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Rafael Moura Azevedo

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RAFAEL MOURA AZEVEDO

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Resumo

Esta tese é composta de três artigos sobre finanças. O primeiro tem o título "Nonparametric Option Pricing with Generalized Entropic Estimators " e estuda um método de apreçamento de derivativos em mercados incompletos. Este método está relacionado com membros da família de funções de Cressie-Read em que cada membro fornece uma medida neutra ao risco. Vários testes são feitos. Os resultados destes testes sugerem um modo de definir um intervalo robusto para preços de opções. Os outros dois artigos são sobre anúncios agendados em diferentes situações. O segundo se chama "Watching the News: Optimal Stopping Time and Scheduled Announcements"e estuda problemas de tempo de parada ótimo na presença de saltos numa data fixa em modelos de difusão com salto. Fornece resultados sobre a otimalidade do tempo de parada um pouco antes do anúncio. O artigo aplica os resultados ao tempo de exercício de Opções Americanas e ao tempo ótimo de venda de um ativo. Finalmente o terceiro artigo estuda um problema de carteira ótima na presença de custo fixo quando os preços podem saltar numa data fixa. Seu título é "Dynamic Portfolio Selection with Transactions Costs and Scheduled Announcement"e o resultado mais interessante é que o comportamento do investidor é consistente com estudos empíricos sobre volume de transações em momentos próximos de anúncios.

Palavras-chave: Apreçamento de Opções, Métodos Não-Paramétricos, Anúncios Agendados, Tempo de Parada Ótimo, Problemas de Portfólio Ótimo, Métodos Numéricos.

Abstract

This thesis is comprised of three articles about finance. The first one has the title "Nonparametric Option Pricing with Generalized Entropic Estimators" and studies a pricing method in incomplete markets. This method is linked to members in the Cressie-Read family function with each one providing one risk-neutral measure. Several tests are performed. The results suggest a way to define a robust intervals for option prices. The others two articles are about scheduled announcements in different settings. The second one is titled "Watching the News: Optimal Stopping Time and Scheduled Announcements" and studies optimal stopping times problems in the presence of a jump at a fixed time. It provides results concerning the optimality to whether stop or not just before the news. It applies such results to the optimal time to exercise time of an American Option and to the optimal time to sell an asset. Finally the third article numerically studies an optimal portfolio problem with fixed cost when the price may jump at a fixed date. It is called "Dynamic Portfolio Selection with Transactions Costs and Scheduled Announcement" and the most interesting result is that the trading behavior of the investor is consistent with empirical findings for trading volume around announcements.

Keywords: Option Pricing, Nonparametric Methods, Scheduled Announcements, Optimal Stopping Time, Optimal Portfolio Problems, Numerical Methods.

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Chapter 1

Introduction

This thesis is comprised of three articles about finance. The first one studies a derivative pricing method in incomplete markets, the second one characterizes optimal stopping time problems in the presence of scheduled announcement and the third article solves numerically an optimal portfolio problem with fixed cost for each transaction in the presence of scheduled announcements.

The first article is a joint work with Caio Almeida and has the title: "Nonparametric Option Pricing with Generalized Entropic Estimators". It investigates a generalization of a nonparametric pricing method proposed in Stutzer (1996). Originally, the method obtains a risk-neutral measure from a price history of the underlying asset and uses the entropy concept found in Information Theory (equivalent to the physical entropy). Such concept is used as a way to define distance between two probability distributions. Our work uses a generalization of the entropy distance by applying the Cressie-Read (CR) family function. In this case, each element in the CR family provides a different risk-neutral measure. It allows us to define an interval of possible prices for derivatives similar to what is proposed in Bernardo and Ledoit (2000) and Cochrane and Saa-Requejo (2000).

The method only makes very general hypothesis about the data generating process (DGP) and is applied in the same way whether the prices are obtained from the Black-Scholes Model, Jump-diffusion model or real world data. Thus the first article's main goal is to assess if the method is able to give a good estimate of derivatives prices for different DGPs. We test it for the Black-Scholes (B&S) model, for Heston model and for an affine jump diffusion model. The later model is the closest to the real world data as argued by some authors (for instance, as argued by Broadie *et al.* (2007)). It is an incomplete model and we use the parameters estimated by Eraker *et al.* (2003) using only the S&P 500 Index data and by Broadie *et al.* (2007) using option data.

We obtain encouraging results. The numerical tests for B&S model suggest that some member of the CR family provides the theoretical derivative price. This is indeed the case and we show that the SDF implied by the B&S model is the same as one implied by the non-parametric pricing method for a specific member of the CR family. The numerical tests for the other two models are encouraging also. In particular, the tests for the Jump-Diffusion model suggest that the prices provided by the method are close to the theoretical ones for several members in CR family. Nonetheless, in this case, the optimal member seems to depend upon the derivative maturity.

The second and third articles are about scheduled announcements in different settings. There is a voluminous literature about this issue with articles published in the most important journals of finance, economics and accounting (see Bamber *et al.* (2011) for a review). We are interested in the price behavior and the trading volume around that type of news. The information contained in it is incorporated into securities' prices very quickly. The bulk of the change can be seen within 5 minutes after the announcement in some markets. Such behavior suggests we can model the information arrival with a jump in the price process, i.e., the price as function of time is continuous except for a few points (the jumps). The last two articles model the scheduled announcement as a jump in the price process occurring (with positive probability) at a fixed time known by the agents. The articles consider also situations in which the price doesn‘t jump but the price process parameters may change at a fixed date.

The article “Watching the News: Optimal Stopping Time and Scheduled Announcements” studies optimal stopping times problems in the presence of a jump at a fixed time. It characterizes situations in which it is not optimal to stop just before the jump. The results may be applied to the most diverse situations but the paper focus on finance. Note that such type of problems arises naturally in the context of American Options. It is the first application and it is shown that it is never optimal to exercise the option just before the announcement if the payoff is convex. The second application studies the problem of selling an asset in the presence of fixed cost. Depending upon the type of the jump or the utility function, it is not optimal to sell just before the news. A numerical solution is provided for a particular case. The last application is consistent with agents that need to sell the asset for exogenous reason but has time discretion.

The last article titled as “Dynamic Portfolio Selection with Transactions Costs and Scheduled Announcement” is a joint work with Marco Bonomo. It numerically studies an optimal portfolio problem in continuous time when the price may jump at a fixed date. At a given time the investor chooses how much to consume and whether to balance his/her portfolio or not. When there is a trading, i.e., when the investor rebalances the portfolio, it is necessary to pay a fee. We model this fee as a fixed transaction cost implying that the investor rebalances the portfolio infrequently. It is a combined Stochastic Control with Impulse Control and we solve it numerically with a relatively recent method developed in Chancelier et al. (2007). We solve three situations: a jump with high average and low variance, a jump with high variance, and no jump but a random change in risk-free rate. An interesting trading pattern emerges. The first situation is the most striking: the investor has a low chance to transact a little before the announcement but has very high chance to trade just before and just after it paying the fixed cost twice. This is consistent with Chae‘s empirical findings on trading volume behavior around announcements (Chae (2005)). He finds that the trading volume is lower than normal for 10 to 3 days before the scheduled announcements. Then the trading activity is greater than normal the day before the announcement remaining high for a few days after it.

Chapter 2

Nonparametric Option Pricing with Generalized Entropic Estimators

Chapter Abstract[1]

Pricing options in arbitrage-free incomplete markets is a challenging task since there are potentially infinite risk-neutral measures, each giving an alternative price. In such context, we analyze a large family of Cressie Read entropic discrepancy functions. Each discrepancy implies a risk-neutral measure that takes into account specific combinations of higher moments of the underlying return process. Based on a simulation experiment with a DGP for the underlying asset given by a realistic jump-diffusion process, we test the ability of these risk-neutral measures to approach theoretical option prices for different moneyness and maturities. A specific subset of Cressie Read measures is identified to be the most appropriate to price options under the adopted jump-diffusion model. We make use of this subset of measures to suggest robust price intervals for options as opposed to single prices.

Keywords: Risk-Neutral Measure, Option Pricing, Nonparametric Estimation, Robustness, Minimum Contrast Estimators, Cressie–Read Discrepancies.

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2.1 Introduction

The most important information embedded in financial instruments is the state price density (SPD), or the Arrow-Debreu state prices. Arrow-Debreu securities are very simple instruments that pay one unit on one specific state of nature and zero elsewhere, and they are very useful to price any new or exotic financial instrument. The estimation of such SPDs has been a very important topic of research within the financial economics community[2].

As documented by Ait-Sahalia and Lo (1998), there are different ways to estimate state price densities implicit in financial instruments. Some methods focus on the underlying asset price dynamics, others in specifying parametric forms for the state price density, and others in nonparametric estimation of the state price density. Ait-Sahalia and Lo (1998) suggest that nonparametric methods are valuable since they allow for the data to indicate important features of the distributions of financial instruments.

[1] Joint work with Caio Almeida.

[2] See for instance Jarrow and Rudd (1982) , Rubinstein (1994) , Jackwerth and Rubinstein (1996) , Stutzer (1996), Ait-Sahalia and Lo (1998) , and Ait-Sahalia and Duarte (2003).

In an important work on nonparametric pricing, Stutzer (1996) proposed the method of Canonical Valuation (CV) to price interest rate derivatives. Given a panel of asset pricing returns, CV chooses from the set of all risk neutral measures that price those assets, the one that is closest on the Kullback Leibler (KLIC) sense, to the empirical distribution of the returns. Stutzer (1996) applied his method to returns simulated from a controlled "Black-Scholes world"and compared its performance to the Black & Scholes model adopting historical volatility to price options. Results were encouraging: Even not having direct information on the DGP process, the CV method only slightly underperforms the Black-Scholes model. In addition, when applied to real returns data, the CV method produces implied volatilities that follow a smile pattern similar to that of real world option prices.

Subsequently, researchers have identified two important dimensions to further explore the CV method. First, the necessity of testing more realistic returns DGPs in order to verify the robustness of the method to non-Gaussian distributions. Second, the interest in suggesting alternative ways of choosing a risk-neutral measure to price options, given that CV suggests only one specific way based on KLIC. These two venues have been explored separately in the literature.

On the exploration of more robust DGPs, Gray and Newmann (2005) tested the CV method adopting the stochastic volatility model proposed by Heston (1993) as the underlying return process. In their analysis, CV outperforms the Black-Scholes model with historical volatility. More recently Haley and Walker (2010) suggested the adoption of members of the Cressie-Read family of discrepancy functions (Cressie and Read (1984)) as alternative ways of measuring distance in the space of probabilities. They compared the performance of three specific members of this family (Empirical Likelihood, KLIC, and Euclidean Distance) when the DGP for returns was either the log-normal Black-Scholes environment or Heston‘s stochastic volatility environment. They identified that performance of each estimator depends on the number of outliers, and on the options' maturity, and that the best nonparametric estimator changes depending on combinations of those characteristics. They further identified that the Empirical Likelihood estimator achieved overall the best results.

In this paper, we make the following contributions with respect to the previous literature. First, we build on the work of Almeida and Garcia (2009, 2012) and Haley and Walker (2010) by proposing a comprehensive analysis of the Cressie-Read family of discrepancies. Almeida and Garcia (2009, 2012) showed that the Cressie Read family is extremely rich including an infinity of functions each generating a risk-neutral measure that represents a specific nonlinear function of the original returns. In fact, each discrepancy corresponds to a risk-neutral measure coming from an optimization problem with specific weights given to skewness, kurtosis and other higher moments of returns. This is important because in the context of incomplete markets, where the number of states is larger than the number of assets, the CV method is only one specific way of choosing a risk-neutral measure from the infinity of possible measures implied by the observed data. We advocate here in favor of a more robust treatment of option pricing in the spirit of Cochrane and Saa-Requejo (2000) , Bernardo and Ledoit (2000) and Cerny (2003), by providing intervals of prices instead of point values that are usually obtained through a specific parametric model and therefore subject to model misspecification. If the real unknown DGP process for the underlying asset in equity markets leads to fat tailed and skewed returns, analyzing the sensitivity of risk-neutral measures to higher moments of returns can give us important insights on the pricing of derivatives in incomplete markets.

Our second contribution is to propose a more complete DGP process for the underlying asset to analyze the nonparametric option pricing methods based on the Cressie Read family of discrepancies[3]. It is now well documented that returns in equity markets and many other

[3]Previous papers that analyze the performance of the CV method and its variations make use of either a log-normal or Heston stochastic volatility model as DGP. See Gray and Newmann (2005) and Haley and Walker (2010).

markets contain jump components in addition to stochastic volatility. For this reason we choose to work with a jump-diffusion process first suggested by Bates (2000) that is pervasively adopted in the financial economics literature.

There are several studies in the literature on the estimation and testing of affine jump-diffusion models. Bates (2000) finds evidence against some specifications with pure stochastic volatility components or only jumps in returns and is favorable to a model that presents stochastic volatility and correlated jumps in returns and volatility (SVCJ model). Eraker *et al.* (2003), Chernov et al. (2003) and Eraker (2004) estimate variations of the above-mentioned SVCJ model using US stock market data and / or short panels of option prices. Broadie et al. (2007) complements their work by estimating the risk-neutral parameters of the SVCJ model using simultaneously derivatives and spot price data, including a much larger set of option prices in the estimation process.

In the present work, due to its extensive use and to the favorable evidence with respect to its performance, we adopt the SVCJ model as the DGP for returns of the underlying asset. In addition to the favorable evidence, there are some other practical advantages in adopting the SVCJ model. It acknowledges several characteristic of the real world data, there are reliable estimates of objective and risk-neutral parameters of this model available (see Broadie et al. (2007)), and closed-form formulas for option prices (apart from solving an integral) (see Duffie et al. (2000)) that are much less costly and more accurate than Monte Carlo methods. In addition, Broadie and Kaya (2006) describe a method to sample from the exact distribution of the SVCJ model, therefore avoiding the bias introduced on simulations of stochastic volatility models using Euler discretization schemes (see, for example, Duffie and Glynn (1996)).

Our results suggest that when the DGP process is given by the SVCJ model, there is no specific element within the Cressie Read family that performs best when considering options with different moneyness and time to maturity (see Table 2.4). However, an interesting feature of our analysis is that the best Cressie Read probability measures in terms of Mean Absolute Percentage Errors (MAPE) or Mean Percentage Errors (MPE) of options, are in a narrow range of elements of the family ($\gamma \in [-2, -0.9]$ for MAPE and $\gamma \in [-3.7, -1]$ for MPE). Those regions include the Empirical Likelihood risk neutral measure ($\gamma = -1$) found to have the best pricing performance in previous studies that did not include jumps. However, they also include other measures that put more weights on higher moments of returns. Since pricing errors are small for a large range of option maturities and moneynesses, and since the DGP process for the underlying asset is recognized to capture well empirical features of the US equity market, we use these results to suggest a new method to price options. The idea is in the spirit of Cochrane and Saa-Requejo (2000) and Bernardo and Ledoit (2000) to provide intervals of prices for options as opposed to a unique price determined by a specific option pricing model.

In the paper, as a secondary but interesting contribution, we also present an analytical result that shows that the best Cressie Read estimator, when the DGP process follows a log-normal Black and Scholes type distribution, is a specific function of the three parameters that define risk-premia (μ, r, σ) on the Black and Scholes model (see Appendix 2.A). This clearly generalizes Stutzer (1996) who analyzed numerically the properties of KLIC when pricing options under the Black and Scholes model. We show, in particular, that when the parameters adopted by Stutzer (1996) are used the best Cressie Read estimator is not KLIC ($\gamma = 0$) but an element very close to the Empirical Likelihood estimator ($\gamma = -1$).

The rest of the paper is organized as follows: Section 2.2 describes the Canonical Valuation and its extension using the Cressie-Read family. Section 2.3 defines and discusses the SCVJ model and its simulation. Section 2.4 presents the results of a monte carlo experiment when the DGP of the underlying asset follows a SVCJ model. Section 2.5 briefly describes the Cochrane and Saa-Requejo (2000), Bernardo and Ledoit (2000) methodologies and compare those methodologies to our robust price intervals for options. Section 2.6 concludes.

2.2 Risk-Neutral Measures via Canonical Valuation or Cressie-Read

Given a probability space (Ω, F, P), suppose that we are interested in pricing derivatives on a certain underlying asset, whose prices $p_{\tilde{t}}$ are observed under the probability measure P. An assumption of absence of arbitrage guarantees the existence of at least one risk-neutral measure Q equivalent to P under which the discounted price of any asset is a martingale (see Duffie (2001)). In particular, considering an European call with h days to maturity, its price at time t will be given by:

$$C = E_t^Q \left[\frac{\max(p_{t+h} - B, 0)}{(1+r_f)^h} \right], \tag{2.1}$$

where E_t^Q is the conditional expectation operator under Q, B is the option strike, r_f is the risk-free rate for one day, and p_{t+h} is the price of the underlying asset at time $t+h$.

Assuming stationarity and ergodicity of the underlying asset returns under P, we adopt a historical time series of its prices $\{p_{\tilde{t}}\}_{\tilde{t}=t-nh,t-(n-1)h,\dots t-h,t}$ to generate a discrete version of the future price distribution. Each possible future outcome has empirical probability $\pi_k = \frac{1}{n}$ under P and is defined by $p_{t+h}^{(k)} = p_t * R_k$, where R_k is the k_{th} historical return $R_k = \frac{p_{t_0+kh}}{p_{t_0+(k-1)h}}, k = 1, \dots, n$.

In such a context, a sample version of the conditional expectation in Eq. (2.1) can be written as:

$$C = \sum_{k=1}^{n} \pi_k^Q \frac{\max(R_k p_t - B, 0)}{(1+r_f)^h} \tag{2.2}$$

where π_k^Q is the probability of the $k_t h$ outcome under a discrete version of the risk-neutral measure Q.

If the number of observed states n is bigger than the number of primitives assets, the market is incomplete and in general there is an infinity of risk-neutral measures. In this setting, the pricing problem becomes how to properly choose one specific measure π_k^Q from the set of existing risk-neutral measures.

Stutzer (1996) suggested the Canonical Valuation method that consists in choosing the risk-neutral measure π^Q that is closest to the equiprobable objective measure[4] π, by minimizing the Kulback-Leibler Information Criterion (KLIC) between π^Q and π:

$$I(\pi^Q, \pi) = \sum_{k=1}^{n} \pi_k^Q \log\left(\frac{\pi_k^Q}{\pi_k}\right) \tag{2.3}$$

One possible generalization (Walker and Halley (2010), Almeida and Garcia (2009)) is to substitute the KLIC by a more general function that captures the Cressie-Read (CR) family of discrepancies:

$$CR_\gamma(\pi^Q, \pi) = \sum_{k=1}^{n} \pi_k \frac{\left(\frac{\pi_k^Q}{\pi_k}\right)^{\gamma+1} - 1}{\gamma(\gamma+1)} \tag{2.4}$$

where γ defines one function in the CR family.

Note that the KLIC is a particular case of CR:

$$\lim_{\gamma \to 0} CR_\gamma(\pi^Q, \pi) = I(\pi^Q, \pi). \tag{2.5}$$

[4] This is an optimal non-parametric estimator of the objective distribution given some conditions. See Bahadur et al. (1980, sec 3) .

Finally, making π equiprobable ($\pi_k = 1/n, \forall k$) the optimization problem becomes:

$$\begin{aligned}
\pi^{Q^*} &= \arg\min CR_\gamma(\pi^Q, 1/n) && (2.6)\\
& \quad s.t. \\
\sum_{k=1}^{n} \pi_k^Q &= 1 && (2.7)\\
\pi_k^Q &> 0 && (2.8)\\
\frac{1}{(1+r_f)^h} \sum_{k=1}^{n} \pi_k^Q R_k &= 1 && (2.9)
\end{aligned}$$

The first two restrictions guarantee that π^Q is a probability measure, and the last one is a pricing equation that guarantees that it is a risk-neutral measure when primitive assets are the risk-free and the underlying asset.

If the set of known prices at t includes one option with premium $\widetilde{C}$ and strike $\widetilde{B} \neq B$ this information may be added to the set of restrictions:

$$\widetilde{C} = \sum_{k=1}^{n} \pi_k^Q \frac{\max\left(R_k P_t - \widetilde{B}, 0\right)}{(1+r_f)^h}. \tag{2.10}$$

and in this case the primitive assets include not only the risk-free and underlying asset, but also the observed option with price $\tilde{C}$.

2.2.1 The Dual Problem

The number of variables n on the optimization problem above depends on the size of the time series of returns adopted to approximate the future price distribution. In general this number is large what imposes some difficulties on the implementation of this problem. Fortunately Borwein and Lewis (1991) show that this type of convex problem can be solved in a usually much smaller dimensinal dual space. In this case it is possible to show that the moment conditions (Euler Equations) that generate Lagrange Multipliers on the primal problem become the active variables λ on the following dual concave problem:

$$\widehat{\lambda} = \arg\sup_{\lambda\in\Lambda} -\frac{1}{\gamma+1} \sum_{k=1}^{n} \left(1 + \gamma\lambda\left(R_k - (1+r_f)^h\right)\right)^{\left(\frac{\gamma+1}{\gamma}\right)} \tag{2.11}$$

with $\Lambda = \left\{\lambda \in R | \left(1 + \gamma\lambda\left(R_k - (1+r_f)^h\right)\right) > 0 \text{ for all } k\right\}$.

The first order conditions on the problem above allow us to recover the implied risk neutral probability via the following formula:

$$\pi_k^{\gamma,Q} = \frac{\left(1 + \gamma\widehat{\lambda}\left(R_k - (1+r_f)^h\right)\right)^{1/\gamma}}{\sum_{i=1}^{n}\left(1 + \gamma\widehat{\lambda}\left(R_i - (1+r_f)^h\right)\right)^{1/\gamma}} \tag{2.12}$$

In the case of finding a risk-neutral measure that prices the underlying asset and risk-free rate, the dual problem becomes a simple one-dimensional optimization problem.

In what follows below we give a portfolio interpretation for the dual optimization problem that will be important to economically motivate the choice of some specific implied risk neutral measures to price options.

Dual Problem, Utility Pricing and Representative Agent

The dual problem may be interpreted as an optimal portfolio problem. In a interesting way, if the underlying asset is the market portfolio we can relate our work to macro-finance literature[5]. One strand of this literature discusses the ability of equilibrium models to explain option prices. After the 1987's crash it seems crucial to incorporate jumps in the market portfolio process in order to accommodate the smirk in the implied volatility. Moreover a rare disaster in consumption might explain the equity premium puzzle as Barro (2006) argues convincingly with an international panel data.

There is an intimate relation between risk-neutral measure and dynamic equilibrium models. In particular, if one knows the data generating process of asset prices and the risk-neutral density, it is possible to infer the preference of a representative agent in an equilibrium model of asset prices (see Ait-Sahalia and Lo, 1998). The simplest equilibrium model we can relate to our setting, with a *fixed Cressie Read discrepancy*, is a two period model whose agents have the same utility function, consume only on the second period and where the market is complete. The only source of risk is the market portfolio process and each agent is endowed with a fraction of it. In this case, we have a representative agent utility demanding the whole market portfolio and nothing else.

There is a well known beautiful relationship relating the optimization problem of finding an optimal risk-neutral measure in the space of probability measures and solving a representative agent model[6]. It turns out that the latter is the objective function of the dual problem of the former, i.e., the dual problem defined by Equation 2.11 may be interpreted as an optimal portfolio problem with HARA-type utility as shown in Almeida and Garcia (2009):

$$u(W) = -\frac{1}{1+\gamma}(1-\gamma W)^{\frac{\gamma+1}{\gamma}}, \tag{2.13}$$

where

$$W = W_0\left[R_f + \sum \widehat{\lambda}_j (R_j - R_f)\right], \tag{2.14}$$

W_0 is the initial wealth and R_f is the gross risk free rate, R_j is the gross return of the j-th asset and we have the restriction $1-\gamma W > 0$. The connection between the above problem and the dual problem is evident if we define $\widehat{\lambda}$ as

$$\lambda = \frac{-\widehat{\lambda}}{1-\gamma W_0 R_f}, \tag{2.15}$$

and re-write the utility maximization problem as:

$$\sup_{\lambda\in\Lambda} E\left[u(W)\right] = \sup_{\lambda\in\Lambda}\left\{u(W_0 r_f) E\left[(1+\gamma\lambda(R-R_f))^{\frac{\gamma+1}{\gamma}}\right]\right\}. \tag{2.16}$$

2.3 Stochastic Volatility Models with Jumps

As mentioned before, we follow Bates (2000) adopting in this study the stochastic volatility model with correlated jumps as the DGP process for equity returns. It is defined by two stochastic differential equations respectively for the price and volatility of the underlying asset. The jumps in the price and in the volatility process happen at the same time and are therefore correlated. The equity index price, S_t, and its spot variance, V_t, solve:

$$dS_t = S_t\mu dt + S_t\sqrt{V_t}dW_t^s + d\left(\sum_{n=1}^{N_t} S_{\tau_n^-}[e^{Z_n^s}-1]\right) \tag{2.17}$$

[5] Of particular interest here are: Riez (1987), Bates (2000), Barro (2006),Barro et al. (2010), Backus et. al (2011) and Benzoni *et al* (2011) .

