently form Bernard et al. (2007), our framework permit us to determinate closed solutions for all endogenous variables.

We also made a numeric exercise, where we showed many of the theoretical results and how endogenous variables change when trade costs fall. In this numerical exercise we constructed a weighted average mark-ups, thus, it do vary when the economy moves from autarky to an open economy. The result is that when the market has a tougher environment, the weighted average mark-up is smaller.

Although the model present constant average mark-up, our framework don’t present constant mark-ups. It is closer to the reality than assuming constant mark-ups for all firms. Hence, using our model, empirical studies will still use an constant average mark-up, but the model will comport the data, that will have different mark-ups. We believe that it could be an important contribution.

We used indirect utility to evaluate welfare gains. However, other papers used the compensating variation associated with a change in trade costs ((Arkolakis et al., 2012), Arkolakis, Costinot, Donaldson, and Rodríguez-Clare (2012), Arkolakis et al. (2010)), Arkolakis, Costinot, Donaldson, and Rodríguez-Clare (2012) have as result that welfare gains are lower when mark-ups are allowed to vary because there is incomplete pass-through of changes in marginal costs from firms to consumers, this means that firms tend to raise their mark-ups.

Finally, for future study, we could calculate welfare using the compensating variation associated with a change in trade costs to see if our model is in line with Arkolakis, Costinot, Donaldson, and Rodríguez-Clare (2012) or not. We also suggest making different numeric exercises, we only analysed the case where countries were symmetric. Other interesting area to research is to do empirical investigations to test our model.
References


Arkolakis, C., A. Costinot, and A. Rodríguez-Clare (2010). Gains from trade under monopolistic competition: A simple example with translog expenditure functions and pareto distributions of firm-level productivity. mimeo.


Proposition 2.1. The mass of available goods in each industry depends only on the substitutability among varieties:

\[ N_h = \frac{1}{\gamma_h (\ln \hat{p}_h - \ln p_h)} = \frac{1}{\gamma_h \bar{\mu}(k)}. \] (27)

Proof. First, we need to find \( \tilde{\ln p}_h \). Using the equation for \( p_h(\varphi) \) and the following property of lambert function \( W \), that \( \ln |W(x)| = \ln x - W(x) \forall x > 0 \), we have that,

\[ \ln p_h = \ln \left( \frac{(w_s)^{\beta_h} (w_u)^{1-\beta_h}}{\varphi_h} \right) - \mu_h(\varphi). \]

Then, we have that,

\[ \tilde{\ln p}_h = \ln \left( \frac{(w_s)^{\beta_h} (w_u)^{1-\beta_h}}{\varphi_h} \right) - \bar{\mu}(k) = \ln \hat{p}_h - \bar{\mu}(k). \]

Rearranging this we have that \( \ln \hat{p}_h - \tilde{\ln p}_h = \bar{\mu}(k) \). From definition of \( \hat{p}_h \) we have that

\[ N_h = \frac{1}{\gamma_h (\ln \hat{p}_h - \ln p_h)}. \]

Substituting \( \ln \hat{p}_h - \tilde{\ln p}_h = \bar{\mu}(k) \) we have the result. \( N_h \) is constant and equal for both countries. \( \square \)

Proposition 2.2. Expenditures on entry investment employment are equal to profits in each sector.

\[ N_{Eh} f_E (w_s)^{\beta_h} (w_u)^{1-\beta_h} = \Pi_h. \]

Proof. Total profit of industry \( h \), \( \Pi_h \) is

\[ \Pi_h = N_h \int_{\varphi_h}^{\infty} \pi_h(\varphi) g(\varphi | \varphi \geq \varphi_h) d\varphi = N_h \gamma_h I_{h\hat{\chi}}(k). \]

Using FEC, we have that:

\[ N_{Eh} f_E (w_s)^{\beta_h} (w_u)^{1-\beta_h} = N_{ph} \frac{\psi_h I_h}{\varphi_h} = N_{ph} \left( \frac{\varphi}{\varphi_h} \right)^k \gamma_h I_{h\hat{\chi}}(k) = \Pi_h. \]

\( \square \)

Proposition 2.3. Equilibrium wages defined only on comparative advantage parameters.

\[ \frac{w_u}{w_s} = \frac{1 - (\alpha \beta_x + (1 - \alpha) \beta_y) L_s}{(\alpha \beta_x + (1 - \alpha) \beta_y) L_u}. \]
Proof. Using Labour market clearing conditions, we have that \( L_{kh} = L_{kh}^{Dp} + L_{kh}^E \). Using Shepard’s Lemma and cost functions, we can calculate entering labour demand and producing labour demand, for \( k = s \):

\[
L^E_{sh} = N_h f_{E}(w_u/w_s)^{1-\beta_h}.
\]

And,

\[
L^{Dp}_{sh}(\varphi) = \frac{\beta_h (w_u/w_s)^{(1-\beta_h)}}{\varphi} y_h(\varphi).
\]

Then,

\[
L^{Dp}_{sh} = N_h \int_{\varphi_h}^{\infty} L^{Dp}_{sh}(\varphi) g(\varphi | \varphi > \varphi_h) d\varphi
= N_h \frac{\beta_h \gamma_h I_h \tilde{\phi}(k)}{w_s}.
\]

Where \( \tilde{\phi}(k) = k \int_{1}^{\infty} \frac{W(xe)-1}{W(xe)} \frac{1}{x^{s+1}} dx \). Then,

\[
L_{sh} = \delta N_{ph} f_{E}(w_u/w_s)^{1-\beta_h} + N_h \frac{\beta_h \gamma_h I_h \tilde{\phi}(k)}{w_s}.
\]

Using the definition of \( N_{ph} \) and the cutoff \( \varphi_h \), we have that;

\[
N_{ph} = N_h \left( \frac{\varphi_h}{\varphi} \right)^k = N_h \frac{\gamma_h \tilde{\chi}(k) I_h}{\delta f_{E} w_s^{\beta_h} w_u^{1-\beta_h}}.
\]

Thus, substituting it and \( N_h \) in the last equation, we have that,

\[
L_{sh} = \frac{\beta_h I_h}{w_s \tilde{\mu}(k)} (\tilde{\chi}(k) + \tilde{\phi}(k)).
\]

We know that, \( \tilde{\chi}(k) + \tilde{\phi}(k) = \tilde{\mu}(k) \) and \( w_s L_s = w_s (L_{sx} + L_{sy}) \), thus;

\[
w_s L_s = \beta_x \alpha I + \beta_y (1-\alpha) I
= (\alpha \beta_x + (1-\alpha) \beta_y) (w_s L_s + w_u L_u)
\]

Rearranging;

\[
\frac{w_u}{w_s} = \frac{1 - (\alpha \beta_x + (1-\alpha) \beta_y) L_s}{(\alpha \beta_x + (1-\alpha) \beta_y) L_u}.
\]

\[\square\]

Claim 3.1. There is no opportunity of arbitrage in the economy.

1. \( p_{Xh}(\varphi)/\tau_h < p_{Dh}(\varphi) \). There is no profitable export resale by a third party of a good produced and sold in a country.

2. \( p_{Dh}(\varphi)/\tau_h < p_{Xh}(\varphi) \). There is no profitable resale of a good exported to a country, back in its origin country.
Proof. 1. We have that $\partial \mu_r (\varphi, \varphi_r) / \partial \varphi_r < 0$, thus, $\mu_X (\varphi) < \mu_D (\varphi)$. Then we have that:

$$\frac{p_{Xh}(\varphi)}{\tau_h} = (1 + \mu_X (\varphi)) \frac{(w_s)_{\beta h} (w_u)_{1-\beta h}}{\varphi} < (1 + \mu_D (\varphi)) \frac{(w_s)_{\beta h} (w_u)_{1-\beta h}}{\varphi} = p_{Dh}(\varphi).$$

2.

$$p_{Xh}(\varphi) = W \left( \frac{\varphi}{\varphi_{Xh}} \right) \frac{(w_s)_{\beta h} (w_u)_{1-\beta h}}{\varphi} = W \left( \frac{\varphi_{Dh}}{\varphi_{Dh}} \right) \frac{\varphi_{Xh} (w_s)_{\beta h} (w_u)_{1-\beta h}}{\varphi} = p_{Dh}^{\star} \left( \frac{\varphi_{Dh}}{\varphi_{Xh}} \right)$$

In the second equality we use equation (17), now we will use what we already proved in item one and repeat this procedure to find our result.

$$p_{Dh}^{\star} \left( \frac{\varphi_{Dh}}{\varphi_{Xh}} \right) > p_{Xh}^{\star} \left( \frac{\varphi_{Dh}}{\varphi_{Xh}} \right) \frac{1}{\tau_h} = p_{Dh} \left( \frac{\varphi_{Dh}}{\varphi_{Xh}} \right) \frac{1}{\tau_h} > p_{Dh}(\varphi) \frac{1}{\tau_h}.$$  

In the second equality we used equations (18) and (19). Note that now we have $p_{Dh}(\varphi) / \tau_h < p_{Xh}(\varphi)$.