[6] See Follmer and Schweizer (2010) and references therein.

$$dV_t = \kappa_v (\theta_v - V_t) dt + \sigma_v \sqrt{V_t} dW_t^v + d\left(\sum_{n=1}^{N_t} Z_n^v\right) \tag{2.18}$$

$$\mu = r_t - \delta_t + \gamma_t - \overline{\mu}_s \lambda \tag{2.19}$$

$$\overline{\mu} = exp(\mu_s + \sigma_s^2/2) - 1 \tag{2.20}$$

where $(dW_t^s,\, dW_t^v)$ is a bi-dimensional Brownian motion with $E[dW_t^s dW_t^v] = \rho t$, N_t is the number of jumps until time t described as a Poisson process with intensity λ; Z_n^v is the $n_t h$ jump in volatility with an exponential distribution with mean μ_v; Z_n^s is associated to the $n_t h$ jump in price with a normal distribution conditional on Z_n^v with mean $(\mu_s + \rho_s Z_n^v)$ and variance σ_s^2; τ_n is the time of $n_t h$ jump, r is the risk-free rate, δ is the dividend yield and γ is the equity premium.

We choose this model mainly because it acknowledges several characteristics found in the empirical works (see Backus, Chernov and Martin, 2012) and because there are reliable estimates of its parameters for the equity market[7].

2.3.1 Chosen Parameters

Note that the market is incomplete whenever there is stochastic volatility and/or jumps with only one underlying and one risk-free asset. A direct consequence is the existence of an infinity many risk-neutral measures consistent with the prices of such assets.[8] The usual way to deal with this issue is to parameterize the possible changes of measure and make specific assumptions about the distributions of jumps. In this context, we follow Duffie *et al.* (2000) and Broadie *et al.* (2007) by considering the following stochastic differential equations under the risk-neutral measure:

$$dS_t = S_t \mu^Q dt + S_t \sqrt{V_t} dW_t^s(Q) + d\left(\sum_{n=1}^{N_t(Q)} S_{\tau_n^-} [e^{Z_n^s(Q)} - 1]\right) \tag{2.21}$$

$$dV_t = \kappa_v^Q (\theta_v - V_t) dt + \sigma_v \sqrt{V_t} dW_t^v(Q) + d\left(\sum_{n=1}^{N_t} Z_n^v(Q)\right) \tag{2.22}$$

$$\mu^Q = r_t - \delta_t - \overline{\mu}_s^Q \lambda^Q \tag{2.23}$$

$$E[dW_t^s(Q) dW_t^v(Q)] = \rho t \tag{2.24}$$

The objective measure's simulations uses the objective measure parameters from Table 2.1. The "correct"option prices are calculated with the formula obtained by Duffie *et al.* (2000) using the risk-neutral parameters from Table 2.2 and SDEs that appear in Equations (2.21) to (2.24). Those parameters are borrowed from estimations in Eraker *et al.* (2003) for the objective measure and from Broadie *et al.*(2007) for the risk-neutral measure. The risk-free rate r is the one year average of the 1-year T-Bill during the year 2000 and the initial volatility is $\sqrt{V_0} = 0.19$.

The absolute continuity requirement implies that some parameters (or combination of parameters) ashould be the same under both measures. In particular in our case, this is true for σ_v, ρ and the product $\kappa_v \theta_v$. We also consider that the arrival intensity is a constant, that $Z_n^s(Q)$ has a normal distribution $N(\mu_s^Q, (\sigma_s^Q)^2)$ and that $Z_n^v(Q)$ has an exponential distribution with mean μ_v^Q.

[7] See Eraker *et al.* (2003), Chernov et al. (2003), Eraker (2004), and Broadie et al. (2007).

[8] In fact, the Girsanov theorem imposes very weak conditions for the jump-distribution change of measure. See for instance the appendix in Pan (2002).

2.3.2 Exact Simulation

It is well known that approximating a continuous time price process by a discrete time process may generate bias on the final price. In general, this bias decreases as the number of steps increases. For the Euler scheme, under certain conditions described by Kloeden and Platen (1992), there is a first order convergence rate. Nonetheless, stochastic volatility processes do not satisfies such conditions. In fact, Broadie and Kaya (2006) find that the bias may be very large in some cases even if a large number of steps are used.

For this reason, we work with exact sampling from the SVCJ by using the method described by Broadie and Kaya (2006). Notice that for the times between jump arrivals, the process behaves exactly as the Heston (1993) model. Therefore, after sampling the jump times and sizes (τ_n,Z^s_n and Z^v_n), there is only the additional necessity of simulating a Heston-type model. To see how this can be done, write the stock price and the variance at t as:

$$S_t = S_u \exp\left[\mu(t-r) - \frac{1}{2}\int_u^t V_s ds + \rho\int_u^t \sqrt{V_s}dW^v_s + \sqrt{1-\rho^2}\int_u^t \sqrt{V_s}dW^2_s\right], \tag{2.25}$$

$$V_t = V_u + \kappa_v\theta_v(t-u) - \kappa_v\int_u^t V_s ds + \sigma_v\int_u^t \sqrt{V_s}dW^v_s, \tag{2.26}$$

where dW^v_t and dW^2_t are independent and dW^s_t is decomposed as:

$$dW^s_t = \rho dW^v_s + \sqrt{1-\rho^2}dW^2_s. \tag{2.27}$$

Cox *et al.*(1985) show that V_t conditional on V_u has a non-central chi-squared distribution. Broadie and Kaya (2006) find the Laplace transform of the distribution of $\int_u^t V_s ds$. The inversion of the Laplace transform can be performed in a optimized way by using the numerical integration method described by Abate and Whitt(1992). After sampling those two quantities, it is possible to obtain $\int_u^t \sqrt{V_s}dW^v_s$. Note that it is easy to sample S_t as its distribution is lognormal conditional on $\int_u^t \sqrt{V_s}dW^v_s$, $\int_u^t V_s ds$ and on V_u. For more details see Broadie and Kaya (2006).

2.4 Results

This section reports results concerning the applicability of the Generalized Entropic Estimators to option pricing in the Black-Scholes-Merton (B&S) and SVCJ models. The SVCJ model is analyzed due to its strong ability to fit stylized facts of the US equity market while the B&S model is adopted because it allows us to obtain analytical properties of the Generalized Entropic Estimators that are helpful in interpreting results from previous studies.

Each model provides a theoretical option price[9] to be benchmarked by our nonparametric Cressie Read risk-neutral measures. A Monte Carlo study generates different realizations for the path of the underlying asset allowing us to approximate the probability distributions of the option pricing errors. For each DGP process we analyze two statistics based on the error probability distributions: The mean percentage pricing error (MPE) and the mean absolute percentage pricing error (MAPE).

Now, consider pricing an European call option on a company X with maturity h, assuming that a history of the stock prices represent the only available information. In our study, the time series of prices (or returns) will be simulated from each adopted DGP.

For a given model (or DGP), we draw a time series of returns with fixed length (in our case, 200 monthly returns) from the model-implied return distribution. Using those returns, we use

[9] The premiums are given as the no-arbitrage price implied by the risk-neutral parameters fixed for each model (B&S, Heston, or SVCJ model).

the Cressie Read method with different values of γ indexing the $CR_\gamma(\cdot)$ function to price the option. For each of 71 equally spaced γ's ranging from -5 to 2 we obtain one option price[10].

The procedure above is repeated 5000 times in order to obtain a distribution for the pricing errors and calculate the MPE and MAPE reported.

2.4.1 Black and Scholes Model

We first test the method adopting the B&S enviroment with the same parameters used in previous works (Stutzer (1996), Gray and Neumann (2005) and Haley and Walker (2010)). The objective measure drift for the stock price is given by $\mu = 10\%$, the volatility by $\sigma = 20\%$, and the risk-free rate by $r = 5\%$. The stochastic differential equation followed by the price is:

$$\frac{dS_t}{S_t} = \mu dt + \sigma dW_t. \tag{2.28}$$

implying a log-normal distribution for stock prices under the probability measure P.

Figure 1 superimposes 20 graphs of MPE versus γ for the entropic method applied to the B&S model. Each graph corresponds to a call option with different maturity and/or moneyness. An apparently striking feature of this picture is that all graphs cross the horizontal line around $\gamma \approx -0.8$. This suggests that in principle the true risk-neutral measure under the specific log-normal DGP might be precisely estimated using our nonparametric family for some γ close to -0.8. We provide theoretical results (see Appendix 2.A) showing that in fact if we fix the Cressie Read parameter γ equal to:

$$\gamma^* = -\frac{\sigma^2}{\mu - r} \tag{2.29}$$

the corresponding Cressie Read risk-neutral measure coincides with the risk-neutral measure implied by the B&S model. That is, from a theoretical viewpoint, if prices satisfies the B&S log-normal dynamics the best Generalized Entropic Estimator will be the one with γ^* from Equation (2.29). Substituting the parameters used in our simulations at Equation (2.29) we obtain $\gamma^* = -0.8$.

Note that both numerical and theoretical results indicate that the most appropriate γ under the B&S DGP is -0.8. This is in accordance to results obtained in previous papers[11] which have found a negative MPE when using $\gamma = 0$ or $\gamma = 1$ and a positive very close to zero MPE when using $\gamma = -1$[12].

2.4.2 SVCJ Model

As pointed out in section 2.3 the SVCJ is a fairly parsimonious model that accomodates several important characteristics of equity prices. It accomodates a realistic process for volatility and a simultaneous jump in price and volatility. We proceed to simulations using the parameter estimates in Eraker *et al.* (2003) and Broadie *et al.* (2007) and price a set of call options with different maturities and moneynesses. The price errors are obtained comparing the prices given by our Generalized Entropic Estimators and the theoretical prices given in closed form by an application of the techniques found in Duffie *et al.* (2000).

Unlike the B&S model, the simulations suggest that there is no clear element of the CR family function for which the MPE is zero. Table 2.4 indicates that the best estimator varies

[10] There are some difficulties to obtain the implied risk-neutral measure for some $\gamma > 0$. For instance, the solution for the optimization problem defined by Equations (2.6)-(2.9) may not exist with $\pi_k^Q > 0$ for some $\gamma > 0$.

[11] See Table 1 on page 994 in Haley and Walker (2010) and Table 1 on page 7 in Gray and Newmann (2005).

[12] Note that $\gamma = -1$, $\gamma = 0$, and $\gamma = 1$ correspond, respectively, to Empirical Likelihood, Euclidean Divergence and KLIC.

with maturity. For instance, the gammas that minimize MPE are close to $\gamma \approx -3.2$ and $\gamma \approx -1.1$ for maturities equal to 1-month and 12-months respectively[13].

On the other hand, table 2.4 indicates that the lowest MAPE is inside a narrower interval $\gamma \in (-2.1, -0.9)$, apart from the two cheapest options. Again it seems a good strategy to consider a set of prices given by the method with an interval of γ's.

It is also possible to draw more informative graphs of MPE (or MAPE) versus γ's for call options with a given maturity and moneyness. These graphs are shown in appendix 2.B for the B&S, Heston and SVCJ models[14].

The overall pattern is that the MPE has a negative slope and it is flat for short maturities. For the B&S and SVCJ models, it crosses the horizontal axis ($MPE = 0$) for a γ within the interval $(-3.7, -.07)$. On the other hand, for the Heston model it does not cross the horizontal axis, at least for $\gamma \in [-5, 0]$ but it approaches zero for γ's close to zero, which represent the Canonical Valuation estimator. Since the SVCJ model represents the Heston model with an extra term for jumps, we concentrate our analysis on only the B&S model (due to the possibility of obtaining analytical results) and on the SVCJ model due to its generality.

2.5 Robust Price intervals

We test the method for different models with typical (or estimated) parameters from U.S. stock market. The performance in terms of MPE is good and relatively similar for the B&S and SVCJ models. In general, the average pricing error is zero for some discrepancy function within the Cressie Read family in almost all cases. The only exception is found on the SVCJ model for the deep out-the-money option with short maturity.

The discrepancy function with best performance in general depends on the DGP, moneyness and option maturity. We show that the best performance γ for the B&S model depends on the model parameters (but not on maturity). On the other hand, simulations suggest that the optimal γ varies with maturity and moneyness for the SVCJ model.

Now, in the spirit of Cochrane and Saa-Requejo (2000) and Bernardo and Ledoit (2000) we propose an alternative way to look at this family of discrepancies. Instead of trying to obtain an optimal dependence of γ on pricing errors based on an specific option pricing model (like B&S, Heston or SVCJ), we suggest using this family to provide intervals of prices for options. These intervals would be obtained by applying the method to an interval of γ's. This would correspond to giving option prices compatible with a set of different HARA utility functions. In particular, this method could be useful for pricing options in illiquid markets or over-the-counter derivatives.

2.5.1 No-Arbitrage Price Interval

The no-arbitrage price interval only requires that there are no-arbitrage opportunities in the market. The no-arbitrage price interval at time t for a European call option with maturity $t+h$ and strike B may be defined by the relations:

$$C \leq S_t, \tag{2.30}$$

$$C \geq \max\left\{0, S_{t+h} - \frac{B}{(1+r^f)^h}\right\} \tag{2.31}$$

[13] For S/B = 1.125 and 1-month maturity MPE is zero for $\gamma \approx -3.7$. Nonetheless the graph MPE versus γ has a very small slope and therefore is more prone to statistical error linked to Monte Carlos studies. See Appendix 2.B for more details.

[14] We also include the Heston model since it isolates the effects of stochastic volatility on our nonparametric risk-neutral measures. In addition, we include it as a matter of completion since as said before, it has been previously analyzed by Grey and Newman (2005) and Waley and Walker (2010).

where $C = E\left[m * \max\left\{0, S_{t+h} - B\right\}\right]$ is the call premium and m is a Stochastic Discount Factor (SDF hereafter) that prices the underlying and risk-free assets.

Note that the call option price can not be higher than the current stock price otherwise one could sell the option, buy the stock to hedge generating an immediate arbitrage. Similarly, if the price is smaller than $S_{t+h} - \frac{B}{\left(1+r^f\right)^h}$ one could buy the option sell the stock and lend $\frac{B}{\left(1+r^f\right)^h}$ generating again an arbitrage[15].

As a matter of convenience we introduce the concept of SDF here, keeping in mind that a given SDF is equivalent to the following risk-neutral measure[16]:

$$m_i = \frac{1}{\left(1+r^f\right)^h} \frac{\pi_i^Q}{\pi_i} \tag{2.32}$$

where m_i is the SDF value at state i.

Note that despite being robust, such price interval isn't very useful in practice since it is too wide. For this reason, some authors have tried to define tighter intervals guided by some set of economic arguments.

2.5.2 Good Deal and Gain-Loss Ratio

Cochrane and Saa-Requejo (2000) define a price interval that rules out too good opportunities. By too good they mean strategies that achieve very high Sharpe ratios. They define an upper bound for the Sharpe ratio and find a price interval that is consistent with this bound.

In order to implement their idea, they use the well known Hansen and Jagannathan (1991) bounds to relate the Sharpe ratio of an arbitrary strategy to the variance of an arbitrary admissible SDF:

$$\frac{E\left[R - \left(1+r^f\right)\right]}{\sigma\left(R\right)} \leq \left\{\frac{\sigma\left(m\right)}{E\left[m\right]}\right\} \tag{2.33}$$

where $\sigma\left(\cdot\right)$ is the standard deviation $\sigma(R) = \sqrt{E\left[\left(R - E\left[R\right]\right)^2\right]}$, R is the return on a strategy and m is any admissible SDF that prices R.

Suppose we want to price an European call option with maturity $t+h$ and strike B, and that we only know the underlying asset price at time t and the risk-free rate r^f. In this case, their price interval is defined by $\left[\underline{C}, \overline{C}\right]$ with:

$$\overline{C} = \max_m E\left[mX\right]$$

$$\underline{C} = \min_m E\left[mX\right] \tag{2.34}$$

where m satifies the following conditions for both optimization problems:

$$m > 0 \tag{2.35}$$

$$E\left[mS_{t+h}\right] = S_t \tag{2.36}$$

$$E\left[m\right] = \frac{1}{(1+r^f)^h} \tag{2.37}$$

$$\sigma(m) \leq \frac{H}{(1+r^f)^h}. \tag{2.38}$$

where H is the maximum Sharpe ratio, $X = \max\left\{0, S_{t+h} - B\right\}$ is the call payoff and $\sigma(m) = \sqrt{E\left[\left(m - E\left[m\right]\right)^2\right]}$.

[15] See Hull (2011) for a detailed explanation.

[16] Assuming for the sake of simplicity that the economy has discrete states of nature.

Similarly, Bernardo and Ledoit (2000) rule out too good trading opportunities by using a different notion for good opportunity. They define the Gain-Loss ratio with respect to a SDF m^* (the benchmark SDF) for an excess payoff (a payoff whose price is zero). The SDF m^* does not need to price correctly the assets and it is used only as a way to define this attractiveness measure. For an excess payoff X^e, the Gain-Loss ratio with respect to m^* is

$$L = \frac{E\left[m^*\left(X^e\right)^+\right]}{E\left[m^*\left(X^e\right)^-\right]} \tag{2.39}$$

where $(X^e)^+ = \max\{X^e, 0\}$ is the positive part of the payoff X^e (the gain) and $(X^e)^- = \max\{-X^e, 0\}$ is the negative part of the payoff X^e (the loss). Note that if m^* prices correctly X^e we have that $L = 1$ because

$$\begin{aligned} E\left[m^* X^e\right] &= 0 \\ E\left[m^*\left(X^e\right)^+ - m^*\left(X^e\right)^-\right] &= 0 \\ E\left[m^*\left(X^e\right)^+\right] &= E\left[m^*\left(X^e\right)^-\right] \end{aligned}$$

leading to

$$L = \frac{E\left[m^*\left(X^e\right)^+\right]}{E\left[m^*\left(X^e\right)^-\right]} = 1. \tag{2.40}$$

In order to implement their method, the authors prove a duality result that implies the following bound:

$$\frac{E\left[m^*\left(X^e\right)^+\right]}{E\left[m^*\left(X^e\right)^-\right]} \leq \frac{\sup_i\left(\frac{m_i}{m_i^*}\right)}{\inf_i\left(\frac{m_i}{m_i^*}\right)} \tag{2.41}$$

where m is any admissible SDF, that is, it correctly prices X^e. Note that it defines a kind of variational measure. For instance, if m^* is constant the right hand side of the above equation would be the ratio of the maximum value to the minimum value of the SDF m:

$$\frac{\sup_i\left(m_i\right)}{\inf_i\left(m_i\right)} \text{ for } m^* \text{ constant.} \tag{2.42}$$

In order to price any new asset, they define an upper bound to the Gain-Loss ratio and implement this idea using the inequality above. For instance, suppose again that we want to price an European call option with maturity $t+h$ and strike B, and we only know the underlying asset price at t and the risk-free rate r^f. Then we would have:

$$\overline{C} = \max_m E\left[mX\right] \tag{2.43}$$

$$\underline{C} = \min_m E\left[mX\right] \tag{2.44}$$

where m satisfies the following conditions for both optimization problems:

$$\begin{aligned} m &> 0 && (2.45)\\ E\left[mS_{t+h}\right] &= S_t && (2.46)\\ E\left[m\right] &= \frac{1}{\left(1+r^f\right)^h} && (2.47)\\ \frac{\sup\left(\frac{m_i}{m_i^*}\right)}{\inf\left(\frac{m_i}{m_i^*}\right)} &\leq \overline{L} && (2.48) \end{aligned}$$

where $\overline{L}$ is the maximum Gain-Loss ratio and $X = \max\{0, S_{t+h} - B\}$ is the call payoff.

Note that the difference between the two methods for the present application appear specifically in Equations (2.38) and (2.48).

2.5.3 Robust Price Interval for $\gamma \in [-4, -0.5]$

In this section, based on the good performance of the Cressie Read implied risk-neutral measures under the SVCJ monte carlo experiment, we suggest intervals of prices for options. Our intervals are different from Cochrane and Saa-Requejo (2000) or Bernardo and Ledoit (2000).While they restrict the family of SDFs fixing one criterion like variance of Gain-Loss ratio, we consider several different discrepancies compatible with dual HARA functions to obtain our price intervals.

Our intervals are robust in the sense that they do not rely on an specific option pricing model that would give a unique price for any option.

Table 2.6 shows price intervals for option prices with different maturities and moneynesses for $\gamma \in [-4, -0.5]$. The underlying asset price at t is $S = 100$ for all cells but the strike B and maturity h change.

We intend to use those intervals to verify if they contain prices of real options written on the S&P 500 index.

2.6 Conclusion

In this work we study the performance of a non-parametric option pricing method when the underlying asset follows a realistic jump-diffusion model. We try to find the risk-neutral measure within a certain class of entropic measures that best proxies, from an option pricing perspective, a given DGP process for the underlying asset.

We simplify solving a minimization problem in the space of risk-neutral measures by solving a optimal portfolio problem on the dual space of returns of the underlying process. By simulating the jump-diffusion process proposed by Bates (2000) with parameters following recent studies in the option pricing literature (Eraker *et. al* (2003), and in Broadie *et. al* (2007)), we show that the most appropriate entropic risk-neutral measures are very sensitive to higher moments of returns in the dual space (γ's ranging between -2 and -1).

We conclude by proposing price intervals for option prices that are obtained by focusing on an interval of γ's that parameterize our entropic family. Such intervals are compatible with giving option prices based on a set of HARA utility functions whose average risk-aversion is parameterized by the γ parameter.

From a pricing perspective, our results might be used to provide robust price intervals for derivatives in illiquid and over the counter markets.

2.A Nonparametric Pricing Method Applied to B&S Model: An Exact Estimation

Here we show that nonparametric method defined in section 2.3 provides the correct derivatives prices when applied to the B&S model when using an appropriate discrepancy function. It means that we obtain the implied risk-neutral distribution implied by B&S model when we solve the optimization problem (equations (2.6)-(2.9)) applied to the returns sampled from B&S model. The adequate discrepancy function belongs to the Cressie-Read family and depends upon the parameter of B&S but doesn't depend upon the maturity of the derivative. When the family is defined by the function $CR_\gamma(\cdot)$ defined in equation (2.4) the appropriate γ is

$$\gamma^* = -\frac{\sigma^2}{\mu - r}. \tag{2.49}$$

More precisely we consider the optimization problem applied to the continuous distribution of returns in B&S model. In this case we show that the Radon-Nikodym derivative obtained by the optimization problem is the same as the Radon-Nikodym derivative implied by the B&S

model. It implies that when the method is applied to a finite sample with γ^* we obtain an approximation for the correct Radon-Nikodym derivative.

We begin by writing the Randon-Nikodym derivative in the B&S model as a function of the returns. Then we write the optimization solution of the method in a suitable way and finally we see that the Randon-Nikodym derivative is the solution for method if γ^* is used[17].

In the Black-Scholes-Merton model, we have for the objective measure:

$$\ln\left(\frac{S_t}{S_u}\right) = \left(\mu - \frac{1}{2}\sigma^2\right)(t-u) + \sigma\left(W_t - W_u\right). \tag{2.50}$$

where μ is the continuous expect rate of return, σ is the volatility and W_t is the Wiener process. This implies that

$$W_t - W_u = \frac{1}{\sigma}\left[\ln\left(\frac{S_t}{S_u}\right) - \left(\mu - \frac{1}{2}\sigma^2\right)(t-u)\right] \tag{2.51}$$

or, defining the gross return between t and u as usual

$$R_{u,t} = \frac{S_t}{S_u}, \tag{2.52}$$

we have

$$W_t - W_u = \frac{1}{\sigma}\left[\ln\left(R_{u,t}\right) - \left(\mu - \frac{1}{2}\sigma^2\right)(t-u)\right]. \tag{2.53}$$

In order to change to risk-neutral measure implied by no-arbitrage conditions one may apply the Girsanov's theorem. In this case, the Radon-Nikodym derivative is:

$$Z(t) = \exp\left\{-\theta(W_t - W_0) - \frac{1}{2}\theta^2 t\right\}, \tag{2.54}$$

where

$$\theta = \frac{\mu - r}{\sigma}, \tag{2.55}$$

and r is the risk-free rate. We can write the $Z(t)$ as a function of return:

$$Z(t) = \exp\left\{-\theta\frac{1}{\sigma}\left[\ln\left(R_{0,t}\right) - \left(\mu - \frac{1}{2}\sigma^2\right)t\right] - \frac{1}{2}\theta^2 t\right\} \tag{2.56}$$

$$Z(t) = \exp\left\{\ln\left(\left(R_{0,t}\right)^{-\frac{\theta}{\sigma}}\right) + \frac{\theta}{\sigma}\left(\mu - \frac{1}{2}\sigma^2\right)t - \frac{1}{2}\theta^2 t\right\}$$

$$Z(t) = \left(R_{0,t}\right)^{-\frac{\theta}{\sigma}}\exp\left\{At\right\}. \tag{2.57}$$

where

$$A = \frac{\theta\mu}{\sigma} - \frac{1}{2}\sigma\theta - \frac{1}{2}\theta^2. \tag{2.58}$$

Remember that the Radon-Nikodym derivative in the Girsanov 's theorem is a martingale with $Z_0 = 1$:

$$E\left[Z_t\right] = 1.$$

[17] For the B&S model, the method using only one pricing equation restriction may be regarded as an Esscher transform. Option pricing with Esscher transform are studied in Gerber and Shiu (1994) along with several others authors discussing it. In particular Y. Yao's response (Gerber and Shiu (1994: page 168-173) shows that the Esscher transform obtains the risk-neutral measure implied by the Black-Scholes model using tools from martingale theory. The argument used here are similar to Yao's.