Proposition 3.2. The average prices and ln average prices of domestic and imported goods are equal.

$$\tilde{p}_h = \tilde{p}_{Dh} = \tilde{p}_{Xh} \quad \tilde{p}_h^{\star} = \tilde{p}_{Dh}^{\star} = \tilde{p}_{Xh},$$

$$\tilde{\ln} p_h = \tilde{\ln} p_{Dh} = \tilde{\ln} p_{Xh} \quad \tilde{\ln} p_{Dh}^{\star} = \tilde{\ln} p_{Xh}^{\star}.$$

Proof. First, let’s find $\tilde{p}_{Dh}$. We can write $p_{Dh}(\varphi)$ as

$$p_{Dh}(\varphi) = W \left( \frac{\varphi}{\varphi_{Dh}} \right) \frac{(w_s)_{\beta h} (w_u)_{1-\beta h}}{\varphi}.$$

Then, $\tilde{p}_{Dh}$ is given by,

$$\tilde{p}_{Dh} = \int_{\varphi_{Dh}}^{\infty} W \left( \frac{\varphi}{\varphi_{Dh}} \right) \frac{(w_s)_{\beta h} (w_u)_{1-\beta h}}{\varphi} k \varphi^{k+1} d\varphi.$$  

Applying change of variables, $x = \varphi / \varphi_{Dh}$, we have that,

$$\tilde{p}_{Dh} = v(k) \frac{(w_s)_{\beta h} (w_u)_{1-\beta h}}{\varphi_{Dh}}.$$

Where, $v(k) = k \int_{1}^{\infty} W(xe) / x^{k+2} dx$ is a constant function of the parameter $k$. Analogously, we have that,
\[
\tilde{p}_{Xh} = v(k)\tau_h \frac{(w_s)^{\beta_h}(w_u)^{1-\beta_h}}{\varphi_{Xh}}.
\]

From equation (16), we have that,
\[
\frac{(w_s)^{\beta_h}(w_u)^{1-\beta_h}}{\varphi_{Dh}} = \tau_h \frac{(w_s)^{\beta_h}(w_u)^{1-\beta_h}}{\varphi_{Xh}}.
\]

Thus, \( \tilde{p}_{Dh} = \tilde{p}_{Xh} = \tilde{p}_h \). Repeating this argument, we have that \( \tilde{p}_{Dh} = \tilde{p}_{Xh} = \tilde{p}_h \).

For the second part, we start by finding \( \tilde{\ln p}_{Dh} \). Using the equation for \( p_{Dh}(\varphi) \) that was used before and the property of lambert function \( W \), that \( \ln |W(x)| = \ln x - W(x) \forall x > 0 \), we have that,
\[
\tilde{\ln p}_{Dh} = \ln \left( \frac{(w_s)^{\beta_h}(w_u)^{1-\beta_h}}{\varphi_{Dh}} \right) - \tilde{\mu}(k).
\]

Integrating both sides between \( \varphi_h \) and \( \infty \).
\[
\tilde{\ln p}_{Dh} = \ln \left( \frac{(w_s)^{\beta_h}(w_u)^{1-\beta_h}}{\varphi_{Dh}} \right) - \tilde{\mu}(k).
\]

Analogously, for the prices of imported goods
\[
\tilde{\ln p}_{xh} = \ln \left( \frac{(w_s)^{\beta_h}(w_u)^{1-\beta_h}}{\varphi_{Xh}} \right) - \tilde{\mu}(k).
\]

From relation (16), and we have that \( \frac{(w_s)^{\beta_h}(w_u)^{1-\beta_h}}{\varphi_{Dh}} = \tau_h \frac{(w_s)^{\beta_h}(w_u)^{1-\beta_h}}{\varphi_{Xh}} \). Then, \( \tilde{\ln p}_{Dh} = \tilde{\ln p}_{xh} = \tilde{\ln p}_h \). Again, we can repeat this process and show that, \( \tilde{\ln p}_{Dh} = \tilde{\ln p}_{xh} = \tilde{\ln p}_h \).