This implies

$$E\left[(R_{0,t})^{-\frac{\theta}{\sigma}}\right] = \exp\{-At\} \tag{2.59}$$

where $E[.]$ is the expectation in the objective measure. Moreover, we have by the properties of Radon-Nikodym derivative and risk-neutral measure

$$E[R_{0,t}Z(t)] = \widetilde{E}[R_{0,t}] = e^{-rt}. \tag{2.60}$$

The last two relations will be useful to our purpose.

By the other side, define the option maturity as t and make $R = R_{0,t}$. In section 2.2 we have that the solution of the optimization problem implied by the method may be given as the Radon-Nikodym derivative (see Almeida and Garcia(2012) for more detail):

$$\frac{dQ}{dP} = \frac{(1+\gamma\lambda(R-R_f))^{\frac{1}{\gamma}}}{E\left[(1+\gamma\lambda(R-R_f))^{\frac{1}{\gamma}}\right]}.$$

As we are in the Black and Scholes - Merton model, R is lognormal (see equation (2.50)). We will consider the method using only one pricing relation, i.e., the optimization problem only has the restriction

$$\frac{1}{R_f}E^Q[R] = 1,$$

or, equivalently,

$$\frac{1}{R_f}E\left[\frac{dQ}{dP}R\right] = 1, \tag{2.61}$$

along with $1+\gamma\lambda(R-R_f) > 0$ for all states. If there is λ such that equation (2.61) holds and $1+\gamma\lambda(R-R_f) > 0$ we have that this is the solution. Moreover this solution it is unique. This is so because the dual problem is strictly concave (or, equivalently, the primal problem is strictly convex).

In order to continue, define (implicitly) $\widehat{\lambda}$ as

$$\lambda = \frac{\widehat{\lambda}}{\gamma R_f}.$$

Then we have

$$\frac{dQ}{dP} = \frac{\left(1+\gamma\frac{\widehat{\lambda}}{\gamma R_f}(R-R_f)\right)^{\frac{1}{\gamma}}}{E\left[\left(1+\gamma\frac{\widehat{\lambda}}{\gamma R_f}(R-R_f)\right)^{\frac{1}{\gamma}}\right]}$$

$$\frac{dQ}{dP} = \frac{\left(\frac{1}{R_f}\right)^{\frac{1}{\gamma}}\left(R_f+\widehat{\lambda}(R-R_f)\right)^{\frac{1}{\gamma}}}{\left(\frac{1}{R_f}\right)^{\frac{1}{\gamma}}E\left[\left(R_f+\widehat{\lambda}(R-R_f)\right)^{\frac{1}{\gamma}}\right]}$$

$$\frac{dQ}{dP} = \frac{\left(R_f(1-\widehat{\lambda})+\widehat{\lambda}R\right)^{\frac{1}{\gamma}}}{E\left[\left(R_f(1-\widehat{\lambda})+\widehat{\lambda}R\right)^{\frac{1}{\gamma}}\right]}. \tag{2.62}$$

The trick is to make the ansatz $\widehat{\lambda} = 1$.

$$\frac{dQ}{dP} = \frac{R^{\frac{1}{\gamma}}}{E\left[R^{\frac{1}{\gamma}}\right]}$$

and, in order to find the correct γ, compare the above $\frac{dQ}{dP}$ with $Z(t)$. This suggest to choose $1/\gamma = -\theta/\sigma$ as this makes $\frac{dQ}{dP} = Z(t)$ (where t is the maturity and $E\left[R^{\frac{1}{\gamma}}\right] = \exp\{-At\}$ by equation (2.59)). In order to show that this is the solution it is necessary to verify that $1 + \gamma\lambda(R - R_f) > 0$ and that the equation (2.61) holds. Indeed, $1 + \gamma\lambda(R - R_f) = R/R_f > 0$ a.s. because R is lognormal and noting that $R_f = e^{-rt}$ we have

$$\frac{1}{R_f} E\left[\frac{dQ}{dP} R\right] = \frac{1}{e^{-rt}} E\left[Z(t)R\right] = 1,$$

as expected.

2.B MPE and MAPE for Different Models

In this appendix we provide results concerning the Black-Scholes-Merton (B&S) model, Stochastic Volatility (SV) model (Heston model) and Stochastic Volatility with Correlated Jumps (SVCJ) model. The SVCJ model is described in section 2.3. Here we extends the results in Stutzer (1996), Gray and Neumann (2005) and Haley and Walker(2010) for B&S and SV models by exploring a wider set of discrepancy functions. Ours results are consistent with what they found in their work. The parameters used in B&S model are the same as in these 3 articles and the parameters for SV model is the same as in the last 2 articles. The parameters for SVCJ model are borrowed from Eraker et al. (2003) and Broadie et al. (2007).

Here we show graphs of MPE and MAPE against γ and tables reporting for which γ these graphs has zero MPE and the lowest MAPE. The variable γ defines the Cressie-Read function through the function $CR_\gamma(\cdot)$ (equation (2.4)):

$$CR_\gamma\left(\pi^Q, \pi\right) = \sum_{k=1}^{n} \pi_k \frac{\left(\frac{\pi_k^Q}{\pi_k}\right)^{\gamma+1} - 1}{\gamma(\gamma+1)}.$$

Values used in Haley and Walker (2010) are: $\gamma \to -1$ (Empirical Likelihood), $\gamma \to 0$ (Kullback-Leibler Information Criterion) and $\gamma = 1$ (Euclidean estimator).

2.B.1 The Black and Scholes Model

The price follows the Stochastic Differential Equation (SDE):

$$\frac{dS_t}{S_t} = \mu dt + \sigma dW_t$$

and we choose the parameters $\mu = 10\%$,$\sigma = 20\%$,$r = 5\%$. These parameter are the same as in Stuzter (1996), Gray and Neumann (2005) and Haley and Walker (2010). Figure 2.2 and 2.3 shows the MPE and MAPE as a function of γ for European call options with different combination of maturity and moneyness (price over strike: S/B). We use only one Euler equation in the restrictions (equation (2.9)) and we do not take into account any derivative price.

Figure 2.2 is used to construct figure 2.1 in section 2.4. We superimpose all 20 cells in order to highlight that the MPE is zero for approximatelly the same γ for all combinations of maturity and moneyness considered. It ilustrate the result proved in appendix 2A, i.e., the existence and the value of γ resulting in the risk-neutral measure implied by B&S model. Appendix 2A shows that this γ doesn't depend upon the maturity but varies with the parameters.

Table 2.5 depicts the values of γ in which the MPE is zero (first panel) and MAPE is lowest (second panel). For most entries the MPE is zero for $\gamma = -0.9$. Actually for most entries the MPE is a little greater than zero for $\gamma = -0.9$ and little lower than zero for $\gamma = -0.8$. It implies that the MPE is zero for some γ in between (-0.9,-0.8) for our simulations.

The graph of MAPE against γ has a region almost flat close to the minimum. For instance, the MAPE is equal to the fifth decimal place (MAPE = 0.03895) for $\gamma \in (-0.6; -0.4)$ for at-the-money option with 1 month maturity.

2.B.2 The SV Model

In this model the price follows the SDE:

$$\frac{dS_t}{S_t} = \mu dt + \sqrt{V_t} dW_t^S$$
$$dV_t = k_v (\theta_v - V_t) + \sigma_v \sqrt{V_t} dW_t^v$$
$$E\left[dW_t^S dW_t^v\right] = \rho t$$

and we make use of the parameters in table 2.3. These are the same parametes used in Gray and Neumann (2005) and Haley and Walker (2010). They implicitly consider that the parameters k_v, θ_s, ρ are the same in the risk neutral and objective measure. They don't specify the value of volatility at the begining of the period. We assume that it is the average $V_0 = \theta_v$.

Figures 2.4 and 2.5 shows the MPE and MAPE for the SV model. The simulation suggests that the MPE is decreasing with γ and is positive for $\gamma \in (-5, 0)$. The method has some issues when dealing with $\gamma > 0$ because in this case the optimum probability density risk-neutral measure may be equal to zero for some states. This violates the no-arbitrage constraint.

2.B.3 The SVCJ model

The price follows the SDE

$$\frac{dS_t}{S_t} = \mu dt + \sqrt{V_t} dW_t^S + dJ_t^S$$
$$dV_t = k_v (\theta_v - V_t) + \sigma_v \sqrt{V_t} dW_t^v + dJ_t^v$$
$$E\left[dW_t^S dW_t^v\right] = \rho t$$

where the J_t^S and J_t^v jump at the same time (the jump at prices and volatility occurs at the same time) and the intensity of n-th jump are Z_n^v and $S_{\tau_n}(\widetilde{Z}_n^S - 1)$ where Z_n^v has exponential distribution with mean μ_v and $\widetilde{Z}_n^S$ has lognormal distribution conditional to Z_n^v with mean $\mu_S + \rho_S Z_n^v$ and variance σ_S^2 and

$$\mu = r_t - \delta_t + \gamma_t^{ep} - \overline{\mu}_S \lambda$$
$$\overline{\mu}_S = \exp\left(\mu_S + \frac{\sigma_S}{2} 2\right) - 1$$

where δ_t is the dividend yield, γ_t^{ep} is the equity premium and λ is the jump intensity[18].

We use the parameters estimated in Eraker *et al.*(2003) for the objective measure and the estimations in Broadie *et al.* (2007) for the risk-neutral measure as discussed in section 2.3. Figures 2.6 and 2.7 depict MPE and MAPE as a function of γ. The simulations suggest that the MPE γ in which the MPE is zero varies with maturity. Moreover the MPE seems to be decreasing in γ and that the MAPE doesn't varies very much in a suitable interval.

[18] To be more precise

$$\begin{aligned} dJ_t^S &= d\Sigma_{n=1}^{N_t} \widetilde{Z}_n^S \\ &= d\left(\Sigma_{n=1}^{N_t} S_{\tau_n -}\left[e^{Z_n^S} - 1\right]\right), \end{aligned}$$
$$dJ_t^v = d\left(\Sigma_{n=1}^{N_t} Z_n^v\right).$$

Table 2.1: Objective measure parameters for SVCJ model.

μ	κ_v	θ_v	σ_v	ρ	λ	μ_s	σ_s	μ_v	ρ_s
0.1386	6.5520	0.0136	0.4	-0.48	1.5120	-0.0263	0.0289	0.0373	-0.6

Table 2.2: Risk-neutral measure parameters for SVCJ model.

r	κ_v^Q	θ_v^Q	σ_v	ρ	λ^Q	μ_s^Q	σ_s^Q	μ_v	ρ_s
0.0593	6.5520	0.0136	0.4	-0.48	1.5120	-0.0725	0.0289	0.1333	0.0

Table 2.3: Risk-neutral and Objective measure parameters for SV model.

μ	r	k_v	θ_v	σ_v	ρ	
0.10	0.05	0.03	0.04	0.4	-0.5	

Table 2.4: **Optimal Cressie-Read discrepancy function for SVCJ model.** This table contains the Cressie-Read discrepancy function which attains the minimum error for the SVCJ model. Each cell is associated with an European Option Call with a different combination of moneyness and maturity. The first panel displays the γ in which the method has zero mean percentage error (MPE). The second panel displays the γ in which the method has the lowest mean absolute percentage error (MAPE). The index γ defines the Cressie-Read function through the function $CR_\gamma(\cdot)$ - see equation (2.4). Appendix 2.B shows the graphs where those values are obtained. Pricing errors are based on the method applied to 200 returns draws from appropriate distribution and the mean is obtained by an average of 5000 repetitions. The entries with n/a means that no γ in the range $-(5, 2)$ has zero MPE.

γ **with MPE equal to zero**	Maturity (year)			
	1/12	**1/4**	**1/2**	**1**
S/B = 0.90	n/a	n/a	-2.3	-1.3
S/B = 0.93	-3.1	-2.2	-1.6	-1.2
S/B = 1.00	-3.1	-2.1	-1.5	-1.1
S/B = 1.03	-3.2	-2.0	-1.5	-1.1
S/B = 1.125	-3.7	-2.0	-1.4	-1.0
γ **with minimum MAPE**	Maturity (year)			
	1/12	**1/4**	**1/2**	**1**
S/B = 0.90	n/a	n/a	-2.1	-1.3
S/B = 0.93	-1.8	-1.6	-1.5	-1.1
S/B = 1.00	-1.7	-1.5	-1.4	-1.1
S/B = 1.03	-1.6	-1.5	-1.3	-1.0
S/B = 1.125	-1.4	-1.4	-1.2	-0.9

Table 2.5: **Optimal Cressie-Read discrepancy function for B&S model.** This table contains the Cressie-Read discrepancy function which attains the minimum error for the B&S model. The first panel displays the γ in which the method has zero mean percentage error (MPE). The second panel displays the γ in which the method has the lowest mean absolute percentage error (MAPE). Note that the MPE is zero for $\gamma = -0.9$ for most entries. In matter of fact the MPE is almost zero for $\gamma = -0.9$ and for $\gamma = -0.8$. For the most entries the MAPE is almost the same for some γ close to the lowest one (sometimes it is equal to the fifth decimal place). Each cell is associated with an European Option Call with a different combination of moneyness and maturity. The index γ defines the Cressie-Read function through the function $CR_\gamma(\cdot)$ - see equation (2.4). Pricing errors are based on the method applied to 200 returns draws from appropriate distribution and the mean is obtained by an average of 5000 repetitions.

γ **with MPE equal to zero**	Maturity (year)			
	1/12	**1/4**	**1/2**	**1**
S/B = 0.90	-0.7	-0.8	-0.8	-0.9
S/B = 0.93	-0.9	-0.9	-0.8	-0.9
S/B = 1.00	-0.9	-0.9	-0.8	-0.9
S/B = 1.03	-0.9	-0.9	-0.9	-0.9
S/B = 1.125	-1.0	-0.9	-0.9	-0.9
γ **with minimum MAPE**	Maturity (year)			
	1/12	**1/4**	**1/2**	**1**
S/B = 0.90	.	-0.5	-0.7	-0.8
S/B = 0.93	-0.5	-0.8	-0.7	-0.8
S/B = 1.00	-0.7	-0.7	-0.8	-0.8
S/B = 1.03	-0.3	-0.7	-0.8	-0.9
S/B = 1.125	0.5	-0.3	-0.5	-0.7

Table 2.6: **Price Interval for $\gamma \in [-4, -.5]$ for the SVCJ model** This table contains the price interval for a Call Option in the SVCJ model given by the method when applied with $\gamma \in [-4, -0.5]$. The underlying asset price at t is S=100.00 for all cells but the strike B and maturity h changes. Each cell is shows the interval $(C_{\gamma=-0.5}, C_{\gamma=-4})$ and the theoretical Option value is depicted bellow. The objective and risk-neutral parameters are in table 2.1 and 2.2 respectively. The value of C_γ is calculated as the average Option price for the method applied in 5000 different realization of the process.

	Maturity (year)			
	1/12	**1/4**	**1/2**	**1**
S/B = 0.90	(0.04; 0.04)	(0.64; 0.75)	(2.18; 2.67)	(5.64; 7.18)
	0.05	0.78	2.53	6.31
S/B = 0.97	(1.13; 1.19)	(3.06; 3.39)	(5.44; 6.25)	(9.41; 11.33)
	1.18	3.26	5.82	10.07
S/B = 1.00	(2.53; 2.63)	(4.71; 5.10)	(7.17; 8.08)	(11.17; 13.18)
	2.61	4.92	7.56	11.80
S/B = 1.03	(4.45; 4.57)	(6.58; 7.02)	(9.01; 9.99)	(12.96; 15.02)
	4.54	6.81	9.40	13.57
S/B = 1.125	(11.68; 11.75)	(13.12; 13.55)	(15.14; 16.13)	(18.67; 20.69)
	11.74	13.32	15.48	19.18

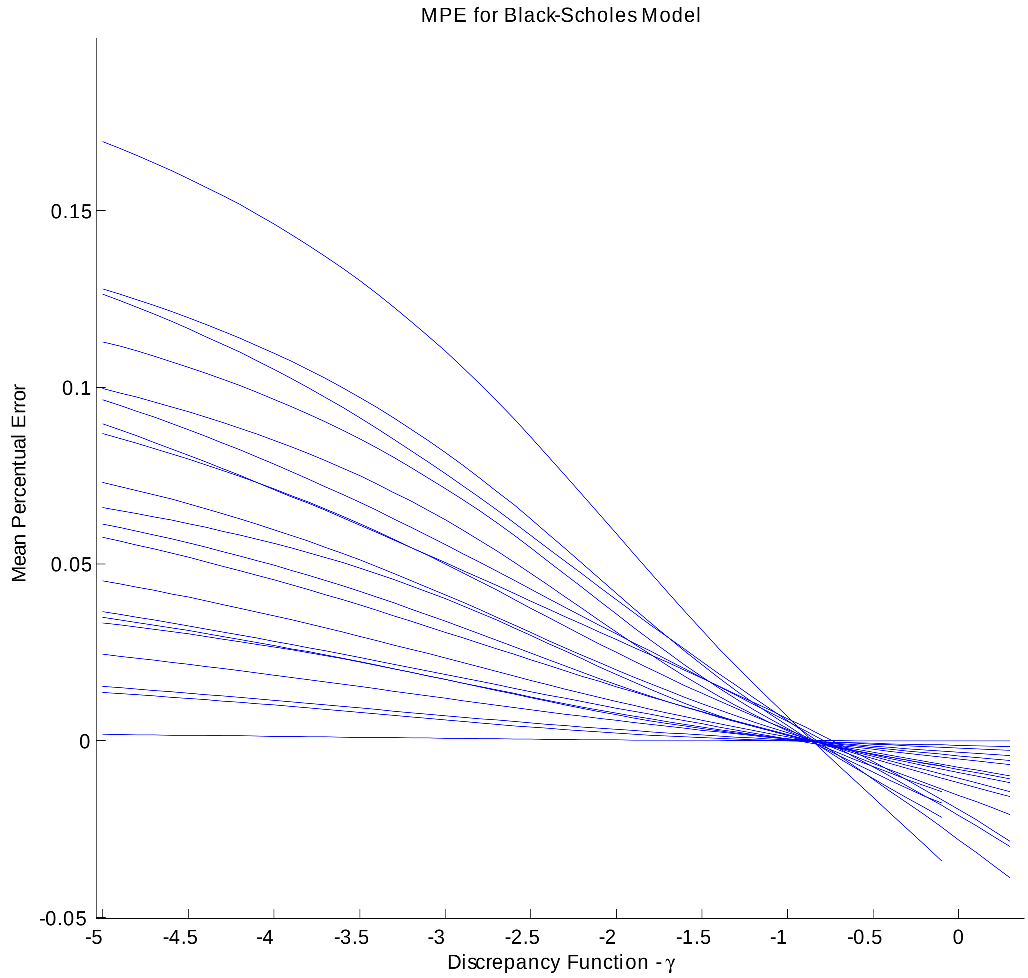

Figure 2.1: Graphs of mean percentage errors (MPE) against γ in the Black-Scholes-Merton model. All graphs crosses the horizontal axis close to $\gamma \approx -0.8$. Each curve is associated with one European Call option with a particular combination of maturity and moneyness. Appendix 2B shows the graphs separately. Pricing errors are based on the method applied to 200 returns draws from appropriate distribution and the mean is obtained by an average of 5000 repetitions. The parameter γ defines the discrepancy function through the function $CR_\gamma(\cdot)$ - (equation (2.4)). Values of interest are: $\gamma \to -1$ (Empirical Likelihood), $\gamma \to 0$ (Kullback-Leibler Information Criterion), $\gamma = 1$ (Euclidean estimator).

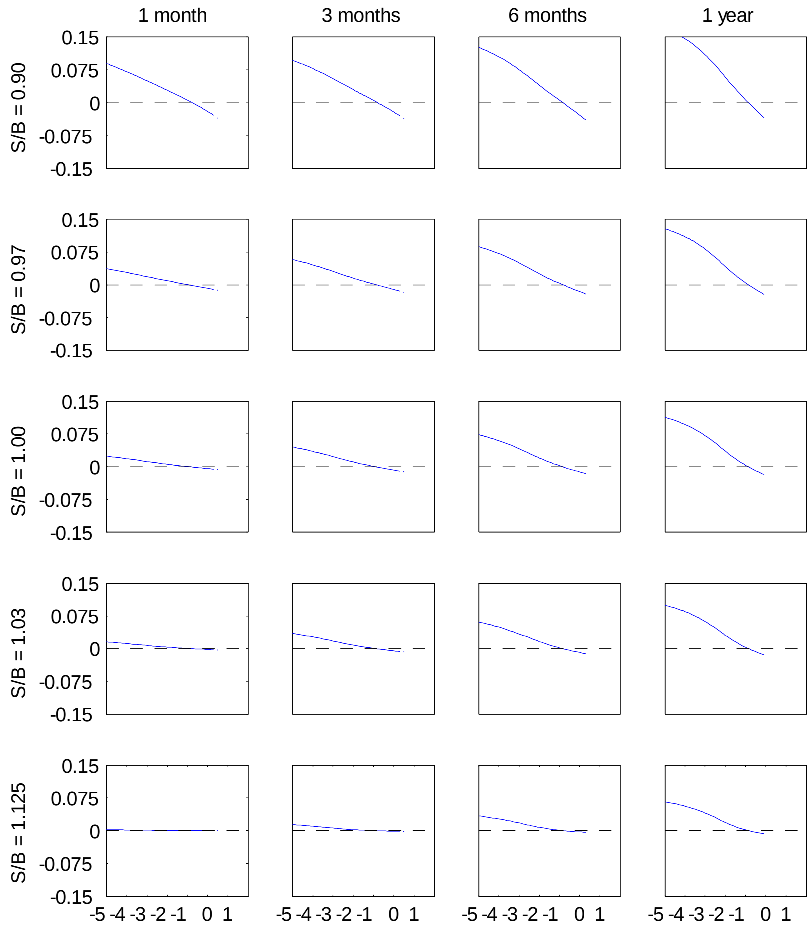

Figure 2.2: Graphs of mean percentage errors (MPE) against γ in the B&S model. All graphs crosses the horizontal axis close to $\gamma \approx -0.8$. Figure 2.1 is obtained superimposing all those cells. Each cell corresponds to the pricing error of a European Call option with a particular combination of maturity and moneyness (S/B). The parameter for B&S model are $\mu = 10\%$, $\sigma = 20\%$ and $r = 5\%$. Pricing errors are based on the method applied to 200 returns draws from appropriate distribution and the mean is obtained by an average of 5000 repetitions. The parameter γ defines the discrepancy function through the function $CR_\gamma(\cdot)$ - see equation (2.4). We use only one Euler equation in the restrictions (equation (2.9)) and don't take into account any derivative price.

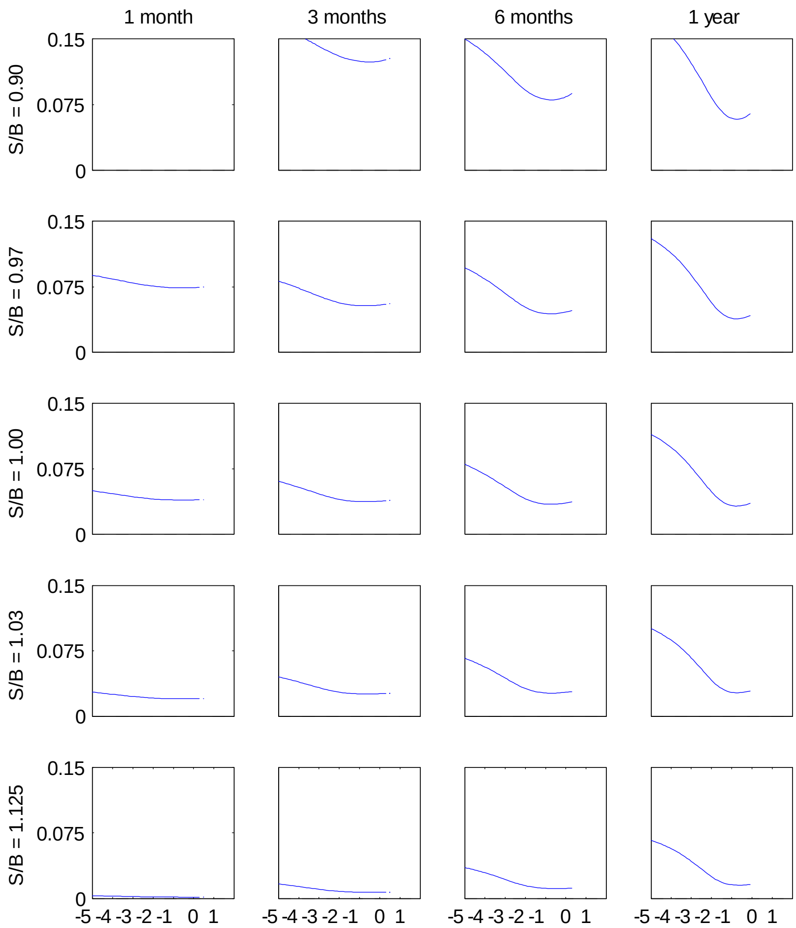

Figure 2.3: Graphs of mean absolute percentage errors (MAPE) against γ in the Black-Scholes-Merton model. Each cell corresponds to the pricing error of a European Call option with a particular combination of maturity and moneyness (S/B). The parameter for B&S model are $\mu = 10\%$, $\sigma = 20\%$ and $r = 5\%$. Pricing errors are based on the method applied to 200 returns draws from appropriate distribution and the mean is obtained by an average of 5000 repetitions. The parameter γ defines the discrepancy function through the function $CR_\gamma(\cdot)$ - see equation (2.4). We use only one Euler equation in the restrictions (equation (2.9)) and don't take into account any derivative price.

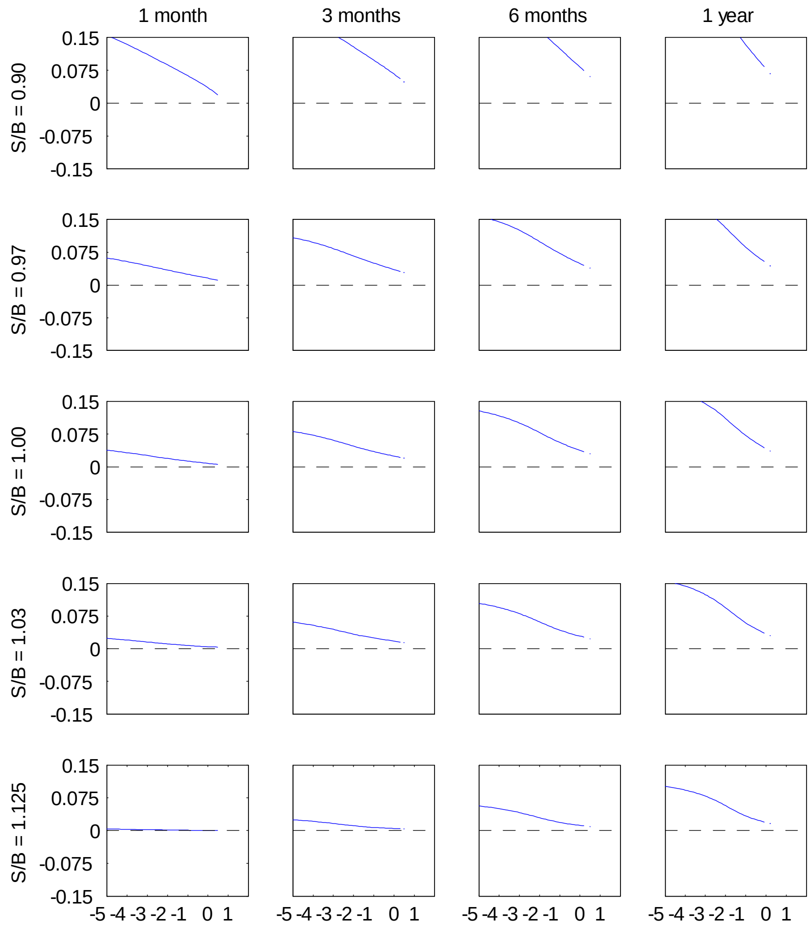

Figure 2.4: Graphs of mean percentage errors (MPE) against γ in the Stochastic Volatility (Heston) model. Each cell corresponds to the pricing error of a European Call option with a particular combination of maturity and moneyness (S/B). Pricing errors are based on the method applied to 200 returns draws from appropriate distribution and the mean is obtained by an average of 5000 repetitions. The parameter γ defines the discrepancy function through the function $CR_\gamma(\cdot)$ - see equation (2.4). We use only one Euler equation in the restrictions (equation (2.9)) and don't take into account any derivative price.

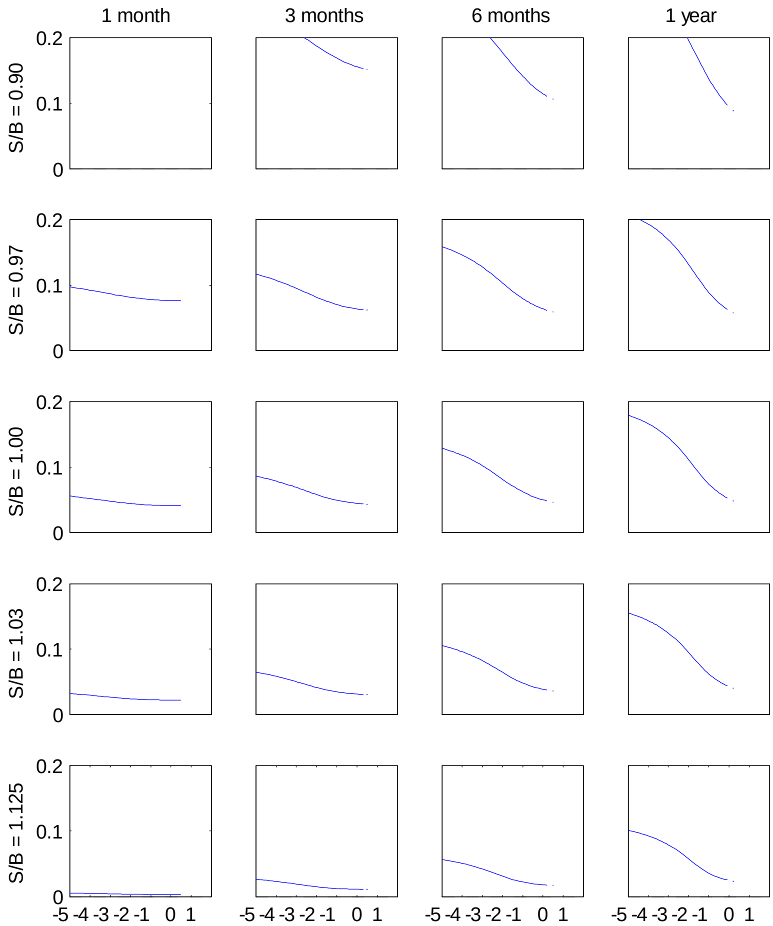

Figure 2.5: Graphs of mean absolute percentage errors (MAPE) against γ in the Stochastic Volatility (Heston) model. Each cell corresponds to the pricing error of a European Call option with a particular combination of maturity and moneyness (S/B). Pricing errors are based on the method applied to 200 returns draws from appropriate distribution and the mean is obtained by an average of 5000 repetitions. The parameter γ defines the discrepancy function through the function $CR_\gamma(\cdot)$ - see equation (2.4). We use only one Euler equation in the restrictions (equation (2.9)) and don't take into account any derivative price.

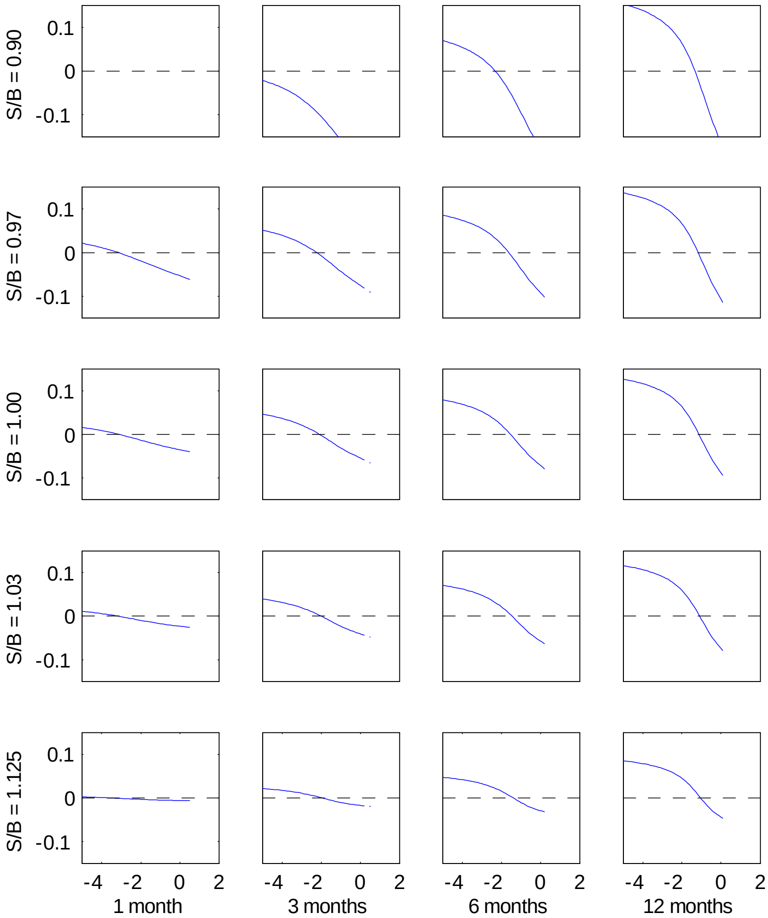

Figure 2.6: Graphs of mean percentage errors (MPE) against γ in the SVCJ model. Each cell corresponds to the pricing error of a European Call option with a particular combination of maturity and moneyness (S/B). Pricing errors are based on the method applied to 200 returns draws from appropriate distribution and the mean is obtained by an average of 5000 repetitions. The parameter γ defines the discrepancy function through the function $CR_\gamma(\cdot)$ - see equation (2.4). We use only one Euler equation in the restrictions (equation (2.9)) and don't take into account any derivative price.

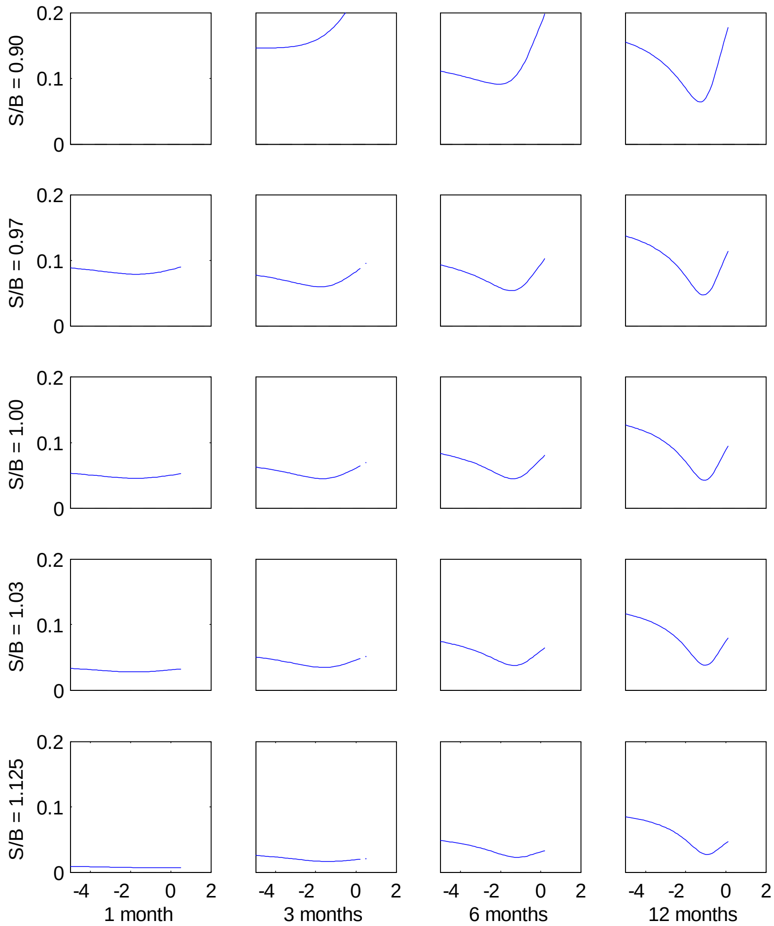

Figure 2.7: Graphs of mean absolute percentage errors (MAPE) against γ in the SVCJ model. Each cell corresponds to the pricing error of a European Call option with a particular combination of maturity and moneyness (S/B). Pricing errors are based on the method applied to 200 returns draws from appropriate distribution and the mean is obtained by an average of 5000 repetitions. The parameter γ defines the discrepancy function through the function $CR_\gamma(\cdot)$ - see equation (2.4). We use only one Euler equation in the restrictions (equation (2.9)) and don't take into account any derivative price.

Chapter 3

Watching the News: Optimal Stopping Time and Scheduled Announcement

Chapter Abstract

The present work studies optimal stopping time problems in the presence of a jump at a fixed time. It characterizes situations in which it is not optimal to stop just before the jump. The results may be applied to the most diverse situations in economics but the focus of the present work is on finance. In this context, a jump in prices at a fixed date is consistent with the effects of scheduled announcements. We apply the general result to the problem of optimal exercise for American Options and to the optimal time to sell an asset (such as a house or a stock) in the presence of fixed cost. In the first application we obtain that it is not optimal to exercise the American Option with convex payoff just before the scheduled announcement. For the second application we obtain that it is not optimal to sell an asset just before the announcement depending upon the utility function and/or the way the prices jump. We provide also a numerical solution for the second application in a particular case.

Keywords: Optimal Stopping Time, Scheduled Announcements, Quasi-Variational Inequality, Jump-Diffusion Models, Numerical Methods in Economics.

JEL Classification Numbers: C6,G1.

3.1 Introduction

Several announcements are scheduled events at which the government, institutions or firms often disclose surprising news. For example, the dates of the Federal Open Market Committee (FOMC) meetings are known in advance[1] and changes in monetary policy are now announced immediately after it. The Federal Reserve Bank determines interest rate policy at FOMC meetings and according to the Bloomberg website[2] "... [the FOMC meetings] are the single most influential event for the markets.". Other macroeconomic data have their release known in advance as well, such as the GDP, CPI, PPI and others. Such information is incorporated into security's prices very quickly. Most of the price changes can be seen within 5 minutes after the announcement[3]. There are similar findings for firms as well. For example, it is common practice

[1] Those dates can be seen at (link accessed at 103/19/2013): http://www.federalreserve.gov/monetarypolicy/fomccalendars.htm.

[2] It may be read at (link at 03/19/2013): http://bloomberg.econoday.com/byshoweventfull.asp?fid=455468&cust=bloomberg-us&year=2013&lid=0&prev=/byweek.asp#top.

[3] See, for instance, Ederington and Lee (1993), Andersen and Bollerslev (1998) or Andersen et. al (2007).

among listed firms to release in advance the dates of the earning announcements. Several authors find a quick move in the markets after the information is released with the bulk of price change in the first few minutes (Pattel and Wolfson (1984)).

In situations where action entails a fixed cost, the economic agents may prefer do nothing most of the time and take some action only occasionally. Empirical studies find such behavior in most diverse fields of economics[4]. Those situations are usually modeled using stochastic control with fixed cost in continuous time. Those problems are called impulse control when the agent takes several actions choosing the time of each one. When the action is taken just once, it is called optimal stopping time problem. The later problem naturally arises when pricing American Options. Oksendal and Sulem (2007) and Stockey (2009) provide a mathematical theory on those problems presenting some important models from the literature.

Our interest is to analyze optimal stopping time problems in the presence of scheduled announcements. We characterize situations where an agent prefers to wait for the information before taking an action. These results may be applied to the most diverse economic situations as the above paragraph suggests, but our focus here is on financial markets. In particular we show that it is never optimal to exercise a class of American Derivatives just before this type of announcement. This class includes very common derivatives such as American calls and puts. Moreover we study the optimal time to sell an asset (such as a house or a stock) in the presence of fixed costs and scheduled announcement. We show that it is not optimal to sell just before the announcements for some cases of utility function and/or jumps characteristics. We provide also a numerical solution for the second application in a particular case.

Several papers model security's prices as a jump-diffusion process in continuous time. The fast price change with news suggests that jumps may be used as a way to incorporate announcements in the price process. It is common to consider the jumps' time as random and unknown before it occur. Nonetheless scheduled announcements don't happen at random dates and they are known in advance. Then we model it as jumps occurring at a fixed and known time[5]. Other empirical findings on prices' behavior may be incorporated in similar fashion. For example, the price volatility may be modeled as an extra continuous time process jumping with news.

Note that the jump is the consequence of some information release impacting the environment or the agent's beliefs about it. In this respect, waiting for the jump is a way to gather more information before taking some action. In some cases there is no substantial risk in waiting for the information so the agent may prefer to act later. In others, waiting is risky as the information may destroy some opportunities. Such interpretation is particularly consistent with evidence in financial markets as announcements usually increase trading activity[6].

Some authors[7] study trading volume behavior around announcements considering investor with exogenous reason for selling an asset. Those investors may have time discretion and may want to avoid trade before an announcement fearing an adverse transaction with a better informed agent. We may add to this literature highlighting that such behavior may be found even without the information asymmetry. As an example, we provide the numerical result for the case in which the price follows a geometric Brownian motion, there is a fixed transaction cost, and the agent is risk-neutral and wants to sell an asset for exogenous reason.

The rest of the article is organized as follows. Section 3.2 presents the results for optional exercise of American Option in the presence of scheduled announcements. The characteristics of

[4] For instance, Bils and Klenow (2004) and Klenow and Kryvtsov (2008) documents the infrequent price changes in retail establishments and Vissing-Jorgensen (2002) finds that households rebalance their portfolio infrequently.

[5] Other authors have a similar modeling strategy. For instance, Dubinsky and Johannes (2006) build an option pricing model incorporating scheduled announcements as jumps occurring at a known date.

[6] There are hundreds of papers about it. It has attracted interest of diverse areas such as economics, finance and accounting. See the seminal work of Beaver (1968) and a review by Bamber et al. (2011). Recent empirical findings in finance includes Chae (2005), Hong and Stein (2007) and Saffi (2009). Some important theoretical work are: Admati and Pfleiderer (1988), Foster and Viswanathan (1990), George et al. (1994).

[7] For instance, see Admati and Pfleiderer (1988), Foster and Viswanathan (1990) or George et al. (1994).

the risk neutral measure allow an easy way to prove the result and provide the basics steps for the more general propositions. Section 3.3 provides the main results in its generality. Section 3.4 provides one application with a numerical result: the optimal time to sell an asset. Section 3.5 presents a discussion and Section 3.6 summarizes the findings and points towards future work. The most technical proofs are in the appendix 3.A and the numerical algorithm‘s details is in Appendix 3.B.

3.2 Optimal Exercise for American Options

The goal of the present section is twofold: to provide a simple demonstration in a particular case and to give a contribution to the optimal exercise of American Options. We show that it is never optimal to exercise just before a scheduled announcement in some common situations. What simplifies the proof is the existence of the risk-neutral measure. The demonstration here gives the guidelines for the general case. We have one empirical implication in this section: if the agents are rational then no exercise is made a little before the announcement for American Option with convex payoff (and absence of arbitrage).

In general, for put options there is a region in which it is better to exercise and the premium is the same as the payoff. Do not exercise at time t means a premium greater than the payoff at t. A jump in a fixed date increases the uncertainty around it and it seems reasonable that the issuer raises the premium. This would imply a smaller region of prices where it is optimal to exercise. In this sense, our results would be intuitive and its interest lays in that the exercise regions shrink to an empty set. Nonetheless, to the best of our knowledge, this reasoning is not necessarily true. For instance, Ekstrom (2004) shows that for a class of American Options the premium increases with volatility but the proposition isn't applied to American puts.

It is not straightforward to infer what happens in the neighborhood of an announcement for the exercise of American Options. Pattel and Wolfson (1979), (1981) find empirically that the implied volatility increases close to announcements, i.e., other things constant, there is an increase in the premium for Europeans calls and puts. On the other hand American calls have usually the same premium as its European counterparty. It is not the case when there are dividends payments because it may be advantageous to exercise just before the payment.

The modeling of a scheduled earning announcement as a jump is taken by Dubinsky and Johannes (2006). They consider a jump-diffusion model with stochastic volatility, apply it to a set of equities and try to measure empirically some definitions of uncertainty about the news. Similarly Pattel and Wolfson (1979), (1981) try to gauge the uncertainty with a generalization of the Black-Scholes-Merton model in which the stock volatility varies deterministically over time. In their generalization the implied volatility increases as the option approaches the announcement date, and drops to a constant after it .

3.2.1 Example: American Put Option on a Black-Scholes-Merton Model with Scheduled Announcement

This subsection introduces the notation and presents a concrete example. Suppose we have an American put on a stock with 60 days maturity of and that the next FOMC meeting will happen in 30 days and will define a new interest rate. Suppose further that the actual interest rate is 1% and that the uncertainty about the meeting implies an interest rate of 0.75%, 1.00% or 1.25% after it. Let T_M be the time of maturity (60 days) T_A be the time of the scheduled announcement (the end of the FOMC meeting in 30 days) and S_t be the price of my equity at time t. We model the price process as in the Black-Scholes-Merton environment but with a jump in price at T_A and a change in the interest rate at T_A, i.e., the price follows geometric Brownian motion and (in the risk-neutral measure) it reads:

$$dS_t = r_t S_t dt + \sigma S_t d\widetilde{B}_t + \Delta S_{T_A} \chi_{\{t=T_A\}}, \tag{3.1}$$

$$S_0 = z_0 \tag{3.2}$$

where z_0 is a constant, $\chi_{\{t=T_A\}}$ is the indicator function

$$\begin{aligned} \chi_{\{t=T_A\}}(t) &= 0 \text{ if } t \neq T_A \\ &= 1 \text{ if } t = T_A, \end{aligned} \tag{3.3}$$

$$r_t = r_{BA} \text{ if } t < T_A \text{ (Before Announcement)}, \tag{3.4}$$

$$r_t = r_{AA} \text{ if } t \geq T_A \quad \text{(After Announcement)}, \tag{3.5}$$

r_{BA} is a constant, r_{AA} is a random variable whose realization is not known before T_A, σ is the constant volatility and $\widetilde{B}_t$ is the Wiener process in the risk-neutral measure. The risk-free rate after announcement r_{AA} has a discrete distribution with 3 possible (and equiprobable) outcomes: 0.75%, 1.00% or 1.25%. Moreover, the price process is continuous before and after T_A but has a jump at T_A of

$$\Delta S_{\,T_A} = \zeta S\ (T_A-) \tag{3.6}$$

where $S_{\,(T_A)^-}$ is the left limit of the price process

$$S_{\,(T_A)^-} = \lim_{t \to (T_A)^-} S_t, \tag{3.7}$$

ζ has a lognormal distribution[8] and $\Delta S_{\,T_A}$ is the jump's size:

$$\Delta S_{\,T_A} = S_{\,T_A} - \lim_{t \to (T_A)^-} S_t. \tag{3.8}$$

In order to compute the American put's premium we shall consider the early exercise feature and that the option holder uses it optimally. As we are in the risk-neutral measure, we compute the present value expectation using the discounting

$$e^{-\int_0^\tau r_s ds} \tag{3.9}$$

where τ is the exercise time. If $\tau \leq T_A$ we have the discount as $e^{-r_{BA}\tau}$, otherwise we have $e^{-r_{BA}T_A - r_{AA}(\tau - T_A)}$. The premium for a given strategy τ is

$$\widetilde{E}\left[e^{-\int_0^\tau r_s ds}\,(K - S(\tau))^+\right] \tag{3.10}$$

where K is the strike, $\widetilde{E}\,[\cdot]$ denotes the expectation in the risk-neutral measure and $(x)^+ = \max\{0, x\}$. As we seek the maximum, we have

$$v(z_0) = \max_{\tau \leq T_M} \widetilde{E}\left[e^{-\int_0^\tau r_s ds}\,(K - S(\tau))^+\right] \tag{3.11}$$

where τ is a stopping time, T_M is the maturity and $v(z_0)$ is the premium at $t = 0$ when $S(0) = z_0$.

We lost the Markov property held by the Black-Scholes-Merton model when we introduced a scheduled announcement. Nonetheless, we still have something similar. For $t < T_A$, all we know about the distribution after t is contained in the price level. For conditional expectation this implies that

$$\widetilde{E}\,[\cdot|\mathcal{F}_t] = \widetilde{E}\,[\cdot|S_t = z] \quad \text{for } t < T_A. \tag{3.12}$$

[8] To be precise about the information structure, we shall define the probability space $\left(\Sigma, \Omega, \widetilde{P}\right)$ along with the filtration $(\mathcal{F}_t)_0^{T_M}$. Let the price process be right-continuous and the portfolios be left-continuous. The realization of ζ and r_{AA} aren't known before T_A, i.e., these information belong to $\mathcal{F}_{T_A}$ but not to $\mathcal{F}_t$ if $t < T_A$.

Note that we are considering only the risk neutral measure $\widetilde{P}$, i.e., we only need to know the jump size and change distributions in this measure.