**Proposition 3.3.** The mass of available goods in each industry depends only on the substitutability among varieties;
\[
N_h = \frac{1}{\gamma_h(\ln \hat{p}_h - \ln \tilde{p}_h)} = \frac{1}{\gamma_h \hat{\mu}(k)} = N_h^*.
\]

**Proof.** From 3.2 we know that,
\[
\tilde{\ln p}_h = \ln \left( \frac{(w_s)^{\beta_h}(w_u)^{1-\beta_h}}{\varphi_{Dh}} \right) - \tilde{\mu}(k) = \ln \hat{p}_h - \tilde{\mu}(k).
\]

Rearranging this we have that \( \ln \hat{p}_h - \tilde{\ln p}_h = \tilde{\mu}(k) \). From definition of \( \hat{p}_h \) we have that
\[
N_h = \frac{1}{\gamma_h(\ln \hat{p}_h - \ln \tilde{p}_h)}.
\]

Substituting \( \ln \hat{p}_h - \tilde{\ln p}_h = \tilde{\mu}(k) \) we have the result. \( N_h \) is constant and equal for both countries.

**Proposition 3.4.** Expenditures on entry investment employment are equal to profits in each sector.
\[
N_{Eh}f_E(w_s)^{\beta_h}(w_u)^{1-\beta_h} = \Pi_h.
\]
Proof. Total profit of industry \( h \), \( \Pi_h \), include profit on domestic market and on export market. It is

\[
\Pi_h = \int_{\varphi_Dh}^{\infty} \pi_Dh(\varphi)g(\varphi|\varphi \geq \varphi_Dh) \, d\varphi + \int_{\varphi_Xh}^{\infty} \pi_Xh(\varphi)g(\varphi|\varphi \geq \varphi_Xh) \, d\varphi
\]

\[
= N_Dh \gamma \, I_h \, \tilde{\chi}(k) + N_Xh \gamma \, I^*_h \, \tilde{\chi}(k)
\]

Using FEC, we have that;

\[
N_{eh} \int_{w_s}^{w_u} \left( \frac{\psi I_h}{\varphi_{rh}} + \frac{\psi^* I_h}{\varphi_{rh}^*} \right)
\]

\[
= N_{eh} \left[ \left( \frac{\varphi}{\varphi_{Dh}} \right)^k \gamma \, I_h \, \tilde{\chi}(k) + \left( \frac{\varphi}{\varphi_{Xh}} \right)^k \gamma^* I_h \, \tilde{\chi}(k) \right]
\]

\[
= \Pi_h
\]

Proposition 3.5. There exists a unique costly trade equilibrium referenced by the equilibrium vector, \( \{ \varphi_{Dx}, \varphi^*_{Dx}, \varphi_{Dy}, \varphi^*_{Dy}, \varphi_{Xx}, \varphi^*_{Xx}, \varphi_{Xy}, \varphi^*_{Xy}, p_{Dx}(\varphi), p^*_{Dx}(\varphi), p_{Dy}(\varphi), p^*_{Dy}(\varphi), p_{Xx}(\varphi), p^*_{Xx}(\varphi), p_{Xy}(\varphi), p^*_{Xy}(\varphi), \mu_{Dx}(\varphi), \mu^*_{Dx}(\varphi), \mu_{Dy}(\varphi), \mu^*_{Dy}(\varphi), \mu_{Xx}(\varphi), \mu^*_{Xx}(\varphi), \mu_{Xy}(\varphi), \mu^*_{Xy}(\varphi), R_x, R_y, R^* x, R^* y, w_s, w_u, w^*_s, w^*_u \} \).

Proof. We choose \( w_s = 1 \) as numerary. Then we use labour market clearing conditions, Shepard’s Lemma and cost functions to determine labour demand, we have that;

\[
L^E_{sh} = N_{eh} \int_{w_s}^{w_u} \beta_h(\frac{\psi \varphi}{\psi_{rh}} + \frac{\psi \varphi^*}{\psi_{rh}^*}) \, \frac{1}{\psi_{rh}^{1-\beta_h}}
\]

\[
L^D_{sh}(\varphi) = \frac{\beta_h(\frac{\psi \varphi}{\psi_{Dh}})^{1-\beta_h}}{\psi_{Dh}(\varphi)}
\]

\[
L^X_{sh}(\varphi) = \frac{\tau_h \beta_h(\frac{\psi \varphi}{\psi_{Xh}})^{1-\beta_h}}{\psi_{Xh}(\varphi)}
\]

Then,

\[
L^D_{sh} = N_{Dh} \int_{\varphi_{Dh}}^{\infty} L^D_{sh}(\varphi)g(\varphi|\varphi > \varphi_{Dh}) \, d\varphi
\]

\[
= N_{Dh} \frac{\beta_h \gamma \, I_h \, \tilde{\phi}(k)}{w_s}
\]

\[
L^X_{sh} = N_{Xh} \int_{\varphi_{Xh}}^{\infty} L^X_{sh}(\varphi)g(\varphi|\varphi > \varphi_{Xh}) \, d\varphi
\]

\[
= N_{Xh} \frac{\beta_h \gamma \, I^*_h \, \tilde{\phi}(k)}{w_s}
\]