On the other hand, after $t \geq T_A$ all information is contained in $S_t = z$ and $r_{AA} = r$ and we have

$$\widetilde{E}\left[\cdot|\mathcal{F}_t\right] = \widetilde{E}\left[\cdot|S_t = z, r_{AA} = r\right] \quad \text{for } t \geq T_A. \tag{3.13}$$

To what follows, we need to define the premium for other dates. For $t < T_A$ denote it by[9] $V_{BA}(t,z)$:

$$V_{BA}(t,z) = \max_{t\leq\tau\leq T_M} \widetilde{E}\left[e^{-\int_t^\tau r_s ds}\left(K - S(\tau)\right)^+ |S(t) = z\right] \tag{3.14}$$

and for $t \geq T_A$

$$V_{AA}(t,z,r) = \max_{t\leq\tau\leq T_M} \widetilde{E}\left[e^{-\int_t^\tau r_s ds}\left(K - S(\tau)\right)^+ |S(t) = z, r_{AA} = r\right]. \tag{3.15}$$

In the present work, we want to study the exercise behavior just before T_A and we do it through the optimal stopping time τ. A decision to stop should depend only upon the past information, i.e., if the agent wants to exercise at t this decision is make using the information $\mathcal{F}_t$. But the all relevant information is in the value of S_t (and $r_{AA} = r$ if $t \geq T_A$). Then, for each t (and each r_{AA} after T_A) we have a set of prices that makes optimal the exercise and in this case the premium is $V_{BA}(t,z) = (K - S(t))$. We call this the stopping set[10]

$$\mathbf{S}_{BA} = \left\{(t,z); V_{BA}(t,z) = (K-z)^+\right\} \text{ for } t < T_A, \tag{3.16}$$

$$\mathbf{S}_{AA} = \left\{(t,z,r); V_{AA}(t,z,r) = (K-z)^+\right\} \text{ for } t \geq T_A. \tag{3.17}$$

On the other hand, we have the price region where it is not optimal to exercise, i.e., the continuation set where the premium is greater than the payoff[11]

$$\mathbf{C}_{BA} = \left\{(t,z); V_{BA}(t,z) > (K-z)^+\right\} \text{ for } t < T_A, \tag{3.18}$$

$$\mathbf{C}_{AA} = \left\{(t,z,r); V_{AA}(t,z,r) > (K-z)^+\right\} \text{ for } t \geq T_A. \tag{3.19}$$

In this model, it is not optimal to stop just before the announcement and we show this below. In the next subsection we give sufficient conditions for not being optimal to exercise (stop) just before the announcement for a generic model, i.e., for each z there is $\varepsilon > 0$ such that $(T_A - \varepsilon, z) \in \mathbf{C}_{BA}$.

3.2.2 Generic Problem

Let Z_t be a n+m-dimensional defined as:

$$Z_t = (S_t, X_t) \tag{3.20}$$

[9] We could do simply:

$$V(t,z,r) = \max_{t\leq\tau\leq T_A} \widetilde{E}\left[e^{-\int_t^\tau r_i ds}\left(K - S(\tau)\right)|S(t) = z, r_s = r\right]$$

considering the interest rate another process that jumps with the announcement. Nonetheless we want to emphasize the role of the announcement.

[10] Actually, the stopping set $\mathbf{S}$ shall be defined as

$$\mathbf{S} = (\mathbf{S}_{BA} \times r_{BA}) \cup \mathbf{S}_{AA}$$

where $\times$ denotes cartesian product.

[11] Again, the continuation region $\mathbf{C}$ shall be defined as

$$\mathbf{C} = (\mathbf{C}_{BA} \times r_{BA}) \cup \mathbf{C}_{AA}.$$

where S_t is a n-dimensional process for assets prices satisfying the stochastic differential equation (SDE hereafter) in the real world (objective measure):

$$dS_t = S_t\alpha(S_t, X_t, \theta_t)dt + S_t\sigma(S_t, X_t, \theta_t)dB_t + \Delta S_{T_A}\chi_{\{t=T_A\}} \tag{3.21}$$

X_t is a m-dimensional vector satisfying the SDE:

$$dX_t = \alpha_X(S_t, X_t, \theta_t)dt + \sigma_X(S_t, X_t, \theta_t)dB_t + \Delta X_{T_A}\chi_{\{t=T_A\}}, \tag{3.22}$$

B_t be a n+m-dimension Wiener process, α, α_X, σ, σ_X satisfies usual regularity conditions (see Oksendal and Sulem (2007), Theorem 1.19), $t \geq 0$ and θ_t is a set of parameters satisfying

$$\theta_t = \theta_{BA} \quad \text{Before the Announcement,} \tag{3.23}$$

$$\theta_t = \theta_{AA} \qquad \text{After the Announcement,} \tag{3.24}$$

where θ_{AA} is a random variable known after the announcement. Note that the process X_t isn't a price process. For instance, in the stochastic volatility model (as in Heston (1993) for instance) the volatility is a process but it is not a price process. It implies that it isn't (in general) a martingale under the risk-neutral measure. The process may include jumps as well but we do not consider it here explicitly in order to simplify the exposition. This broad specification includes, for instance, the Black and Scholes model, Merton model and the class of Affine Jump-Diffusion models as in Duffie et al. (2000).

The scheduled announcement is made at $T_A > 0$ and there is a jump in $(S_{\,T_A}, X_{\,T_A})$:

$$\Delta S_{\,T_A} = S_{\,T_A} - \lim_{t \to (T_A)^-} S_t, \tag{3.25}$$

$$\Delta X_{\,T_A} = X_{\,T_A} - \lim_{t \to (T_A)^-} X_t, \tag{3.26}$$

along with a change in the parameters as

$$\theta_t = \theta_{BA} \text{ for } t < T_A, \tag{3.27}$$

$$\theta_t = \theta_{AA} \text{ for } t \geq T_A \tag{3.28}$$

with θ_{AA} known only for $t \geq T_A$.

We assume that there is a risk-neutral measure. In the absence of arbitrage this is indeed true (see, for instance, Duffie (2001)). Under this measure, we have that the asset prices satisfies the SDE:

$$dS_t = r_t S_t dt + S_t\sigma(S_t, X_t, \theta_i)d\widetilde{B}_t + \Delta S_{T_A}\chi_{\{t=T_A\}} \tag{3.29}$$

and X_t:

$$dX_t = \widetilde{\alpha}_X(S_t, X_t, \theta)dt + \sigma_X(S_t, X_t, \theta_i)d\widetilde{B}_t + \Delta X_{T_A}\chi_{\{t=T_A\}}. \tag{3.30}$$

where r is the instantaneous interest rate assumed constant for simplicity[12], B_t be a n+m-dimension Wiener process in the risk neutral measure and $\widetilde{\alpha}_X$, σ_X satisfies regularity conditions ((see Oksendal and Sulem (2007), Theorem 1.19)). We assume further that the jump at T_A, ΔZ_{T_A},is a random variable that depends only upon $Z\,(T_A-)$ (as in the multiplicative case of equation (3.6)) and that the future distribution of the economy only depends upon the actual state of the economy. We express the last assumption with the equation:

$$\widetilde{E}\left[\cdot|\mathcal{F}_t\right] = \widetilde{E}\left[\cdot|\left(S(t), X(t)\right) = z, \theta_t = \theta\right]. \tag{3.31}$$

[12] We can model the interest rate process as well as in done the example above. Nonetheless nothing changes in the proof of the proposition.

where z is a n+m dimensional constant and θ is a constant set of parameters.

The price of American Option is obtained defining an optimal stopping problem in the risk neutral measure. Let $g : R^n \to R$ denote the option's payoff and let $T_M > T_A$ be the maturity. Then we have for the option's premium:

$$V_{BA}(t,z) = \max_{t\le\tau\le T_M} \widetilde{E}\left[e^{-\int_t^\tau r_s ds} g(S_\tau)\,|Z(t)=z\right] \quad \text{for } t<T_A, \tag{3.32}$$

$$V_{AA}(t,z,\theta) = \max_{t\le\tau\le T_M} \widetilde{E}\left[e^{-\int_t^\tau r_s ds} g(S_\tau)\,|Z(t)=z, \theta_{AA}=\theta\right] \quad \text{for } t\ge T_A, \tag{3.33}$$

where V is the premium. Note that we make the assumption that g only depends upon S_t.

3.2.3 Results for Convex American Options

The simplification in the American Option case comes mainly by two simple equalities we stablish now. The prices and the premium follow a martingale in the risk-neutral measure. In particular, for $t < T_A \le u$ we have[13]

$$e^{-rt}y = \widetilde{E}\left[e^{-ru}S_u|Z_t=(y,x)\right] \quad \text{for } t<T_A\le u, \tag{3.34}$$

$$e^{-rt}V_{BA}(t,z) = \widetilde{E}\left[e^{-ru}V_{AA}(u,Z_u,\theta_u)|Z_t=z\right] \quad \text{for } t<T_A\le u. \tag{3.35}$$

If $u = T_A$ we can make the limit:

$$\begin{aligned} e^{-rt}y &= \widetilde{E}\left[e^{-ru}S_u|Z_t=(y,x)\right] \\ \lim_{t\to(T_A)^-} e^{-rt}y &= \lim_{t\to(T_A)^-} \widetilde{E}\left[e^{-ru}S_u|Z_t=(y,x)\right] \\ e^{-rT_A}y &= \widetilde{E}\left[e^{-rT_A}S_{T_A}|Z_{(T_A)^-}=(y,x)\right] \end{aligned} \tag{3.36}$$

or[14]

$$y = \widetilde{E}\left[S_{T_A}|Z_{(T_A)^-}=(y,x)\right] \tag{3.37}$$

and for the same reason

$$\lim_{t\to(T_A)^-} V_{BA}(t,z) = \widetilde{E}\left[V_{AA}(T_A,Z_{T_A},\theta_{T_A})|Z_{(T_A)^-}=z\right]. \tag{3.38}$$

The above 2 equations is what make the proof easier. We will implicitly impose that $V_{BA}(t,z)$ is continuous in t close to T_A. Although we can avoid this assumption, it simplifies the proof.

Proposition 1 *Consider the model defined in the risk-neutral measure by the equations (3.21)-(3.30) along with the distribution of θ_{AA} and the jumps in T_A. Consider further an American Option with maturity $T_M > T_A$ whose g is a convex function of S_t. Moreover, assume that it is not optimal to execise at T_A with positive probability in the risk-neutral measure. Then for each z there is $\varepsilon > 0$ such that it is never optimal to exercise the option at time $t \in (T_A-\varepsilon, T_A)$ if $Z_t = z$. In other words, it is never optimal to exercise just before the announcement.*

[13] In the general case we should use $e^{-\int_0^t r_s ds}$ instead of e^{-rt}.

[14] The step where the limit enters on the expectation needs to be better defined. More explictly, make

$$\begin{aligned} \widetilde{E}\left[\cdot|\lim_{t\to(T_A)^-} Z_t=(y,x)\right] &= \widetilde{E}\left[\cdot|\lim_{t\to(T_A)^-} Z_t=(S_t,X_t);S_t=y;X_t=x\right] \\ &= \widetilde{E}\left[\cdot|\lim_{t\to(T_A)^-} \mathcal{F}_t;S_t=y;X_t=x\right] \end{aligned}$$

and we shall define $\lim_{t\to(T_A)^-}\mathcal{F}_t$ as an increasing set limit

$$\lim_{t\to(T_A)^-}\mathcal{F}_t = \cup_{n=1}^{\infty}\left(\mathcal{F}_{T_A-\frac{1}{n}}\right).$$

Proof. What we want to show is that

$$\lim_{t \to (T_A)^-} V_{BA}(t,z) > g(y) \tag{3.39}$$

with $z = (y, x)$ because the above limit means that exists $\varepsilon > 0$ such that

$$V_{BA}(t-\varepsilon, z) > g(y) \tag{3.40}$$

and the strict inequality is a suficient (and a necessary) condition to not exercise,i.e, (t, z) belongs to the continuation region.

Being not optimal to execise at T_A with positive probability implies that

$$\widetilde{E}\left[V_{AA}(T_A, Z_{T_A}, \theta_{T_A}) | Z_{(T_A)^-} = z\right] > \widetilde{E}\left[g(S_{T_A}) | Z_{(T_A)^-} = z\right] \tag{3.41}$$

because we have the strict inequality $V_{AA}(T_A, Z_{T_A}, \theta_{T_A}) > g(S_{T_A})$ with positive probability and the inequality $V_{AA}(T_A, Z_{T_A}, \theta_{T_A}) \geq g(S_{T_A})$ with certainty.

Finally, in order to obtain the inequality (3.39), we just need to do[15]:

$$V_{T_A-} = \widetilde{E}_{(T_A)^-}\left[V_{T_A}\right] > \widetilde{E}_{(T_A)^-}\left[g(S_{T_A})\right] \geq g\left(\widetilde{E}_{(T_A)^-}\left[S_{T_A}\right]\right) = g(y). \tag{3.42}$$

$$V_{T_A-} > g(y)$$

where $V_{T_A-} = \lim_{t \to (T_A)^-} V_{BA}(t,z)$ and $\widetilde{E}_{(T_A)^-}\left[V_{T_A}\right] = \widetilde{E}\left[V_{AA}(T_A, Z_{T_A}, \theta_{T_A}) | Z_{(T_A)^-} = z\right]$. ■

In the Black-Scholes-Merton model without dividend payment but with this kind of news, we have that the exercise feature for American call is worthless and premium is equal to the European one with the same characteristics. Moreover, for options where the exercise feature has some value, this proposition means that the premium will increase at least in some set of prices.

A crucial assumption is the possibility of no exercise after the announcement. If you know that you will exercise anyway after the news release, why bother to wait for it? Actually it is reasonable to have at least a small chance to not exercise after the announcement. For instance, one may think that the jump has a lognormal distribution. In this case any (open) interval of S has a positive probability to occur.

On the other hand, there is a greater chance to exercise after the announcement. This is a consequence of the jump and the change in the price process at the announcement. In the next sections we analyze this more deeply. For instance, the modeling approach we use for timing the selling of an asset is quite similar to the above problem.

[15] Or, in a more complete notation, we have with $z = (y, x)$:

$$\begin{aligned}
\lim_{t \to (T_A)^-} V_{BA}(t,z) &= \widetilde{E}\left[V_{AA}(T_A, Z_{T_A}, \theta_{T_A}) | Z_{(T_A)^-} = z\right] \\
&> \widetilde{E}\left[g(S_{T_A}) | Z_{(T_A)^-} = z\right] \\
&\geq g\left(\widetilde{E}\left[S_{T_A} | Z_{(T_A)^-} = z\right]\right) \\
&= g(y).
\end{aligned}$$

3.3 Optimal Strategies Close to Announcement

We established in the previous section some results for American Options when the payoff is convex and there is a risk-neutral measure. In this section we relax those assumptions characterizing general models that use optimal stopping time with a random change at a known and fixed time. We simplify some definitions here using a notation similar to Shreve (2000) in order to have a more readable text but in Appendix 3A we give a full account.

Let T_A be the time of announcement, $Z_t = (Y_t, X_t)$ be a n+m-dimensional[16] process where Y_t is n-dimensional that doesn't jump at T_A a.s., and X_{T_A} is a m-dimensional process that jumps with a positive probability at T_A:

$$Z_t = (Y_t, X_t), \tag{3.44}$$

$$dZ_t = \alpha(Z_t)dt + \sigma(Z_t)dB_t + \Delta Z_{T_A}\chi_{\{t=T_A\}}, \tag{3.45}$$

$$Z(0) = z_0 \tag{3.46}$$

$$\begin{aligned} X(T_A) &= X(T_A-) + \Delta X(T_A) \\ Y(T_A) &= Y(T_A-) \qquad \text{a.s.} \end{aligned} \tag{3.47}$$

where α and σ are function satisfying some regularity conditions ensuring the existence of strong solution (see Oksendal and Sulem (2007), Theorem 1.19), B_t is a n+m-dimensional Wiener Process and $\Delta X(T_A)$ has a probability distribution depending upon the information $\mathcal{F}_{T_A-}$. Assume that the process has the properties:

$$E\left[\cdot|\mathcal{F}_t\right] = E\left[\cdot|Z_t = z\right], \tag{3.48}$$

i.e., all the information relevant for the distributions after t is summed up in the value of state variables at t: $(t, Z_t = z)$.

Let $f : \mathbb{R}^{n+m} \to \mathbb{R}$ and $g : \mathbb{R}^{n+m} \to \mathbb{R}$ be continuous functions satisfying regularity conditions (see Oksendal and Sulem (2007), Chapter 2) and suppose $f \geq 0$ and $g \geq 0$. The optimal stopping problem at time 0 is to find the supremum:

$$v(z) = \sup_{\tau\in\Upsilon} E^y\left[\int_0^\tau f((Z(t))dt + g(Z(\tau))\chi_{\{\tau<\infty\}}\right] \tag{3.49}$$

where $\chi_{\{\tau<\infty\}}$ is the indicator function and at time t:

$$V(t,z) = \sup_{\tau\geq t} E\left[\int_t^\tau f((Z(t))dt + g(Z(\tau))\chi_{\{\tau<\infty\}}|Z_t = z\right]. \tag{3.50}$$

Note that the change in parameters here are inside the process X_t implicitly. For instance, the risk-free rate of the example in section 3.2 may be regarded as one of the dimensions in X_t.

We make the assumption that the random variable $\Delta X(T_A)$ depends only upon $Z(T_A-)$, i.e., given $Z(T_A-)$ the jump $\Delta X(T_A)$ is independent of $Z(T_A - s)$ for any $s > 0$. Section 3.2 provides an example in which

$$X(T_A) = X(T_A-)\zeta \tag{3.51}$$

[16] All the proofs consider a probability space $(\Omega, \mathcal{F}, P)$ and the filtration $\mathcal{F}_t$ and there is no change when $Z(t)$ is a jump diffusion in $\mathbb{R}^{n+m}$ given by

$$dZ(t) = b(Z(t),\theta(t))dt + d(Z(t),\theta(t))dB(t) + \int_{\mathbb{R}^K} \gamma(Z(t^-), z, \theta(t^-))\widetilde{N}(dt, dz), \tag{3.43}$$

where the jump is explicitly now. We should define also the solvency region. It is an open set $S \subset \mathbb{R}^{n+m}$. In order to simplify the exposition we consider $S = \mathbb{R}^{n+m}$ (all space) and omit it in the main text.

where ζ is independent and follows a lognormal. Another assumption (satisfied by the example in section 3.3) relates to a continuity property for the jump:

$$\lim_{s \to T_A} Z^{s,z}(T_A) = z + \Delta Z(T_A) \quad \text{a.s..} \tag{3.52}$$

We want to characterize the continuation region just before T_A and in particular we want to give sufficient conditions for the case when it is never optimal to stop just before the announcement. In the present context we need something similar to the Equation (3.38):

$$V_{T_A-} = \widetilde{E}_{(T_A)^-}[V_{T_A}].$$

Indeed we have the following:

Lemma 2 (L1) *Consider the model described in the present section. Assume further that condition C2 is true (see appendix 3A), that the value function V exists and that $V(T_A, z)$ is lower semi continuous in z. Then:*

$$\lim \inf_{t \to T_A-} V(t,z) \geq E[V(T_A, Z_{T_A}) | Z_{T_A-} = z]. \tag{3.53}$$

The proof is technical and is left for the appendix 3A. The condition C2 guarantees that certain stopping times exists. This condition may hold quite generally but we were not able to prove it. The lower semi-continuity (l.s.c.) property isn't very restrictive. Indeed, as there are no jump after T_A, a sufficient condition is that g should be l.s.c. (see Oksendal (2003) Chapt. 10). The continuity property on the jump at T_A is quite general also.

3.3.1 Main Results

Here we characterize situations in which it is not optimal to stop just before the scheduled announcement. This is true if

$$\lim \inf_{t \to T_A} V(t,z) > g(z) \tag{3.54}$$

because in this case there is $\varepsilon > 0$ such that

$$V(t,z) > g(z) \quad \text{for } t \in (T_A - \varepsilon, T_A). \tag{3.55}$$

It is useful to define three regions. The first one is the set D_p where it is not optimal to stop at T_A with positive probability. In other word, z belongs to this set if the value function $V(T_A, Z_{T_A})$ is greater than $g(T_A, Z_{T_A})$ with positive probability.

Definition 1 *Define the set D_p as*

$$D_p = \left\{ z \in \Re^{n+m} | P[V(T_A, Z_{T_A}) > g(T_A, Z_{T_A}) | Z_{T_A-} = z] > 0 \right\}. \tag{3.56}$$

The other two sets relate only to the function g and the jump. For the elements in the set $D_>$ it is better to stop just after the announcement than just before (when comparing only those two options), i.e., for $z \in D_>$ we have that $E[g(T_A, Z_{T_A}) | Z_{T_A-} = z] > g(z)$. Similarly, for the element in $D_\geq$, the agent prefer to stop just after than just before or may be indifferent, i.e., for $z \in D_\geq$ we have that $E[g(T_A, Z_{T_A}) | Z_{T_A-} = z] \geq g(z)$. Those sets may be defined using the concept of certainty equivalence as well (note that the certainty equivalent state $c(z)$ is not unique in some cases).

Definition 2 *The certainty equivalent $c(z)$ is defined implicilty by the equation*

$$g(c(z)) = E\left[g\left(T_A, Z_{T_A}\right)|Z_{T_A-} = z\right]. \tag{3.57}$$

Definition 3 *Define the set $D_>$ as*

$$D_> = \left\{z \in \Re^{n+m}|E\left[g\left(T_A, Z_{T_A}\right)|Z_{T_A-} = z\right] > g(z)\ \right\} \tag{3.58}$$

or, equivalently

$$D_> = \left\{z \in \Re^{n+m}|g\left(c(z)\right) > g(z)\ \right\} \tag{3.59}$$

Definition 4 *Define the set $D_\geq$ as*

$$D_\geq = \left\{z \in \Re^{n+m}|E\left[g\left(T_A, Z_{T_A}\right)|Z_{T_A-} = z\right] \geq g(z)\ \right\}. \tag{3.60}$$

or, equivalently

$$D_\geq = \left\{z \in \Re^{n+m}|g\left(c(z)\right) \geq g(z)\ \right\}. \tag{3.61}$$

With those definition we can now enunciate the main proposition. It basically states that it is not optimal to stop just before the scheduled announcement in two situation: if the state variable z belongs to $D_>$ or if $z \in D_\geq \cap D_p$.

Proposition 3 *Consider the model defined in the present section and assume as true the hypothesis of lemma L1. Then, it is not optimal to stop just before the announcement if $z = Z(T_A-)$ belongs to $D_>$, i.e.:*

$$\lim_{t \to T_A} \inf V\left(t, z\right) > g\left(z\right) \quad \textit{for } z \in D_>. \tag{3.62}$$

Moreover if $Z(T_A-) = z \in D_\geq \cap D_p$ then it is not optimal to stop just before T_A, i.e.,

$$\lim_{t \to T_A} \inf V\left(t, z\right) > g\left(z\right) \quad \textit{for } z \in D_\geq \cap D_p. \tag{3.63}$$

Proof. It generalizes the same steps we did in the previous section:

$$\begin{aligned}
\lim_{t \to T_A} \inf V\left(t, z\right) &\geq E\left[V\left(T_A, Z_{T_A}\right)|Z_{T_A-} = z\right] && (3.64)\\
&\geq E\left[g\left(Z_{T_A}\right)|Z_{T_A-} = z\right] && (3.65)\\
&\geq g\left(c(z)\right) && (3.66)\\
&\geq g\left(z\right). && (3.67)
\end{aligned}$$

Then, for $z \in D_>$ the inequality in the last line is strict. Moreover, for $z \in D_p$ the inequality is strict in the second line. Finally, for both cases (i.e., for $z \in D_>$ and for $z \in D_\geq \cap D_p$):

$$\lim_{t \to T_A} \inf V\left(t, z\right) > g\left(z\right). \tag{3.68}$$

■

Recall that in order to define D_p we need to know the value function at T_A. However we can find a subset of D_p using only the model primitives and use this set instead of D_p in the above proposition.

Note that if $z \in D_p$ then $P\left[Z_{T_A} \in \mathbf{C}|Z_{T_A-} = z\right] > 0$ where $\mathbf{C} = \left\{(t, z) \in \Re \times \Re^{n+m}|V\left(t, z\right) > g\left(z\right)\right\}$ is the continuation region. The Proposition 2.3 in Oksendal and Sulem (2007) defines a subset of the continuation region using only the primitives of the model. Using this subset instead of $\mathbf{C}$ allows us to find a smaller set $U_p \subset D_p$ not using the value function at T_A.

Definition 5 *Define the set U_p as*

$$U_p = \left\{ z \in \Re^{n+m} | P\left[Z_{T_A} \in U | Z_{T_A-} = z\right] > 0 \right\}. \tag{3.69}$$

where

$$U = \left\{ z \in \Re^{n+m} | Ag + f > 0 \right\}$$

and A is the generator function associated to process Z_t.

In several situations the generator A may be replaced by the differential operator

$$Af(z) = \sum_i \alpha_i(z) \frac{\partial f}{\partial z_i}(z) + \sum \left(\sigma\sigma^T\right)_{ij} \frac{\partial^2 f}{\partial z_i \partial z_j}(z)$$

where σ^T is the transpose of σ. The next section provides an example. For details about the operator A we refer to Oksendal and Sulem (2007). With the set U we may establish the corollary:

Corollary 4 *Suppose the hypotheses of proposition above are satisfied. If $Z(T_A-) = z \in D_{\geq} \cap U_p$ then it is not optimal to stop just before T_A, i.e.,*

$$\lim_{t \to T_A} \inf V(t, z) > g(z) \quad \textit{for } z \in D_{\geq} \cap U_p. \tag{3.70}$$

In several cases, D_p or $D_{>}$ is all space (or both). It is true, for instance, if g is convex, the jump size expectation is zero ($E\left[Z_{T_A} | Z_{T_A-} = z\right] = z$) and it isn't optimal to exercise at T_A with positive probability. This is the case for American Options with convex payoff in the risk-neutral measure. Moreover, if $g(z) = g(y)$, i.e. if the payoff doesn't depends upon variables that jumps at T_A, then $D_{\geq}$ is all space.