Again, we have that \( \tilde{\phi}(k) = k \int_1^{\infty} \frac{W(x-1)}{W(x)} \frac{1}{x^{1+1}} \, dx = \tilde{\mu}(k) - \tilde{\chi}(k) \).
Then,
\[ L_{sh} = N_{Eh} f_E \beta_h (w_u/w_s)^{1-\beta_h} + N_{Dh} \frac{\beta_h \gamma_h I_h \tilde{\phi}(k)}{w_s} + N_{Xh} \frac{\beta_h \gamma_h I'_h \tilde{\phi}(k)}{w_s}. \]

And,
\[ w_s L_{sh} = N_{Eh} f_E \beta_h (w_u)^{1-\beta_h} + N_{Dh} \beta_h \gamma_h I_h \tilde{\phi}(k) + N_{Xh} \beta_h \gamma_h I'_h \tilde{\phi}(k). \]

Analogously,
\[ w_u L_{ah} = N_{Eh} f_E (1 - \beta_h)(w_s)^{1-\beta_h} + N_{Dh} (1 - \beta_h) \gamma_h I_h \tilde{\phi}(k) + N_{Xh} (1 - \beta_h) \gamma_h I'_h \tilde{\phi}(k). \]

Calculating for Foreign Country is analogously. Then, we will have a system of equations:
\[
\begin{align*}
    w_s T_s &= N_{Ex} f_E \beta_x (w_s)^{\alpha_x}(w_u)^{1-\beta_x} + N_{Dx} \beta_x \gamma_x I_x \tilde{\phi}(k) + N_{Xx} \beta_x \gamma_x I'_x \tilde{\phi}(k), \\
    w_u T_u &= N_{Ey} f_E \beta_y (w_s)^{\alpha_y}(w_u)^{1-\beta_y} + N_{Dy} \beta_y \gamma_y I_y \tilde{\phi}(k) + N_{Xy} \beta_y \gamma_y I'_y \tilde{\phi}(k), \\
    w_s T'_s &= N_{Ex}^* f_E \beta_x^* (w_s^*)^{\alpha_x}(w_u^*)^{1-\beta_x} + N_{Dx}^* \beta_x \gamma_x I_x^* \tilde{\phi}(k) + N_{Xx}^* \beta_x \gamma_x I'_x^* \tilde{\phi}(k), \\
    w_u T'_u &= N_{Ey} f_E \beta_y^* (w_s^*)^{\alpha_y}(w_u^*)^{1-\beta_y} + N_{Dy}^* \beta_y \gamma_y I_y^* \tilde{\phi}(k) + N_{Xy}^* \beta_y \gamma_y I'_y^* \tilde{\phi}(k).
\end{align*}
\]

Solving this system for wages, we will find the wage vector \([1, w_u, w_s^*, w_u^*] \] as functions only of comparative advantage parameters, exogenous parameters and endowments.

Using the wage vector, we can find the cutoffs, \([\varphi_{Dx}, \varphi_{Dx}', \varphi_{Dy}, \varphi_{Dy}', \varphi_{Xx}, \varphi_{Xx}', \varphi_{Xy}, \varphi_{Xy}'] \].

Once we have the cutoffs, we can use,
\[ \mu_{rh}(\varphi, \varphi_{rh}) = \mathcal{W} \left( \frac{\varphi}{\varphi_{rh}} \right) - 1, \]

and find all markups, \([\mu_{Dx}(\varphi), \mu_{Dx}^*(\varphi), \mu_{Dy}(\varphi), \mu_{Dy}^*(\varphi), \mu_{Xx}(\varphi), \mu_{Xx}^*(\varphi), \mu_{Xy}(\varphi), \mu_{Xy}^*(\varphi)] \).

Now, using mark-ups and wage vector, we find all prices, \([p_{Dx}(\varphi), p_{Dx}^*(\varphi), p_{Dy}(\varphi), p_{Dy}^*(\varphi), p_{Xx}(\varphi), p_{Xx}^*(\varphi), p_{Xy}(\varphi), p_{Xy}^*(\varphi)] \).

Equations (16), (17), (22), (23) and (24), and wage vector, determine \(N_{ph} \) and \(N_{ph}' \), and the mass of entrants firms \(N_{Eh} \) and \(N_{Eh}' \).