Another interesting case is when g is CRRA (Constant Relative Risk Aversion):

$$g(x^0) = \frac{(x^0)^\gamma}{\gamma}. \tag{3.71}$$

where x^0 is a homogeneous scalar function of degree 1 in Z_t, $\gamma \in (0, 1)$ (remember that $g(x^0) \geq 0$) and the jump at T_A is

$$x^0\left(Z_{T_A}\right) = x^0(Z_{T_A-})\xi \tag{3.72}$$

where ξ is independent of Z_{T_A-}. In this case, the certainty equivalent has a nice property. If

$$E\left[\frac{(x^0)^\gamma}{\gamma} | Z_{T_A-} = z\right] = \frac{c^\gamma}{\gamma} \tag{3.73}$$

then

$$E\left[\frac{(x^0)^\gamma}{\gamma} | Z_{T_A-} = 2z\right] = \frac{(2c)^\gamma}{\gamma}. \tag{3.74}$$

We sum up those observations in the following corollary:

Corollary 5 *Suppose the hypotheses of proposition above are satisfied. Then we have:*

(i) if g is increasing, convex and $E\left[Z_{T_A}\right] \geq Z_{T_A-}$ then it is not optimal to stop for $Z_{T_A-} = z \in D_p$;

(ii) If the payoff doesn't depends upon the variable that jumps, i.e., if $g(z) = g(x, y) = g(y)$ then it is not optimal to stop for $Z_{T_A-} = z \in D_p$;

(iii) If the payoff is a CRRA function, i.e., $g(z) = (x_0(z))^\gamma / \gamma$ where $x_0(z)$ is homogeneous scalar function of degree 1 in z, if the jump has the property that $x^0\left(Z_{T_A}\right) = x^0(Z_{T_A-})\xi$ and if $c(1) > 1$ then it is never optimal to stop just before T_A.

3.4 Another Application in Finance

The objective of the present section is twofold. First, it is an example of the above results. It applies the corollaries and defines the generator operator for one particular case. Second, it discusses a possible modeling for an agent who wants to sell an asset highlighting the incentives when there is a scheduled announcement. For the most part we explore the case in which the price doesn't jump with the announcements. It highlights some incentives and makes the results more clear. However in the last subsection we make comments on more general cases.

3.4.1 The Optimal Time to Sell with Transaction Cost

We will consider a problem of one agent (or investor) that wants to sell its portfolio and there is an information being released at a known date T_A. We are interested in his behavior around the date T_A. To be more clear, we want to show that selling just before T_A is less likely in some sense and may never be optimal in some circumstances. To simplify, we will consider that the portfolio has only one asset, the utility is linear and is obtained when the investor sells the portfolio at time τ:

$$J^{\tau}(x) = E^{s,x}\left[e^{-\rho\tau}\left(X(\tau) - a\right)\right] \tag{3.75}$$

where $X(t)$ is the price of the asset at time t, ρ is the discount factor, a is the fixed cost to sell the asset, $E^{s,x}[.]$ is the expectation operator conditional to information $\mathcal{F}_s$ obtained at s when $X(s) = x$, and τ is a stopping time.

The asset follows a Geometric Brownian Motion :

$$dX(t) = X(t^-)\left[\alpha(t)dt + \beta dB(t)\right] \qquad X(s) = x > 0 \tag{3.76}$$

where $B(t)$ is the Wiener process, β and γ are constants, the function $\alpha(t)$ is constant before and after T. The impact of information on market is a random change on the coefficient $\alpha(t)$ at T_A. It is described as:

$$\alpha(t) = \alpha_0 \text{ if } t < T_A \tag{3.77}$$

$$\alpha(t) = \zeta \text{ if } t \geq T_A \tag{3.78}$$

where ζ is a random variable with uniform distribution in the interval $[\underline{\alpha}, \overline{\alpha})$ with $0 < \underline{\alpha} < \overline{\alpha} \leq \rho$, and $\alpha_0 < \rho$.

Note that for $\rho = \alpha$ we have the same problem as pricing American calls.

3.4.2 Solution Without Information Release

The problem without information release is the same as the example 2.5 of Oksendal and Sulem (2007). The only difference is that $\alpha(t) = \alpha_0$ for all t. We'll give the solution here because we will need it later.

Notice that it is never optimal to sell if $\rho < \alpha$ even if the cost a is zero (in this case $J^{\tau=\infty} = \infty$) and obviously it is never optimal to sell the asset if its price X is less than the cost a for any time (eventually the price will be more than a). We will call the continuation region $D_{noNews} \subset \Re^2$ as the set of time and prices that is not optimal to sell the asset (i.e. the continuation region). Oksendal and Sulem (2007) shows that:

$$\mathbf{C}_{noNews} = \{(s, x) : x < x^*\} \tag{3.79}$$

where x^* is defined below and doesn't depend upon time. This is consistent with the assertive that the problem faced by the agent at time s_1 with $X(s_1) = x$ is the same at time s_2 with $X(s_2) = X(s_1) = x$. The solution for $J^* = \sup_\tau J^\tau$ is:

$$J^*(s, x) = e^{-\rho s} C x^{\lambda_1} \qquad \text{if } 0 < x < x^* \tag{3.80}$$

$$J^*(s, x) = e^{-\rho s}(x - a) \qquad \text{if } x^* \leq x \tag{3.81}$$

where λ_1 is the solution of

$$0 = -\rho + \alpha\lambda_1 + \frac{1}{2}\beta\lambda_1(\lambda_1 - 1) \tag{3.82}$$

and

$$x^* = \frac{\lambda_1 a}{\lambda_1 - 1}, \tag{3.83}$$

$$C = \frac{1}{\lambda_1}(x^*)^{1-\lambda_1}. \tag{3.84}$$

Finally, if $\alpha = \rho$, it is never optimal to sell the asset and $J^*(s,x) = J^{\tau=\infty} = xe^{-\rho s}$.

3.4.3 When It Is Not Optimal to Sell Close to T

When $\alpha(t)$ changes randomly at T, the continuation region is no longer constant over time. Nonetheless for $s \geqslant T_A$ the optimization problem is the same as in the previous section and is never optimal to sell in the region:

$$\{(s, x, \alpha) : x < x^*(\alpha), s \geqslant T_A\}. \tag{3.85}$$

Notice that we add α to the notation. The solution is the same above:

$$\begin{aligned} J^*(s,x,\alpha) &= e^{-\rho s}C(\alpha)x^{\lambda_1(\alpha)} \qquad \text{if } 0 < x < x^*(\alpha) \text{ and } s \geqslant T_A && (3.86)\\ J^*(s,x,\alpha) &= e^{-\rho s}(x-a) \qquad \text{if } \quad x^*(\alpha) \le x \quad \text{ and } s \geqslant T_A && (3.87)\end{aligned}$$

where $\lambda_1(\alpha)$ is the solution of

$$0 = -\rho + \alpha\lambda(\alpha) + \frac{1}{2}\beta\lambda(\alpha)(\lambda(\alpha) - 1) \tag{3.88}$$

and

$$x^*(\alpha) = \frac{\lambda(\alpha)a}{\lambda(\alpha) - 1}, \tag{3.89}$$

$$C(\alpha) = \frac{1}{\lambda(\alpha)}(x^*)^{1-\lambda(\alpha)}. \tag{3.90}$$

Note that the payoff depends on the prices that does't jump. Then we have the case in the item (ii) of the corollary 5 and it is only necessary to characterize D_z. But this is easy, if $x^*(\alpha) > x = X_{T_A-}$ with positive probablity, then $x \in D_z$. This is true if

$$x < \sup_{\underline{\alpha}<\alpha\le\overline{\alpha}} \{x^*(\alpha)\} = x^*(\overline{\alpha}). \tag{3.91}$$

We confirm this result below solving it numerically.

And If the Solution After T_A Isn't Known?

In general the solution after T_A isn't known. On those cases it is possible to characterize subset of the inaction region (see Oksendal and Sulem (2007) for details). For this purpose, define the generator operator A as

$$Ag(s,x) = \frac{\partial g}{\partial s} + \alpha(s)x\frac{\partial g}{\partial x} + \frac{1}{2}\beta x^2\frac{\partial^2 g}{\partial x^2}, \tag{3.92}$$

where $g = e^{-\rho\tau}(X(\tau) - a)$ and define the set U as:

$$U = \left\{(x, s, \alpha) \in \mathbb{R}^+ \times \mathbb{R}^+ \times \mathbb{R}^+ | Ag + f > 0\right\} \tag{3.93}$$

where $f = 0$ in our problem. The proposition 2.3 in Oksendal and Sulem (2007) tell us that $U \subset \mathbf{C}$, i.e., it is never optimal to stop when $(x, s, \alpha) \in U$. We find that:

$$Ag + f = e^{-\rho s}\left((\alpha - \rho)x + \rho a\right) \tag{3.94}$$

and U is:

$$U_\alpha = \left\{(x, s, \alpha) | x < \frac{\rho a}{\rho - \alpha}\right\}. \tag{3.95}$$

Realize that if $\alpha(s) = \rho$, the continuation region after T is:

$$\{(s, x) : x < \infty, s > T_A\}. \tag{3.96}$$

3.4.4 Numerical Solution

Algorithm Overview

Oksendal and Sulem (2007) provide a sufficient conditions for a function to be a solution of the above problem. Those conditions are called integrovariational inequalities for optimal stopping time and are characterized by the formulas

$$\max\left(A\phi, g - \phi\right) = 0 \tag{3.97}$$

$$\mathbf{C} = \left\{(s, x) \in R^+ \times R^+ | \phi(s, x) > g(s, x)\right\} \tag{3.98}$$

along with regularity conditions, where A is defined as above. Realize that the problem isn't only to find ϕ, but to find the region $\mathbf{C}$ as well, i.e., finding the right boundary conditions is part of the problem .

We want to solve it numerically using some kind of finite difference approximation for the operator A. Nonetheless, the usual methods cannot be applied directly because the boundary conditions aren't defined from the outset. In pricing American Options, it is common to overcome this difficult using the so called Projected Successive Over Relaxation, a generalization of the Gauss–Seidel method. Nonetheless, we will use a policy iteration algorithm provided by Chancelier et al. (2007). We detail the method in appendix 3B but we give an overview here.

In our case this is done by considering a rectangular grid. The equation above is rewritten as $\max\left(A_h\phi_h, g_h - \phi_h\right) = 0$ and $\mathbf{C}_h = \{(s, x) \text{ belongs to grid} | \phi_h(s, x) > g_h(s, x)\}$ [17].This problem is equivalent to a better behaved one, defined as:

$$\phi_h = \max\left(\left[I_h + \frac{\xi A_h}{1 + \xi\rho}\right]\phi_h, g_h\right) \tag{3.99}$$

where $0 < \xi \leq \min \frac{1}{|(A_h)_{ii} + \rho|}$, and I_δ is the identity operator ($I_h v_h = v_h$). The solution is found iteratively: in the first iteration, define D_h^1 and solve $\frac{\xi A_h}{1+\xi\rho}\phi_h^1 = 0$ for $(s, x) \in \mathbf{C}_h^1$ defining $\phi_h^1 = g_h(s, x)$ for $(s, x) \notin \mathbf{C}_h^1$. In the second iteration, define D_h^2 as the points in the grid that $\left(I_h + \frac{\xi A_h}{1+\xi\rho}\right)\phi_h^1 > g_h(s, x)$, then solve $\frac{\xi A_h}{1+\xi\rho}\phi_h^2 = 0$ for $(s, x) \in \mathbf{C}_h^2$ defining $\phi_h^2 = g_h(s, x)$ for $(s, x) \notin \mathbf{C}_h^2$. Keep iterating until it converges. Chancelier et. al. (2007) shows that this procedure converges to the right solution.

For $s < T_A$ we assume that

$$\lim_{s \to T_A -} \phi_h(s, x) = E\left[\phi_h(T_A, x)\right]. \tag{3.100}$$

[17] The subscript δ denotes the approximation of functions or operators defined on the grid.

Table 3.1: Two Parameters Configurations.

Parameter	Case 1	Case 2
α	0.1	0.1
σ	0.4	0.4
ρ	0.12	0.12
a	10	10
T	10	10
$\underline{\alpha}$	0	0.095
$\overline{\alpha}$	0.11	0.11

We don't prove this statement but lemma L1 implies that $\lim_{s\to T_A-}\phi_h(s,x) \geq E\left[\phi_h(T_A,x)\right]$. Then we are assuming a lower bound if the equality in equation (3.100) does not hold. In this case the numerical solution would have a downward bias when compared to the true solution. This bias lead to a smaller continuation before the announcement. As some of ours analysis are based on how big is $\mathbf{C}$ before the announcement our results are conservative.

Numerical Results

Solution is found for two configurations of parameters (see table table 3.1). Notice that the only differences in the two cases are the parameters $\underline{\alpha}$.

The figure 3.1 shows the region $\mathbf{C}^1$. It is interesting to compare $\mathbf{C}^1$ with the continuation region $\mathbf{C}^{noNews}$ for the problem without information release and the same parameters. To this end a dashed horizontal line at price $x^*(\alpha = 0.1) = 104.24$ represents the upper boundary of $\mathbf{C}^{noNews}$. We can separate three interesting regions in the time. When the information is far (in our case, for $t = 0$) $\mathbf{C}^1$ is similar to $\mathbf{C}^{noNews}$, but lays a little below. Then, $\mathbf{C}^1$ make an U shape and finally increases getting close to price $x^*(\overline{\alpha} = .11) = 204.1211$ at the time T_A. The figure 3.2 shows the difference between the value functions for parameter in case 1 (table 3.1) and for the model without information release with contour curves[18] for $z = V_1 - V_{noNews}$. For $z > 0$ it means that $V_1 > V_{noNews}$ and it happen only at a small region close to T_A. For the most part $z = 0$ or $z < 0$.

For the most part of time the agent isn't better off when compared to the case without announcement. This is explained by the choice of the parameter $\underline{\alpha}$ as zero. In this case, it is much more likely that the parameter α_T will be less than α by a good amount[19], making the agent worse off. This effect is damped when the announcement is far because it is more likely to sell the asset before T. When the time is close to the announcement the agent will probably sell the asset in an adverse environment because α_T will probably be lower. Nonetheless, when the price is "high"(i.e. the price is close to the boundary of D^{sim1}) for a time close to the news, others incentives enter into play. In this case, the agent would sell the asset for this "high"price but can wait a little to see if the realization of α_T makes him better off. In a good realization, the agent probably will "make some money"taking more time to sell the asset. In a bad realization the investor sells it right away, and the "loss"taken to wait a little is probably small. In other words, on those situation, it is worth to wait a little for more information.

The figures 3.3 and 3.4 are of the same type as figures 3.1 and 3.2, respectively. The odds now are in favour to make α_T higher than α in a good amount. The agent now is always better off when compared with the case without information release. Realize that $\mathbf{C}^{noNews} \subset \mathbf{C}^1$ and

[18] A contour line (also isoline) of a function of two variables is a curve along which the function has a constant value.

[19] A good amount, when compared to the possibles α_T higher than α.

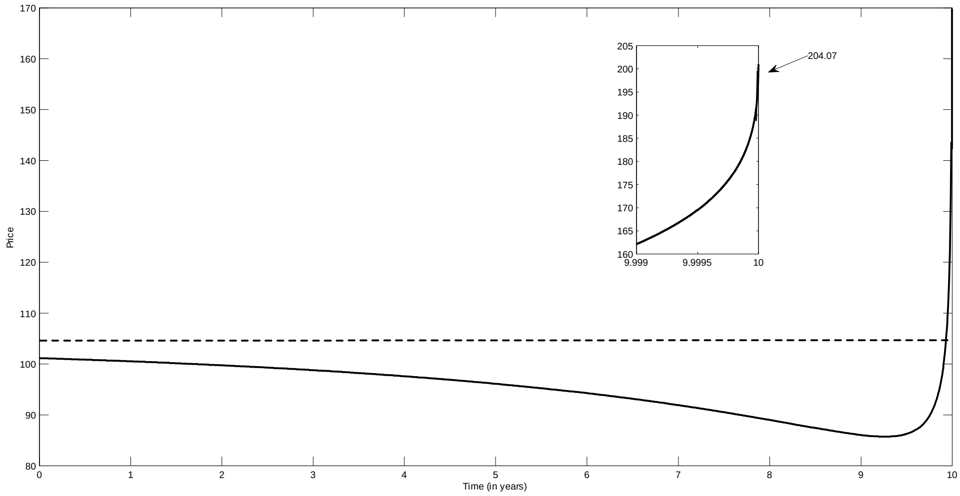

Figure 3.1: The figure shows the continuation regions for the parameters in table 3.1, case 1. The solid line and the dashed line represents the upper boundary of $\mathbf{C}^1$ and $\mathbf{C}^{noNews}$ respectively. The inside graph shows a more detailed simulation close to the announcement.

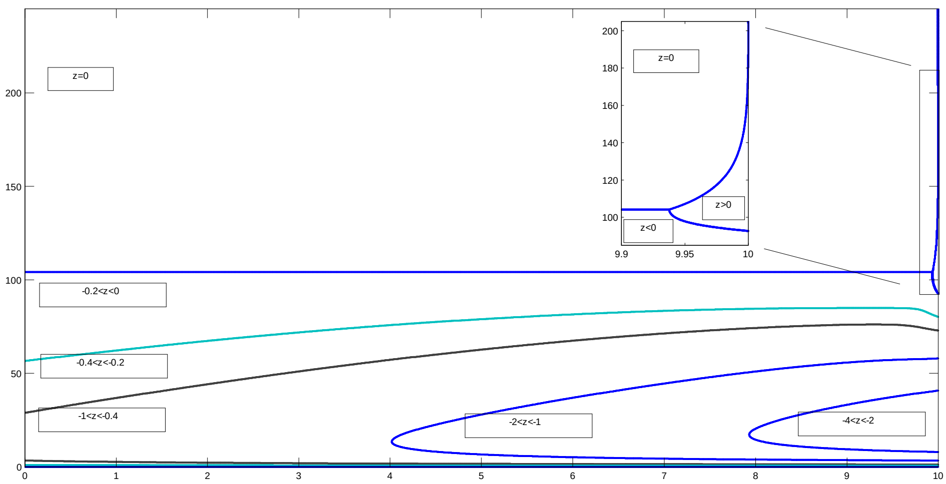

Figure 3.2: Contour line (or isoline) for $z = V_1 - V_{noNews}$. Realize that z is greater than zero only in a small region.

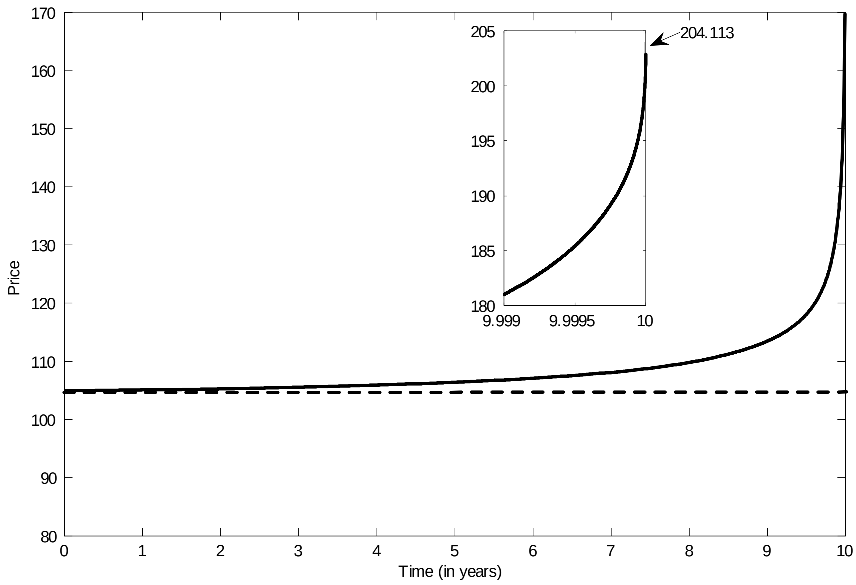

Figure 3.3: Continuation regions for the numerical solution for parameters in case 2, table 3.1. The solid line and the dashed line represents the upper boundary of $\mathbf{C}^2$ and $\mathbf{C}^{noNews}$ respectively. The inside graph shows a more detailed simulation close to the announcement.

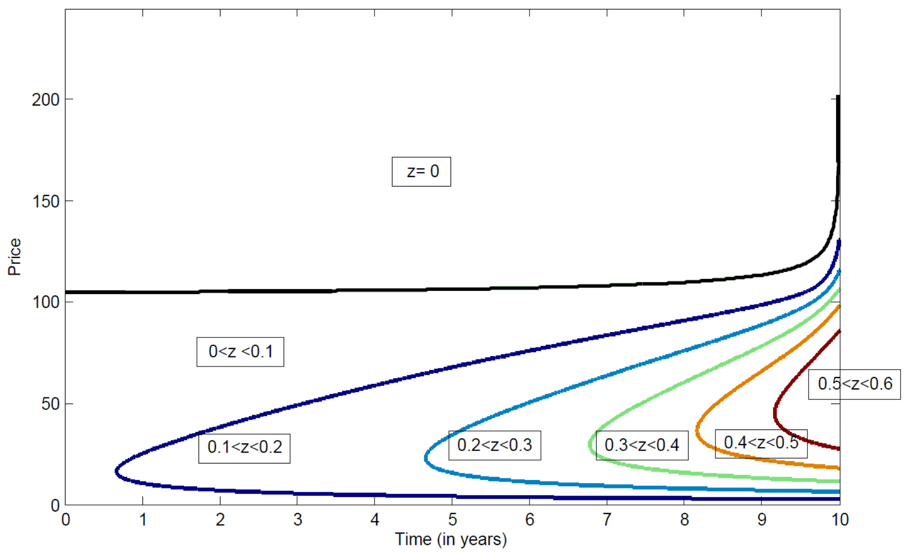

Figure 3.4: Contour line (or isoline) for $z = V_2 - V_{noNews}$. Realize that $z \geq 0$ in all region.

that the boundary increases monotonically with time until T_A. When the news is far from being released, $\mathbf{C}^1$ is similar to $\mathbf{C}^{noNews}$ and value function is just a little bit higher. For "high "prices it may be worth to wait a little more as the incentive to sell is weakened. As the announcement gets closer, the possibility of sell at even higher prices if $\alpha_T > \alpha$ makes the continuation region get wider at a faster pace.

In both cases, the boundary of continuation region increases and gets close to $x^*(\overline{\alpha} = .11) = 204.1211$ as the time gets close to T_A.

3.4.5 Interpretations

This behavior illustrates the incentive an agent face when trying to sell an asset given that he/she knows the price won't jump but the process will change somehow. That simplification has the purpose of intepret some incentives avoiding the analysis of the effect of jumps. In this case, the main benefit is to wait a little more and sell for a better price. If the news affects negatively the trend of the price, usually it is better to sells immediately after the news. If we can summarize the result in one statement, it would be that the agent prefers to sell with more information as long as waiting for such thing has low a risk.

We considered a special case where price doesn't jump and the agent is risk-neutral. More generally the results applies if the agent has a CRRA utility function and the price jumps with positive average (big enough to account for risk aversion). It is interesting to mention that Bamber et al. (1998) finds that only at one quarter of time the prices had a sudden impact. Then it is probable the investors are in a situation between the no jump and the case with a positive average jump.

3.5 Discussion

Under mild conditions, optimal stopping time problems entail a time and state dependent rule: it is optimal to stop whenever the process goes out the continuation region. It implies a higher chance to stop at jumps regardless it happens at fixed or random times. On the other hand, it is harder going out the continuation region when it is bigger (in general) and the main message of the present work is that it is indeed bigger just before a fixed jump for some common situations. In other words, it is less probable to stop before a fixed (and known) jump time when compared to "normal" times for some common cases. Moreover, it is possible to predict this behavior without solving the problem in some cases by applying the generator operator to the reward function.

Such time state dependent rules may arise in several economic situations. For instance it is true in resetting price models with menu cost or optimal portfolio problems with fixed cost. Although those problems may be considered as a sequence of optimal stopping time, we are analyzing here the simplest case of single stopping. This might be a good way to model agents who wants to sell an asset (such as a house or a stock) specially in the presence of fixed cost.

Based on empirical evidence, it is reasonable to assume that prices jump (with positive probability) when relevant information hits the markets. It is true for corporate or market events containing relevant information whether it is a scheduled one or not. Then any investor with state dependent strategy has a higher chance to trade at those times or a little after. This might be an important piece in the explanation of higher volume after announcements. Note that there is no need to incorporate information asymmetries or difference in opinion to obtain the time and state dependent rules. Those considerations are also valid for the decrease in volume before the scheduled announcements, especially in the presence of the type of investor analyzed here. They may prefer to trade only after the announcement even if there is no asymmetry or no chance to engage in an adverse transaction before the event with a more informed investor. Another possible incentive is the average positive price change as is documented in the earning announcement premium (see, for instance, Frazzini and Lamont (2007) or Barber *et al.* (2013)).

We focus on the price as the important state because its role and behavior are clearly observed. Nonetheless other state variable may be considered as well. Some investor may focus their strategies on some fundamental signal such as book-to-value or price-to-earnings. It is even possible to consider some qualitative state such as belonging to an index or the existence of some legal issue. Then, even without change in prices, announcements might spur trades after and decrease volume before it.

3.6 Conclusion

In the present work we investigate the optimal stopping time in continuous time models when there is a jump at a fixed and known date. We characterize the continuation region a little before the jump showing that it is better not to stop just before the news in several situations of interest. Moreover in order to verify such characteristic in a model one needs only to apply the generator operator to the reward function without solving the problem.

These results are used to analyze some financial situations as empirical findings suggest that the price jump with positive probability at scheduled announcement. American Options are modeled as an optimal stopping time problem and we show that if the payoff is convex then it is never optimal to exercise just before the announcement. Moreover, we want to add some theoretical observations about the behavior of the volume around the announcements. Several authors stress out the role of agents with exogenous reasons for sell an asset and we model these investors as facing an optimal stopping time problem. Using the general results we argue that such investors may prefer to transact after the announcements. It happens because the agent "wants"to know the changes caused by the announcement and because the agent "wants"to gather the positive premium usually associated with announcements (such as the earnings announcements premium). Moreover we give the numerical solution for the case of a risk neutral investor facing a fixed costs and use a relatively recent numerical method.

Much of the intuition comes from the time and state dependent strategy implied by the optimal stopping times solution. Such strategies are pervasive in economic especially in situations where some sort of cost (e.g., fixed cost) exist. For instance, a portfolio problem similar to Merton (1969) but with fixed cost imply an optimal impulse problem combined with optimal stochastic problem. To analyze those type of problems when there are a jump at fixed and known date are subject of future research.

3.A Precise Definitions and Proofs

The objective of the present appendix is to define precisely the elements of section 3.3 and extend it to the jump-diffusion case. The definitions are quite general but we make clear what assumption is being used. In particular we make precise the general condition the jump at the announcement (time T_A) should satisfy.

The first step towards proving lemma L1 is to show an inequality on $V(t, Z_t)$ where $V(t, z)$ is the Value Function. This inequality is similar to the property of supermartingales. Note that Z_t is the solution of a stochastic differential equation (SDE) and a more complete notation would be $Z_t^{s,z}$ where the superscript s, z means that $Z_t^{s,z}$ is the value of the process at t with the initial condition $Z(s) = z$.

Finally we make the assumption that $V(t, z)$ is lower semi-continuous (l.s.c.) in z for $t = T_A$ and that the jump at T_A has some continuity properties. Npte that the lower semi-continuity property isn't very restrictive. For instance, if g is l.s.c. and the process Z_t has no jump after T_A then $V(t, z)$ is l.s.c. for $t \geq T_A$ (see Oksendal (2003) Chapt. 10). The continuity property on the jump at T_A is quite general also.

3.A.1 Definitions

Consider the probability space $(\Omega, \mathcal{F}, P)$ and the filtration $\mathcal{F}_t$. Fix an open set $S \subset \mathbb{R}^{n+m}$ (the solvency region) and let $Z(t)$ be a jump diffusion cadlag process in $\mathbb{R}^{n+m}$ given by

$$\begin{aligned} dZ(t) &= \alpha(Z(t))dt + \sigma(Z(t))dB(t) + \int_{\mathbb{R}^{n+m}} \gamma(Z(t^-), z')\widetilde{N}(dt, dz'), && (3.101) \\ Z(s) &= z \in \mathbb{R}^{n+m}, && (3.102) \end{aligned}$$

where $b(.)$, $\sigma(.)$ and $\gamma(.)$ are functions such that a unique solution to $Z(t)$ exists (see Oksendal and Sulem (2007), Theorem 1.19), $B(t)$ is the n+m dimensional Wiener process and $\widetilde{N}$ is the compensated Poisson random measure.

The integral incorporates jumps into the process. In order to define the compensated Poisson random measure completely, we define the Poisson random measure $N(t, U)$ as the number of jumps of size $\Delta Z \in U$ (where U is a borel set whose closure doesn't contain the origin) which occur before or at time t. We need the Levy measure also:

$$\nu(U) = E[N(1, U)] \tag{3.103}$$

where U is a borel set whose closure does not contain the origin. There is $R \in [0, \infty]$ where

$$\begin{aligned} \widetilde{N}(dt, dz) &= N(dt, dz) - \nu(dz)\, dt \quad \text{if } |z| < R && (3.104) \\ &= N(dt, dz) \quad \text{if } |z| \geq R && (3.105) \end{aligned}$$

and z is inside the integrand. For more details we refer to Protter (2003) and Oksendal and Sulem (2007).

The process Z_t (recall that $Z_t = Z(t)$) is divided in two process: $X_t \in \Re^m$ that jump with positive probability at T_A; and $Y_t \in \Re^n$ that doesn't jump at T_A almost surelly

$$Z(t) = (Y(t), X(t)), \tag{3.106}$$

$$Y(T_A) = Y(T_A-) \text{ a.s.} \tag{3.107}$$

$$X(T_A) = X(T_A-) + \Delta X(T_A), \tag{3.108}$$

where $\Delta X(T_A) \neq 0$ with positive probability and $\Delta X(T_A)$ is $\mathcal{F}_{T_A}-$measurable random variable. A more complete notation is $Z^{s,z}(t)$ indicating that it is a solution of the SDE in equation (3.101) with the initial condition $Z(s) = z$, i.e.,

$$Z^{s,y}(t) = z + \int_s^t \alpha(Z(t))du + \int_s^t \sigma(Z(u))dB(u) + \int_s^t \int_{\mathbb{R}^{n+m}} \gamma(Z(u^-), z')\widetilde{N}(du, dz'). \tag{3.109}$$

The expecation operator $E^{s,z}[h(Z_t)]$ is defined as[20]

$$E^{s,z}[h(Z_t)] = E[h(Z_t^{s,z})]. \tag{3.110}$$

We make the assumption that the random variable $\Delta X(T_A)$ depends only upon $Z(T_A-)$, i.e., given $Z(T_A-)$ the jump $\Delta X(T_A)$ is independent of $Z(T_A - s)$ for any $s > 0$. Section 3.2 provides an example in which

$$X(T_A) = X(T_A-)\zeta \tag{3.111}$$

where ζ is independent from $X(t)$ for $t < s$ and that the conditional distribution is lognormal. Another assumption (satisfied by the example in section 3.3) relates to a continuity property:

$$\lim_{s \to T_A} Z^{s,z}(T_A) = z + \Delta Z(T_A) \quad \text{a.s.}. \tag{3.112}$$

[20] We will write $E^{y,s}[h(Y(t))]$ and $E[h(Y^{s,y}(t))]$ interchangeably.
I'm following the notation used in Shreve (2004), Stochastic Calculus for Finance II. This expectation is defined on chapter 6, page 266.

Let
$$\tau_{s,z}^{S} = \inf\{t > s | Z^{s,y}(t) \notin S\}. \tag{3.113}$$
For notation sake, τ_S will be used instead of $\tau_{s,z}^{S}$ whenever it is clear which (s, z) is the right one. For instance $E^{s,z}[h(\tau_S)] = E^{s,z}\left[h\left(\tau_{s,z}^{S}\right)\right]$ unless state otherwise explicitly.

Let $f : \mathbb{R}^{n+n} \to \mathbb{R}$ and $g : \mathbb{R}^{n+m} \to \mathbb{R}$ be continuous functions satisfying the conditions:
$$E^{s,z}\left[\int_{s}^{\tau_S} f(Y(t^{-}))dt\right] < \infty \text{ for all } z \in \mathbb{R}^{n+m} \text{ and } s \geq 0 \tag{3.114}$$
and assume that the family $\left\{g(Z(\tau^{-}))\chi_{\{\tau<\infty\}}\right\}$ is uniformly integrable for all $z \in \mathbb{R}^{n+m}$, where $\chi_{\{.\}}$ is the indicator function and $f(Y(t-)) = \lim_{s\to t-} f((Y(s))$. We assume further that $f \geq 0$ and $g \geq 0$.

Let $\Upsilon^{s,z}$ be the set of of all optimal time $s \leq \tau \leq \tau_{s,z}^{S}$ and define the utility (or performance) function as
$$J^{\tau}(s, z) = E^{s,z}\left[\left(\int_{s}^{\tau} f((Z(t))dt + g(Z(\tau))\chi_{\{\tau<\infty\}}\right)\chi_{\{\tau\geq s\}}\right]. \tag{3.115}$$
The general optimal stopping problem is to find the supremum:
$$V(s, x) = \sup_{\tau\in\Upsilon_{s}^{s,y}} J^{\tau}(s, z),\ z \in \mathbb{R}^{n+m}. \tag{3.116}$$
Note that for $s \geq T_A$ we have the same situation as in Oksendal and Sulem (2007), chapter 2, and if there is no jump, it is the same as in Oksendal (2003), chapter 10, and all results therein applies.

3.A.2 Proof of Lemma L1

It is important to emphasize the assumption about the limiting behavior:

Condition 6 (C1) *The jump at T_A has the limiting behavior*
$$\lim_{s\to T_A} Z^{s,z}(T_A) = z + \Delta Z(T_A) \qquad a.s.. \tag{3.117}$$

We need another condition relating the utility function at two different times. For instance, we want to compare J^{τ_1} at s and something like J^{τ_2} at t for $s < t$. However there are some details in how to compare τ_1 and τ_2 as each one belongs to different sets: Υ^{s,z_1} and Υ^{t,z_2} respectively. Another difficulty in the definitions lies on how to relate z_1 and z_2. We solve it by considering $z_1 = z$ and $z_2 = Z_t^{s,z}$ and, in turn, the sets $\Upsilon^{s,z}$ and $\Upsilon^{t,Z_t^{s,z}}$. In this case the stopping time τ_2 may depend upon $Z_t^{s,z}$. In order to obtain our results we conjecture that the following is true:

Condition 7 (C2) *Let $\tau_2(Z_t^{s,z}) \in \Upsilon^{t,Z_t^{s,z}}$. For $s < t$, there is $\tau_1 \in \Upsilon^{s,z}$ such that:*
$$E^{s,z}\left[\chi_{\{\tau_{s,z}^{S}\geq t\}}\left(\int_{t\wedge\tau_1}^{\tau_1} f(Z^{t,z}(t))dt + g(Z^{t,z}(\tau_1))\chi_{\{\tau_1<\infty\}}\right)\right] \geq E^{s,z}\left[\chi_{\{\tau_{s,z}^{S}\geq t\}}\left(J^{\tau_2(Z_t^{s,z})}(t, Z_t^{s,z})\right)\right]. \tag{3.118}$$
where $a \wedge b = \min(a, b)$,

Given the condition C2 (and that $f \geq 0$) we obtain an inequality for $\chi_{\{\tau_{s,z}^{S}\geq s\}}V(t, Z_t^{s,z})$ that is important to what follows:

Lemma 8 *Consider the model defined in the first section of this appendix. If condition C2 holds then we have for $s < t$*
$$V(s, z) \geq E^{s,z}\left[\chi_{\{\tau_{s,z}^{S}\geq t\}}V(t, Z_t^{s,z})\right] \tag{3.119}$$

Proof. There are two cases: $V\left(t, Z_t^{s,z}(\omega)\right) < \infty$ a.s. and $V\left(t, Z_t^{s,z}\right) = \infty$ with positive probability (where $\omega \in \Omega$).

- Case 1: $V\left(t, Z_t^{s,z}(\omega)\right) < \infty$ a.s.:

As $V\left(t, Z_t^{s,z}(\omega)\right) < \infty$ a.s., for each $\varepsilon > 0$ there is $\tau_2\left(Z_t^{s,z}(\omega)\right) \in \Upsilon^{t, Z_t^{s,z}(\omega)}$ with the property

$$J^{\tau_2}(t, Z_t^{s,z}(\omega)) > V\left(t, Z_t^{s,z}(\omega)\right) - \varepsilon. \tag{3.120}$$

and

$$E^{s,z}\left[\chi_{\{\tau_{s,z}^S \geq t\}} J^{\tau_2}(t, Z_t^{s,z})\right] > E^{s,z}\left[\chi_{\{\tau_{s,z}^S \geq t\}} V\left(t, Z_t^{s,z}\right)\right] - \varepsilon E^{s,z}\left[\chi_{\{\tau_{s,z}^S \geq t\}}\right] \tag{3.121}$$

Condition C2 guarantees that for each $\tau_2 = \tau_2(Z_t^{s,z}) \in \Upsilon^{t, Z_t^{s,z}}$ there is $\tau_1 \in \Upsilon^{s,z}$ such that:

$$E^{s,z}\left[\chi_{\{\tau_{s,z}^S \geq t\}}\left(\int_{t\wedge\tau_1}^{\tau_1} f(Z^{t,z}(t))dt + g(Z^{t,z}(\tau_1))\chi_{\{\tau_1<\infty\}}\right)\right] \geq E^{s,z}\left[\chi_{\{\tau_{s,z}^S \geq t\}} J^{\tau_2(Z_t^{s,z})}(t, Z_t^{s,z})\right], \tag{3.122}$$

this implies:

$$E^{s,z}\left[\chi_{\{\tau_{s,z}^S \geq t\}}\left(\int_{t\wedge\tau_1}^{\tau_1} f(Z^{t,z}(t))dt + g(Z^{t,z}(\tau_1))\chi_{\{\tau_1<\infty\}}\right)\right] > E^{s,z}\left[\chi_{\{\tau_{s,z}^S \geq t\}} V\left(t, Z_t^{s,z}\right)\right] - \varepsilon E^{s,z}\left[\chi_{\{\tau_{s,z}^S \geq t\}}\right]$$

then

$$\begin{aligned} &\sup_{\tau_1 \in \Upsilon^{s,z}} E^{s,z}\left[\chi_{\{\tau_{s,z}^S \geq t\}}\left(\int_{t\wedge\tau_1}^{\tau_1} f(Z^{t,z}(t))dt + g(Z^{t,z}(\tau_1))\chi_{\{\tau_1<\infty\}}\right)\right] \\ &> E^{s,z}\left[\chi_{\{\tau_{s,z}^S \geq t\}} V\left(t, Z_t^{s,z}\right)\right] - \varepsilon E^{s,z}\left[\chi_{\{\tau_{s,z}^S \geq t\}}\right]. \end{aligned} \tag{3.123}$$

As this is true for all $\varepsilon > 0$ we have that

$$\sup_{\tau_1 \in \Upsilon^{s,z}} E^{s,z}\left[\chi_{\{\tau_{s,z}^S \geq t\}}\left(\int_{t\wedge\tau_1}^{\tau_1} f(Z^{t,z}(t))dt + g(Z^{t,z}(\tau_1))\chi_{\{\tau_1<\infty\}}\right)\right] \geq E^{s,z}\left[\chi_{\{\tau_{s,z}^S \geq t\}} V\left(t, Z_t^{s,z}\right)\right]. \tag{3.124}$$

Now we need to show that $V(s,z)$ is greater than or equal to the l.h.s. in the above equation.

Note that for any $\tau_1 \in \Upsilon^{s,z}$ we have

$$\begin{aligned} V(s,z) &\geq E^{s,z}\left[\int_s^{\tau_1} f(Z^{t,z}(t))dt + g(Z^{t,z}(\tau_1))\chi_{\{\tau_1<\infty\}}\right] \\ &\geq E^{s,z}\left[\chi_{\{\tau_{s,z}^S \geq t\}}\left(\int_s^{\tau_1} f(Z^{t,z}(t))dt + g(Z^{t,z}(\tau_1))\chi_{\{\tau_1<\infty\}}\right)\right] \\ &\geq E^{s,z}\left[\chi_{\{\tau_{s,z}^S \geq t\}}\left(\int_s^{t\wedge\tau_1} f(Z^{t,z}(t))dt + \int_{t\wedge\tau_1}^{\tau_1} f(Z^{t,z}(t))dt + g(Z^{t,z}(\tau_1))\chi_{\{\tau_1<\infty\}}\right)\right] \\ &\geq E^{s,z}\left[\chi_{\{\tau_{s,z}^S \geq t\}}\left(\int_{t\wedge\tau_1}^{\tau_1} f(Z^{t,z}(t))dt + g(Z^{t,z}(\tau_1))\chi_{\{\tau_1<\infty\}}\right)\right] \end{aligned}$$

where the last inequality is true because $\int_s^{t\wedge\tau_1} f(Z^{t,z}(t))dt \geq 0$ a.s.. As it is valid for any $\tau_1 \in \Upsilon^{s,z}$, it is valid also for the supremum:

$$V(s,z) \geq \sup_{\tau_1 \in \Upsilon^{s,z}} E^{s,z}\left[\chi_{\{\tau_{s,z}^S \geq t\}}\left(\int_{t\wedge\tau_1}^{\tau_1} f(Z^{t,z}(t))dt + g(Z^{t,z}(\tau_1))\chi_{\{\tau_1<\infty\}}\right)\right]$$

and comparing with inequality 3.124 we have finally

$$V(s,z) \geq E^{s,z}\left[\chi_{\{\tau_{s,z}^S \geq t\}} V\left(t, Z_t^{s,z}\right)\right].$$

- Case 2: $V(t, Z_t^{s,z}) = \infty$ with positive probability

For ω in which $V(t, Z_t^{s,z}(\omega)) = \infty$ we have that for $k > 0$ there is $\tau_2(Z_t^{s,z}) \in \Upsilon^{t, Z_t^{s,z}}$ such that

$$J^{\tau_{2(\omega)}}(t, Z_t^{s,z}(\omega)) > k. \tag{3.125}$$

and for ω in which $V(t, Z_t^{s,z}(\omega)) < \infty$ we have $\tau_{2(\omega)} \in \Upsilon^{t, Z_t^{s,z}}$ such that

$$J^{\tau_{2(\omega)}}(t, Z_t^{s,z}(\omega)) > V(t, Z_t^{s,z}(\omega)) - \varepsilon. \tag{3.126}$$

By condition C2 we can make

$$E^{s,z}\left[\chi_{\{\tau_{s,z}^S \geq t\}}\left(\int_{t\wedge\tau_1}^{\tau_1} f(Z^{t,z}(t))dt + g(Z^{t,z}(\tau_1))\chi_{\{\tau_1<\infty\}}\right)\right] \geq E^{s,z}\left[\chi_{\{\tau_{s,z}^S \geq t\}}\left(J^{\tau_2(Z_t^{s,z})}(t, Z_t^{s,z})\right)\right],$$

and

$$E^{s,z}\left[\chi_{\{\tau_{s,z}^S \geq t\}}\left(\int_{t\wedge\tau_1}^{\tau_1} f(Z^{t,z}(t))dt + g(Z^{t,z}(\tau_1))\chi_{\{\tau_1<\infty\}}\right)\right] > kE^{s,z}\left[\chi_{\{\tau_{s,z}^S \geq t\}}\chi_{\left\{V\left(t, Z_t^{s,z}\right)=\infty\right\}}\right].$$

This is possible to make for all $k > 0$. If $E^{s,z}\left[\chi_{\{\tau_{s,z}^S \geq t\}}\chi_{\left\{V\left(t, Z_t^{s,z}\right)=\infty\right\}}\right] > 0$ then we have

$$V(t, z) = \infty. \tag{3.127}$$

On the other hand, if $E^{s,z}\left[\chi_{\{\tau_{s,z}^S \geq t\}}\chi_{\left\{V\left(t, Z_t^{s,z}\right)=\infty\right\}}\right] = 0$,

$$E^{s,z}\left[\chi_{\{\tau_{s,z}^S \geq t\}}\left(\int_{t\wedge\tau_1}^{\tau_1} f(Z^{t,z}(t))dt + g(Z^{t,z}(\tau_1))\chi_{\{\tau_1<\infty\}}\right)\right] \geq E^{s,z}\left[\chi_{\{\tau_{s,z}^S \geq t\}}V(t, Z_t^{s,z}(\omega))\right] - \varepsilon E^{s,z}\left[\chi_{\{\tau_{s,z}^S \geq t\}}\right]$$

and the arguments of the case 1 applies. ■

Now we generalize the lemma L1 to the the jump-diffusion case. First we prove a statement using a sequence of time converging to T_A.

Lemma 9 *Assume as true the conditions in the previous proposition and that $V(T_A, z)$ is measurable in z. Then for any sequence $\{u_i\}_{i=1}^{\infty}$ such that $u_i < T$ and $\lim u_i = T$:*

$$\liminf_{i\to\infty} V(u_i, z) \geq E\left[\liminf_{i\to\infty} V(T_A, Z^{u_i,z}(T_A))\chi_{\{\tau_{u_i,z}^S \geq T_A\}}\right]. \tag{3.128}$$

Proof. Using the lemma above:

$$V(u, z) \geq E^{u,z}\left[V(T_A, Z_{T_A})\chi_{\{\tau_S \geq T_A\}}\right]. \tag{3.129}$$

Remember that

$$E^{u,z}\left[V(T_A, Z_{T_A})\chi_{\{\tau_S > T_A\}}\right] = E\left[V\left(T_A, Z_{T_A}^{u,z}\right)\chi_{\{\tau_S \geq T_A\}}\right]. \tag{3.130}$$

As the inequality (3.129) is valid for all $0 \leq u < T_A$, we have that:

$$\liminf_{i\to\infty} V(u_i, z) \geq \liminf_{i\to\infty} E\left[V\left(T_A, Z_{T_A}^{u_i,z}\right)\chi_{\{\tau_S \geq T_A\}}\right] \tag{3.131}$$

We want to use Fatou's lemma in the next step. Then we need to verify that $V\left(T_A, Z_{T_A}^{u_i,z}\right)\chi_{\{\tau_S \geq T_A\}} \geq 0$ a.s. and that it is measurable. As $f \geq 0$ and $g \geq 0$, we have that $V\left(T_A, Z_{T_A}^{u_i,z}\right)\chi_{\{\tau > T\}} \geq 0$. Moreover, $V\left(T_A, Z_{T_A}^{u_i,z}\right)\chi_{\{\tau_S > T\}}$ is $\mathcal{F}_{T_A}-$measurable random variable as it is a compositions of a measurable function $V(T_A,\cdot)$ with a $\mathcal{F}_{T_A}-$measurable random variable $Z_{T_A}^{u_i,z}$. Then, for any sequence $\{u_i\}_{i=1}^{\infty}$ such that $u_i < T_A$ and $\lim u_i = T_A$ we have that:

$$\lim_{i\to\infty}\inf V(u_i,z) \geq E\left[\lim_{i\to\infty}\inf V\left(T_A, Z^{u_i,z}(T_A)\right)\chi_{\{\tau_S\geq T_A\}}\right]. \tag{3.132}$$

■

The next two lemmas are similar to the lemma L1 in section 3.3. The statement explicitly mentions the solvency region. In the first version of the lemma the solvency region is all the space as is implicitly assumed in section 3.3. In the second version the solvency region may be any open set constant through time.

Lemma 10 (L1') *Consider the model defined in first section of this appendix and assume the conditions C1 and C2 as valid. Moreover assume that $V(T_A,z)\chi_{\{\tau_S\geq T_A\}}$ is $\mathcal{F}_{T_A}-$measurable and lower semi-continuous in z and that the solvency region S is all space. Then:*

$$\lim_{s\to T_A-}\inf V(s,z) \geq E\left[V\left(T_A, z+\Delta Z(T_A,z)\right)\right] \tag{3.133}$$

Proof. By condition C1 we have

$$\lim_{s\to T_A} Z^{s,z}(T_A)(\omega) = z + \Delta Z(T_A)(\omega) \qquad \text{a.s..} \tag{3.134}$$

Then, by properties of l.s.c. function (and noting that $\chi_{\{\tau_S\geq T_A\}} = 1$ because the solvency region is all space) , we have

$$\begin{aligned} \lim_{i\to\infty}\inf V\left(T_A, Z^{u_i,z}(T_A)(\omega)\right)\chi_{\{\tau_S\geq T_A\}} &\geq V\left(T_A, \lim_{i\to\infty}\inf Z^{u_i,z}(T_A)(\omega)\right) * 1 \\ &\geq V\left(T_A, z+\Delta Z(T_A)(\omega)\right) \end{aligned} \tag{3.135}$$

or

$$\lim_{i\to\infty}\inf V\left(T_A, Z^{u_i,z}(T_A)\right) \geq V\left(T_A, z+\Delta Z(T_A)\right) \qquad \text{a.s.} \tag{3.136}$$

Then, the previous lemma implies that

$$\begin{aligned} \lim_{i\to\infty}\inf V(u_i,z) &\geq E\left[\lim_{i\to\infty}\inf V\left(T_A, Z^{u_i,z}(T_A)\right)\chi_{\{\tau_S\geq T_A\}}\right] \\ &\geq E\left[\lim_{i\to\infty}\inf V\left(T_A, Z^{u_i,z}(T_A)\right)\right] \\ &\geq E\left[V\left(T_A, z+\Delta Z(T_A)\right)\right] \end{aligned} \tag{3.137}$$

as this inequality is valid for all sequence $\{u_i\}$ converging to the announcement time $\lim_i u_i = T_A$,then it is also valid for the time limit $\lim s = T_A$.

$$\lim_{s\to T_A-}\inf V(s,z) \geq E\left[V\left(T_A, z+\Delta Z(T_A)\right)\right]. \tag{3.138}$$

■

Lemma 11 (L1") *Consider the model defined in first section of this appendix and assume the conditions C1 and C2 as valid. Assume that $V(T_A,z)\chi_{\{\tau_S\geq T_A\}}$ is $\mathcal{F}_{T_A}-$measurable and lower semi-continuous in z. Moreover, assume that the solvency region S doesn"t depend upon time. Then for $z\in S$ or $z\notin \overline{S}$ (where $\overline{S}$ is the closure of S) we have*

$$\lim_{s\to T_A-}\inf V(s,z) \geq E\left[V\left(T_A, z+\Delta Z(T_A,z)\right)\chi_{\{z\in S\}}\right] \tag{3.139}$$

Proof. If $z \in S$ (recall that S is an open set), then for all ω such that

$$\lim_{s \to T_A} Z^{s,z}(T_A)(\omega) = z + \Delta Z(T_A)(\omega) \tag{3.140}$$

there is s^* such that

$$Z^{s^*,z}(t) \in S \text{ for } s^* \leq t < T_A. \tag{3.141}$$

In this case

$$\begin{aligned} \liminf_{i\to\infty} V\left(T_A, Z^{u_i,z}(T_A)\right)(\omega)\chi_{\{\tau_S \geq T_A\}}(\omega) &= \liminf_{i\to\infty} V\left(T_A, Z^{u_i,z}(T_A)\right)(\omega) \\ &\geq V\left(T_A, z + \Delta Z(T_A)(\omega)\right) \\ &= V\left(T_A, z + \Delta Z(T_A)\right)(\omega)\chi_{\{z\in S\}}(\omega). \end{aligned} \tag{3.142}$$

By other side, if $z \notin S$, it is trivially true that

$$\begin{aligned} \liminf_{i\to\infty} V\left(T_A, Z^{u_i,z}(T_A)\right)(\omega)\chi_{\{\tau_S \geq T_A\}}(\omega) &\geq 0 \\ &= V\left(T_A, z + \Delta Z(T_A)\right)(\omega)\chi_{\{z\in S\}}(\omega). \end{aligned} \tag{3.143}$$

because the value function is greater than zero.