Once we have the pools and cutoffs, we can determine the mass of domestic and exporting firms in both countries. Then, we finally can determine the total revenue on each industry in each country, \([R_x, R_y, R^*_x, R^*_y] \).

\[ \square \]

**Proposition 4.1.** The opening of costly trade increases the steady-state zero-profit-productivity cut-off and average industry productivity in both industries.

1. Other things equal, the increase in the steady-state ZPC and average industry productivity is greater in a country’s comparative industry: \(\Delta \varphi_{Dx} > \Delta \varphi_{Dy} \) and \(\Delta \varphi_{Dy} > \Delta \varphi_{Dx}^* \).

2. Other things equal, the exporting productivity cut-off is closer to the ZPC in a country’s comparative industry: \(\varphi_{Xx}/\varphi_{Dx} < \varphi_{Xy}/\varphi_{Dy} \) and \(\varphi_{Xy}/\varphi_{Dy} < \varphi_{Xx}/\varphi_{Dx}^* \).

41
Proof. Using the FEC for the costly trade and the closed economy, we have that the costly trade expected value of entry is equal to the value for the closed economy plus a positive term reflecting the possibility of firms draws a productivity high enough to export. From the relationship of the cut-offs we have that \( \varphi_{Xh} = \Lambda_h \varphi_{Dh} \), where, \( \Lambda_h = \tau \bar{p}_h/\bar{p}_h^* > 1 \). FEC is monotonically decreasing in \( \varphi_{Dh} \), we have that \( \varphi_{Dh} \) must be higher in the costly trade equilibrium. Average productivity is monotonically increasing in \( \varphi_{Dh} \), so we have that \( \bar{\varphi}_{Dh} \) is higher too.

1. Using the definitions of \( \bar{p}_h, \bar{p}_h^* \) and \( N_h \), we have that,

\[
\frac{\bar{p}_x}{\bar{p}_y} = \frac{e^{\ln p_x}}{e^{\ln p_y}}.
\]

Let’s use a lemma that will be helpful.

Lemma \( \Lambda_x < \Lambda_y \) and \( \Lambda_x^* > \Lambda_y^* \).

Proof. Note that we need to show that \( \Lambda_x/\Lambda_y = \frac{\tau_x \bar{p}_x \bar{p}_y}{\tau_y \bar{p}_y} < 1 \). So, we have to show that:

\[
\frac{\tau_x \bar{p}_x}{\tau_y \bar{p}_y} < \frac{\bar{p}_x^*}{\bar{p}_y^*},
\]

\[
\frac{\tau_x e^{\ln p_x}}{\tau_y e^{\ln p_y}} < \frac{e^{\ln p_x^*}}{e^{\ln p_y^*}},
\]

\[
\ln \left( \frac{\tau_x}{\tau_y} \right) + \ln p_x - \ln p_y < \ln p_x^* - \ln p_y^*.
\]

Using \( \ln p_h \) of closed economy, we have that,

\[
\ln p_x - \ln p_y = (\beta_y - \beta_x) \ln \frac{w_u}{w_s} + \ln \frac{\varphi_y}{\varphi_x}.
\]

We can show that \( \ln(\varphi_y/\varphi_x) = 1/k[\ln(\gamma_y/\gamma_x) + \ln((1 - \alpha)/\alpha)] \), thus:

\[
\ln p_x - \ln p_y = \frac{k + 1}{k} (\beta_y - \beta_x) \ln \frac{w_u}{w_s} + \frac{1}{k} \left[ \ln \frac{\gamma_y}{\gamma_x} + \ln \frac{1}{\alpha} \right],
\]

where \( \beta_x > \beta_y \) and \( 1/k[\ln(\gamma_y/\gamma_x) + \ln((1 - \alpha)/\alpha)] \) is equal in both countries. In the closed economy with a larger relative supply of skilled labour is characterized by higher relative wage of unskilled workers, \( w_u/w_s \). From the last equation, this implies \( \ln p_x - \ln p_y < \ln p_x^* - \ln p_y^* \).

\( \tau_x \) and \( \tau_y \) are both parameters of the model, thus, we make the assumption that \( \ln \left( \frac{\tau_x}{\tau_y} \right) \) is such that \( \ln \left( \frac{\tau_x}{\tau_y} \right) + \ln p_x - \ln p_y < \ln p_x^* - \ln p_y^* \) remains true.

Using the last lemma and the FEC again we have that \( \Delta \varphi_{Dx} > \Delta \varphi_{Dy} \) and \( \Delta \varphi_{Dy}^* > \Delta \varphi_{Dx}^* \).
2. Follows immediately from the above since $\phi_{Xh} = \Lambda_h \phi_{Dh}$. \hfill \Box

**Proposition 4.2.** The opening of costly trade increases steady-state average firm output in both industries, and other things equal the largest increase occurs in the comparative advantage industry.

**Proof.** In autarky, we have that:

$$\tilde{p}_h = \left(\bar{w}_s \bar{w}_u \frac{\beta_h}{\varphi_h} \frac{\varphi_h}{v(k)} \right),$$

where $v_1(k) = \int_1^{\infty} W(x,e) k/x^{k+2} dx$ is a constant. We also have that,

$$\bar{c}_h = \left(\bar{w}_s \bar{w}_u \frac{\beta_h}{\varphi_h} \frac{\varphi_h}{v'(k)} \right),$$

where $v'(k) = k/k + 1$ is a constant. We also have that,

$$\Pi_h = \gamma_h I_h \tilde{\chi}(k).$$

Note that $\Pi_h = \pi_h \phi^k / \varphi^k$.

Using this and the FEC, we have that:

$$\Pi_h = \frac{\phi^k}{\varphi^k} \tilde{\chi}(k) = \tilde{\chi}(k) \varphi(k) = \delta_f E.$$

Analogously, in the costly trade economy we can rewrite the FEC:

$$\left[ \frac{\tilde{y}_{Dh}}{(\varphi_{Dh})^{k+1}} + \frac{\tilde{y}_{Xh} \tau_h}{(\varphi_{Xh})^{k+1}} \right] \varphi^k (v(k) - v'(k)) = \delta_f E.$$

Thus, we have that,

$$\frac{\tilde{y}_h}{\varphi^k} = \frac{\tilde{y}_{Dh}}{\varphi_{Dh}} + \frac{\tilde{y}_{Xh} \tau_h}{\varphi_{Xh}}.$$ (29)

Since $\varphi_{Dh} > \varphi_h$ we can conclude that we must have $\tilde{y}_{Dh} + \tau_h \tilde{y}_{Xh} > \tilde{y}_h$.

This is true for both industries. However, using 4.1 we know that $\varphi_{Dh}$ will be higher in the comparative advantage industry, so this is true for steady-state average firm output too. \hfill \Box

**Proposition 4.3.** The opening of costly trade magnifies ex ante cross-country differences in comparative advantage by inducing endogenous Ricardian productivity differences at the industry level that are positively correlated with Heckscher–Ohlin-based comparative advantage.

**Proof.** From 4.1, there is a larger increase in the zero-profit productivity cut-off, $\varphi_{Dh}$, in the country’s comparative advantage industry, which results in a larger increase in weighted average productivity, $\tilde{\varphi}_{Dh}$, in the comparative advantage industry. Since this is true for both countries, the opening of costly trade results in the emergence of endogenous Ricardian productivity differences at the industry level, which are positively correlated with Heckscher–Ohlin-based comparative advantage $\varphi_{Dx} / \varphi_{Dy} > \varphi_{Dx} / \varphi_{Dy}$. \hfill \Box
Proposition 4.4. The opening of costly trade has three sets of effects on the real income of skilled and unskilled workers:

1. The relative nominal reward of the abundant factor rises and the relative nominal reward of the scarce factor factor falls.

2. The rise in the zero production cutoff reduces average variety prices in both industries and so reduces consumer price indices.

3. The rise in industry productivity cutoff reduces the mass of firms producing domestically, then, it rises consumer price indices. However, the opportunity to import foreign varieties rises the available mass of goods in the economy, and then, it reduces consumer price indices. These two effects combined does not have any effect on consumer price indices.

Proof. 1. The relative unskilled wage under costly trade lies in between the two countries’ autarkic values, converging in each country to the autarkic value as trade costs become infinite and converging to the free trade value as trade costs approach 0. Since the home country is skill abundant and the foreign country is unskilled labour abundant \( (L_s/L_u > L_{s\text{World}}/L_{u\text{World}} > L^*_s/L^*_u) \), the opening of costly trade leads to a rise in the relative skilled wage and a reduction in the relative unskilled wage in the skill-abundant home country.

2. Prices are monotonically decreasing in \( \varphi_{rh} \); from \( 4.1 \) we know that \( \varphi_{rh} \uparrow \), so we have that \( \ln p_h \downarrow \), and consequentially, reduces consumer price indices.

3. We have that \( N_h = N_{Dh} + N^*_{Xh} \) is constant. However, in the closed economy \( N^*_{Xh} = 0 \), thus, the opening of costly trade makes \( N^*_{Xh} > 0 \) and \( N_{Dh}^A > N_{Dh}^C \). The first effect reduces consumer prices indices and second increases. But, combined both effects, \( N_h \) still the same constant and there is no effect in consumer prices indices.

Proposition 4.5.

1. The opening of costly trade results in net job creation in the comparative advantage industry and net job destruction in comparative disadvantage industry.

2. The opening of costly trade results in simultaneous gross job destruction in both industries, so that gross job changes exceed net job changes, and both industries experience excess job reallocation.

Proof. 1. From cost minimization and factor market clearing, the fall in the relative unskilled wage in the skill-abundant country following the opening of costly trade leads to a rise in the share of both skilled and unskilled labour used in the skill-intensive industry. With unchanged factor endowments, this implies net job creation in the comparative advantage industry and net job destruction in the comparative disadvantage industry.

2. We have that,

\[
y_{rh}(\varphi, \varphi_{rh}) = \frac{\gamma_h I}{C_h(\varphi)} \left( \frac{\mu_{rh}(\varphi, \varphi_{rh})}{1 + \mu_{rh}(\varphi, \varphi_{rh})} \right). \]

44
Since $\mu_{rh}(\varphi, \varphi_{rh})$ is monotonically decreasing in $\varphi_{rh}$, we have that $y_{rh}(\varphi, \varphi_{rh})$ also is monotonically decreasing in $\varphi_{rh}$. Thus, we have gross job destruction because $\varphi_{Dh}^{CT} > \varphi_{Dh}^{A}$, and less productivity firms gets out of the market; furthermore the output of the firms that only produce to the domestic market gets smaller. However, from 4.2 we know that average output gets higher, thus, those firms that export in costly trade must produce more and we have gross job creation.

Proposition 4.6. The opening of costly trade reduces $\hat{p}_h$ in both industries, this effect is higher in the comparative advantage industry.

Proof. We can write $\hat{p}_h$ in following way:

$$\hat{p}_h = \frac{(w_s)^{\beta_h} (w_u)^{1-\beta_h}}{\varphi_{Dh}}.$$  

We know that $\varphi_{Dh}^{CT} > \varphi_{Dh}^{A}$, so we have that $\hat{p}_h$ reduces in both industries; $\varphi_{Dh}$ is higher in the comparative advantage industry, thus, the effect is higher in this industry.

Proposition 4.7. The opening of costly trade leads to a larger increase in steady-state creative destruction of firms in comparative advantage industry than in comparative disadvantage industry.

Proof. The steady-state rate of creative destruction corresponds to the steady-state probability of firm failure:

$$\Psi_h = \frac{G(\varphi_{Dh})N_{Eh} + \delta N_{ph}}{N_{Eh} + N_{ph}}.$$  

The equilibrium condition in steady state is $\delta N_{ph} = N_{Eh}$. Thus, substituting $N_{Eh}$:

$$\Psi_h = \frac{\delta(G(\varphi_{Dh}) + 1)}{1 + \delta}.$$  

Thus, $\Psi_h$ is monotonically increasing in $\varphi_{Dh}$; by 4.1 we conclude that $\Psi_h$ grows more in the comparative advantage industry.

Proposition 4.8. The opening of costly trade reduces mark-ups in all surviving firms of the market; this effect is higher to firms in comparative advantage industry. However, average mark-ups does not change in both industries.

Proof. We know that $\partial \mu_{rh}(\varphi, \varphi_{rh})/\partial \varphi_{rh} < 0$; thus mark-ups will reduces to those firms that still in the market with opening of costly trade. By 4.1 we know that this effect will be higher in comparative advantage industry. The average mark-up in both industries in given by a constant function of parameter $k$, $\tilde{\mu}(k)$, and it does not change with the opening of costly trade.

Proposition 4.9. In the opening of costly trade, we have that:

$$N_{Dx}/N_{Xx} < N_{Dy}/N_{Xy} \text{ and } N_{Dx}^*/N_{Xx}^* > N_{Dy}^*/N_{Xy}^*.$$  

45
Proof. Using 4.1, we have that $\frac{\varphi_{Xx}}{\varphi_{Dx}} < \frac{\varphi_{Xy}}{\varphi_{Dy}}$. It means that $\varphi_{Xh}$ is closer to $\varphi_{Dh}$ at comparative advantage industry, thus, the percentage of firms that export will be higher in comparative advantage industry. This is true for the Foreign Country too. Then, we can conclude that $\frac{N_{Dx}}{N_{Xx}} < \frac{N_{Dy}}{N_{Xy}}$ and $\frac{N_{Dx}^*}{N_{Xx}^*} > \frac{N_{Dy}^*}{N_{Xy}^*}$. $\square$