Finally, applying the same steps as in the proof of Lemma L1' we have

$$\liminf_{s\to T_A-} V(s,z) \geq E\left[V\left(T_A, z + \Delta Z(T_A, z)\right)\chi_{\{z\in S\}}\right]. \tag{3.144}$$

■

3.B Numerical Algorithm

In this appendix we describe the numerical algorithm in details for the case studied in section 3.4. The algorithm's properties are developed in Chancelier *et al.* (2007) and are described in Oksendal and Sulem (2007, Chapter 9) as well. First we describe the time invariant case (consistent with $t \geq T_A$) and then we incorporate the time variation.

3.B.1 Discrete Definitions

For $t \geq T_A$ we have the analytical solution but we provide the algorithm for this case and then discuss the difference for $t < T_A$. We shall solve the quasivariational inequality

$$\max\{A\Phi, g - \Phi\} = 0 \tag{3.145}$$

where the generator[21] A is

$$A\Phi = \frac{\partial \Phi}{\partial s} + \alpha x \frac{\partial \Phi}{\partial x} + \frac{1}{2}\beta x^2 \frac{\partial^2 \Phi}{\partial x^2}, \tag{3.146}$$

and define the continuation region

$$\mathbf{C} = \left\{(s,x,\alpha) \in R^+ \times R^+ \times R^+ | \Phi(s,x) > g(s,x)\right\}. \tag{3.147}$$

Later we will define a grid but for now consider a "small"$h > 0$ and $h_t > 0$ and define a discrete version of A as

$$A_h v = \partial_t^{h_t} v + \alpha x \partial_x^h v + \frac{1}{2}\sigma^2 x^2 \partial_{xx}^{2,h} v, \tag{3.148}$$

[21] Strictly speaking, the generator isn't a differential operator. Nonetheless it coincides in the set of twice differentiable functions with compact support. See theorem 1.22 in Oksendal and Sulem (2007).

where

$$\partial_t^{h_t} v(s,x) = \frac{v(s+h_t,x) - v(s,x)}{h_t}, \tag{3.149}$$

$$\partial_x^{h} v(s,x) = \frac{v(s,x+h) - v(s,x)}{h}, \tag{3.150}$$

$$\partial_{xx}^{2,h} v(x,y) = \frac{v(s,x+h) - 2v(s,x) + v(s,x-h)}{h^2}. \tag{3.151}$$

Let $T_h(s,\alpha)$ be the discrete version of a temporal slice of $\mathbf{C}$

$$T_h(s,\alpha) = \left\{ih|e^{-\rho s}\left(A_h\phi - \rho\phi\right) > e^{-\rho s}\widehat{g}(x) - e^{-\rho s}\phi\right\}.$$

where $e^{-\rho s}\widehat{g}(x) = g(s,x)$ and $\Phi(s,x) = e^{-\rho s}\phi(x)$.

Refinements for $t \geq T_A$

In our case, it is possible to make a transformation after T_A

$$\Phi(s,x) = e^{-\rho s}\phi(x) \tag{3.152}$$

and

$$\begin{aligned} A\Phi(s,x) &= \frac{\partial\left[e^{-\rho s}\phi(x)\right]}{\partial s} + e^{-\rho s}\alpha x\frac{\partial\phi(x)}{\partial x} + e^{-\rho s}\frac{1}{2}\beta x^2\frac{\partial^2\phi(x)}{\partial x^2} \\ A\Phi(s,x) &= -\rho e^{-\rho s}\phi(x) + e^{-\rho s}\frac{\partial\left[\phi(x)\right]}{\partial s} + e^{-\rho s}\alpha x\frac{\partial\phi(x)}{\partial x} + e^{-\rho s}\frac{1}{2}\beta x^2\frac{\partial^2\phi(x)}{\partial x^2}. \end{aligned} \tag{3.153}$$

$$A\Phi(s,x) = e^{-\rho s}A\phi - \rho e^{-\rho s}\phi(x) \tag{3.154}$$

Now we have an ordinary differential equation in x. In the region where $A\Phi(s,x) = 0$ we may rewrite

$$A\Phi(s,x) = 0 \tag{3.155}$$

if, and only if

$$A\phi - \rho\phi(x) = 0. \tag{3.156}$$

and in the discrete verstion

$$A_h\phi - \rho\phi(x) = 0 \tag{3.157}$$

$$\alpha x\partial_x^h\phi(x) + \frac{1}{2}\sigma^2 x^2\partial_{xx}^{2,h}\phi(x) - \rho\phi(x) = 0. \tag{3.158}$$

The computer can‘t handle an infinite number of elements. Then we will truncate the problem. Define the grid as $D_h = (ih)$ where $i \in \{0,...,N\}$ and N are large enough to not compromise the results or to entail a small error. It is necessary to define a boundary condition at $x = Nh$. At the boundary of D_h we will consider the Neumann boundary condition

$$\frac{\partial\phi}{\partial x}(Nh) = 0. \tag{3.159}$$

Fortunately this boundary condition is innocuous for the numerical results in section 3.4 because the continuation region is smaller then D_h. Remember that the region U is a sub-set of the continuation region and is defined as

$$U = \left\{(x,s,\alpha) \in \mathbb{R}^+ \times \mathbb{R}^+ \times \mathbb{R}^+ | Ag + f > 0\right\} \tag{3.160}$$

and we can define a discrete version (in our case $f = 0$) at time s

$$U_h(s,\alpha) = \left\{ih|A_h g(ih) - \rho g(ih) > 0\right\}. \tag{3.161}$$

Note that for $t \geq T_A$ the set U_h above doesn't change with time. If possibe D_h shall be greater than U_h (this is indeed the case for the section 4).

Then the integrovariational inequality

$$\max\left\{e^{-\rho s}\left(A_h\phi - \rho\phi\right), g - e^{-\rho s}\phi\right\} = 0 \tag{3.162}$$

may be written as

$$\begin{aligned} A_h\phi\left(ih\right) - \rho\phi\left(ih\right) &= 0 \text{ for } ih \in T_h, & (3.163)\\ e^{-\rho s}\phi &= g \text{ for } ih \notin T_h. & (3.164)\end{aligned}$$

and the slice of the continuation region is defined by

$$T_h(s,\alpha) = \left\{ih|e^{-\rho s}\left(A_h\phi - \rho\phi\right) > e^{-\rho s}\widehat{g}(x) - e^{-\rho s}\phi\right\} \tag{3.165}$$

where $g(s,x) = e^{-\rho s}\widehat{g}(x)$. Note that T_h doesn't depend upon time after T_A.

3.B.2 The Algorithm

After defining the elements, the definition of the algorithm are now in order. Given the solution ϕ it is possible to find the continuation region $T_h(s,\alpha)$. On the other hand, given the continuation region, it is possible to find the solution ϕ. It seems a fixed point problem and one can guess if there is an iteration procedure leading to ϕ. Indeed Chancelier *et al.* (2007) shows that a slight different but equivalent problem has this feature. Instead of using the integrovariational inequality (3.162) one can use a better behaved and equivalent problem

$$\phi_h(x) = \max\left\{\left[I_h + \frac{\xi(A_h-\rho)}{1+\xi\rho}\right]\phi, \widehat{g}\right\} \tag{3.166}$$

where $0 < \xi \leq \min \frac{1}{|(A_h)_{ii}+\rho|}$, and I_δ is the identity operator ($I_h v_h = v_h$).

Again, this implies

$$\begin{aligned} A_h\phi\left(ih\right) - \rho\phi\left(ih\right) &= 0 \text{ for } ih \in T_h, & (3.167)\\ e^{-\rho s}\phi &= g \text{ for } ih \notin T_h. & (3.168)\end{aligned}$$

but the slice of the continuation region is now defined as

$$T_h(s,\alpha) = \left\{ih|\left[I_h + \frac{\xi(A_h-\rho)}{1+\xi\rho}\right]\phi(ih) > \widehat{g}\right\}. \tag{3.169}$$

This difference allows us to define an iteration procedure converging to the right solution:

- (step n, sub-step 1) Given v^n find T_h^{n+1} such that

$$T_h^{n+1}(s,\alpha) = \left\{ih|\left[I_h + \frac{\xi(A_h-\rho)}{1+\xi\rho}\right]\phi(ih) > \widehat{g}\right\}. \tag{3.170}$$

- (step n, sub-step 2) Compute v^{n+1} as the solution of

$$\begin{aligned} A_h v^{n+1}\left(ih\right) - \rho v^{n+1}\left(ih\right) &= 0 \text{ for } ih \in T_h^{n+1}, & (3.171)\\ e^{-\rho s} v^{n+1} &= g \text{ for } ih \notin T_h^{n+1}. & (3.172)\end{aligned}$$

- Repeat the procedure until $\max\left\{abs(v^{n+1} - v^n)\right\}$ less then a predefined error.

The only piece missing is to define v^0 or T_h^0. In this case, it is easier to define $T_h^0 = D_h$ and begin the procedure from sub-step 2. It is shown that $\lim_{n\to\infty} v^n \to \phi$.

Remark 1 *We omit several technical conditions in the above presentation. They hold for the problem we are dealing with and we refer to Oksendal and Sulem (2007) and Chancelier et al. (2007) in order to account for them.*

3.B.3 Modification in the Algorithm for $t < T_A$

We will discretize the time and apply the above algorithm at each slice of time using a implicit scheme. Note that it is necessary to define a boundary condition at $t = T_A$. Remember that we have the analytical solution after T_A. We have for $t = T_A$ the boundary condition

$$\widetilde{\Phi}(T_A, x) = E\left[\Phi\left(T_A, x, \alpha\right)\right], \tag{3.173}$$

$$\begin{aligned}
\widetilde{\Phi}(T_A, x) &= E\left[e^{-\rho s}C(\alpha)x^{\lambda_1(\alpha)}\chi_{\{0<x<x^*(\alpha)\}} + e^{-\rho s}(x-a)\chi_{\{x^*(\alpha)\leq x\}}\right] &(3.174)\\
\widetilde{\Phi}(T_A, x) &= \int_{\underline{\alpha}}^{\overline{\alpha}} \left(e^{-\rho s}C(\alpha)x^{\lambda_1(\alpha)}\chi_{\{0<x<x^*(\alpha)\}} + e^{-\rho s}(x-a)\chi_{\{x^*(\alpha)\leq x\}}\right) d\alpha &(3.175)
\end{aligned}$$

where $C(\alpha)$, $\lambda_1(\alpha)$ and $x^*(\alpha)$ are defined in section 4.

Note that Φ depends upon α and it changes after T_A. Nonetheless, before it doesn't change. Fot $t < T_A$ we omit α in the notation

$$\begin{aligned}
\Phi\left(T_A - nh_t, ih\right) &= \Phi\left(T_A - nh_t, ih, \alpha\left(T_A - nh_t\right)\right) &(3.176)\\
&= \Phi\left(T_A - nh_t, ih, \alpha\left(0\right)\right).
\end{aligned}$$

The grid in the dimension x will be the same for all s and the discretization in time will be given by $T_A - nh_t$. Now the continuation region varies over time, $T_h(s) = T_h(T_A - nh_t)$, and we have

$$\begin{aligned}
A_h\Phi\left(T_A - nh_t, ih\right) &= 0 && \text{for } ih \in T_h(T_A - nh_t), &(3.177)\\
\Phi\left(T_A - nh_t, ih\right) &= g\left(T_A - nh_t, ih\right) && \text{for } ih \notin T_h\left(T_A - nh_t\right), &(3.178)
\end{aligned}$$

with Neumann boundary condition at $x = Nh$

$$\partial_x^h v(s, Nh) = 0 \tag{3.179}$$

and the final condition

$$\Phi\left(T_A, ih\right) = \widetilde{\Phi}(T_A, x). \tag{3.180}$$

Note that we defined the discrete time differential as

$$\partial_t^{h_t} v(s,x) = \frac{v(s+h_t, x) - v(s,x)}{h_t}. \tag{3.181}$$

This entails a implicit scheme when solving the numerical partial differential equation defined in equations (3.177) and (3.178). For instance, given $T_h^0(T_A - h_t)$,we have for $s = T_A - h_t$

$$\begin{aligned}
A_h\Phi(T_A - h_t, ih) &= \partial_t^{h_t}\Phi + \alpha\partial_x^h\Phi + \frac{1}{2}\sigma^2x^2\partial_{xx}^{2,h}\Phi\\
&= \frac{\widetilde{\Phi}(T_A, ih) - \Phi(T_A - h_t, ih)}{h_t} + \alpha x\partial_x^h\Phi(T_A - h_t, ih) + \frac{1}{2}\sigma^2x^2\partial_{xx}^{2,h}\Phi(T_A - h_t, ih)
\end{aligned}$$

and

$$\begin{aligned}
A_h\Phi(T_A - h_t, ih) &= 0 && \text{for } ih \in T_h(T_A - nh_t). &(3.182)\\
\Phi\left(T_A - nh_t, ih\right) &= g\left(T_A - nh_t, ih\right) && \text{for } ih \notin T_h\left(T_A - nh_t\right),
\end{aligned}$$

with the Neumann boundary conditions. Now it is only necessary to use the algorithm defined above in this slice of time.

The problem may be solved sequentially as $\Phi(T_A - nh_t, ih)$ depends upon $\widetilde{\Phi}$ only through $\Phi(T_A - (n-1)h_t, ih)$. Moreover,$\Phi(T_A - (n-1)h_t, ih)$ doesn't depend upon $\Phi(T_A - nh_t, ih)$.

Chapter 4

Dynamic Portfolio Selection with Transactions Costs and Scheduled Announcement

Chapter Abstract[1]

The present work provides a numerical solution to an optimal portfolio problem with fixed cost and scheduled announcement. The scheduled announcements are modeled as a jump occurring at known date. This setting leads to an optimal impulse problem along with a stochastic control problem and the numerical solution uses a recently developed method. This modeling is consistent with the asset price behavior observed in financial markets and the optimal policy agrees with studies reporting the trading volume around announcements. This suggests that the fixed costs are important to understand these findings.

Keywords: Scheduled Announcements, Volume Behavior, Optimal Portfolio, Fixed Cost, Impulse Control, Quasi-Variational Inequality, Numerical Methods in Economics.
JEL Classification Numbers: C6,G11,G12.

4.1 Introduction

Infrequent action taking has been a pervasive empirical finding in economics. It seems common to observe economic agents doing nothing most of the time and taking a substantial action once a while instead of doing several small interventions. Examples include retail establishment choosing the timing and size of price changes, job creation and destruction by firms and infrequent portfolio rebalancing for households. Such behavior is obtained in several models where action entails some sort of cost. This is true in particular for some optimal portfolio problems[2].

The optimal portfolio allocation has been a long-standing subject in the literature and also has a far reaching interest for practitioners. In the context of diffusion processes it has been of interest at least since the seminal works of Merton (1969,1971) and Samuelson (1969) and one important generalization is to incorporate fixed and/or proportional transaction cost. The latter cost is faced by most (if not all) market participants and some investors face the former type

[1] Joint work with Marco Bonomo.

[2] Stockey (2009) and Oksendal and Sulem (2005) presents the mathematical theory and analyzes several important models.

A short list of empirical work are: Bils and Klenow (2004), Klenow and Kryvtsov (2008) for infrequent prices changes; Vissing-Jorgensen (2002) and the references therein for household portfolio behavior.

of cost as well[3]. Moreover several models account for microstructure frictions (such as bid-ask spread) by using fixed costs[4]. Another possible generalization is to incorporate announcements in the price process.

There is voluminous evidence that announcements spur trading, make prices jump and have a long term effect on the price process (see the review by Bamber *et al.* (2011) and references therein). In the present work we are interested in the announcement with release date known in advance. In this case, empirical studies find an increase in trading volume in the day before the scheduled announcement that persists abnormally high for several days after it. However trading volume may be lower than average for the period between 10 to 3 days before the scheduled announcement depending upon the sample considered. For instance, Chae (2005) reports the abnormal low volume for this period while Hong and Stein (2007) report no change in the same period. From a theoretical point of view, several models tried to explain to the rise or fall in the volume before a known in advance information release. Nonetheless, to the best of our knowledge, none was able to address the dynamics that Chae (2005) reported.

We characterize in this work the incentives an investor faces close to a scheduled announcement in the presence of fixed cost in continuous time models. We consider an economic agent that can invest in a risky and risk-less asset and consumes continuously. The consumption is withdrawn from the bank account (risk-less asset) and the rate is chosen continuously. When buying or selling the risky asset the agent has to pay a fee that doesn't depend upon the transaction size. It implies an infrequent action taking and a state dependent rule. We solve the model numerically using a novel method introduced in Chancelier *et al.* (2007). In some arguably common cases, the simulations suggest that the chance of transacting the risky asset has the dynamic discussed in previous paragraph.

Our model builds into the work of Merton (1969), Oksendal and Sulem (2007) and others. The risky asset follows a geometric Brownian motion. One way to incorporate the scheduled announcements is to add a jump (a discontinuity in price path with random size) at a fixed date known by the investor. Another way is to change randomly some parameter of the model such as the risk-free rate at a fixed and known time. We report the numerical results incorporating the news in both ways as they are consistent with empirical observations[5].

We report simulations results incorporating the news with and without jumps. The basic insight when the prices don't jump builds into a menu cost model developed in Bonomo *et al.* (2013). If the agent is close to the time of information arrival, if there is a fixed cost to exercise the control (the menu cost in their model and the fixed fee in ours) and the prices don't jump, then it is usually better to wait for the information. In this case, there is a very low risk in waiting for the information and the gain with a better informed is high. The consequence for volume in this case would be a fall before the announcement and an increase after it. However we believe that such cases are better interpreted as private information arrival changing the beliefs of one market participant and, as such, may not have the influence over market trading volume.

On the other hand, information arriving at same time for all investor is likely to change the price and its process. There is, at least, some chance of a jump in prices as reported by Bamber and Cheon (1995). In this case, a better informed investor (or one with strong opinion) may be modeled with a belief putting low variance in the jump.

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[3] Transaction fees for NYSE can be found at (it was accessed at 4/18/2013): https://usequities.nyx.com/markets/nyse-equities/trading-fees.

[4] For portfolio problems with proportional cost only, see: Magill an d Constantinides (1976), Constantinides (1986), Davis and Norman (1990), Dumas and Luciano (1991), Shreve and Soner (1994), Akian et al.(1996), Sulem (1997); Tourin and Zariphopoulou (1997), Leland (2000); Atkinson and Mokkhavesa (2003).

With fixed cost only, see: Eastham and Hastings (1992), Hastings (1992), Schroder (1995), Korn (1998).

For both types of costs, see: Chancelier et al. (2000), Oksendal and Sulem (2002); Zakamouline (2002), Chellaturai and Draviam (2007).

[5] See the introduction in Azevedo (2013). Note that this paper is incorporated in the present volume as chapter 3.

In the presence of a jump at the announcement, the investor has incentives to prepare the portfolio for it. Usually the best asset position just prior to the scheduled news is different from the best one after it. If the costs are sufficient low, two transactions may occur: one just before and another just after the announcement. The key result is that the investor tends to transact just before the announcement but not a little before. In this case, the chance of trading diminishes until just before the announcement, then there is a high chance to trade just before and again just after the news. This behavior is consistent with trading volume behavior reported by Chae (2005).

Those observations suggest that transaction costs are important to explain the higher volume after the announcement and the mixed behavior before the announcement. The state dependent strategies by themselves can explain the larger volume after the news as the prices and some fundamental quantities change abruptly with positive probability.

The rest of the paper is organized as follows. The next two sections present the optimal portfolio problem and the numerical method overview. The fourth and fifth sections present the results and the discussion and the sixth section concludes. There are two appendices. The first one describes the numerical method in details and second one describes some image filter we used.

4.2 The Portfolio Problem

The present portfolio problem builds into the work of Merton (1969), Oksendal and Sulem (2007) and others. Merton (1969) studies the investor problem with two assets and no transaction costs. In his case it is possible to transact continuously and the optimal portfolio consist of the risky and risk-free assets carried with a fixed proportion regardless of wealth (the Merton line). Davis and Norman (1990) and Shreve and Soner (1994) consider the case with proportional transaction cost only. In this case a no-transaction region arises and depends only on the proportion of assets (not on wealth). The optimal strategy consists of "infinitesimal" trades whenever the proportion hits an upper or lower bound and this behavior prevents the portfolio from going outside the no-transaction region. Oksendal and Sulem (2007) and Chancelier *et al.* (2002) investigate the fixed cost along with proportional cost. As in the previous case the no-transaction region arises but it depends upon the wealth and upon the proportion of asset. More importantly, the trades occur in finite amounts whenever the portfolio hit the no-transaction region boundaries. If the proportional cost is zero the rebalanced portfolio is a function of wealth and nothing else. Otherwise there are two possible rebalanced portfolios for a given wealth. The choice depends upon which part of inaction region's boundary the portfolio hits.

The present paper adds to this literature by considering that a jump or a random change in parameters occurs at a fixed date. We study the optimal portfolio problem in the presence of fixed cost and the present focus is on the behavior around the scheduled announcement. After the jump the problem is the same as in Oksendal and Sulem (2007) with fixed cost only. Although proportional cost could be added, the numerical results are cleaner without it. Chancelier *et al.* (2002) provides a numerical simulation for the case without jump by solving optimal stopping times iteratively. Nonetheless the present work uses the method developed in Chancelier *et al.* (2007) that is based in a fixed point problem as it is more efficient. The algorithm along with some implementation details are described in appendix 4A.

This modeling choice is consistent with the price process observed in practice and several authors provide empirical evidence. Early works such as Beaver (1968) document noticeable price movements at earning announcements and Pattel and Wolfson (1984) find a quick move with the bulk of price change in the first few minutes after the release. Moreover a change in drift may occur as is witnessed by the post-announcement drift that can last up to several months (see, for instance, Bernard and Thomas (1989, 1990)). Similar findings are documented in others markets as well (see Bamber et al. (2011) and reference therein). Note that the practice of scheduled announcements is pervasive in financial market. For instance the dates of the Federal Open Market Committee (FOMC) meeting are known in advance and several listed firms release earning information at a scheduled date.

4.2.1 Definition of The Problem

There are two assets and a scheduled announcement at T_A. X_t denotes the amount of money invested in a risk-free money bank account with the constant interest rate of r and S_t denotes the amount invested in the risky asset (stock). Without any type of control (consumption or transaction), these variables evolves as:

$$dX_t = r_t X_t dt, \text{ with } X(u) = x \text{ and } t \geq u \tag{4.1}$$

and the stock follows

$$dS_t = \alpha_t S_t dt + \sigma_t S_t dW_t + \Delta S_{T_A} \chi_{\{t=T_A\}}, \text{ with } S(u) = y \text{ and } t \geq u \tag{4.2}$$

where r_t, α_t and σ_t are constants before and after the announcement. For some simulation the jump at T_A does not occur, i.e., in some simulations we have that $\Delta S_{T_A} \chi_{\{t=T_A\}} = 0$ with certainty. The values for the risk-free, drift and volatility $(r_t, \alpha_t, \sigma_t)$ before T_A is known since the beginning. In some simulation it remains the same after the announcement but in others theirs value change randomly at T_A. This may be written as

$$(r_{t,}\alpha_t, \sigma_t) = (r_{BA,}\alpha_{BA}, \sigma_{BA}) \text{ if } t < T_A \text{ (Before Announcement)}, \tag{4.3}$$

$$(r_{t,}\alpha_t, \sigma_t) = (r_{AA}, \alpha_{AA}, \sigma_{AA}) \text{ if } t \geq T_A \quad \text{(After Announcement)}. \tag{4.4}$$

where $(r_{AA}, \alpha_{AA}, \sigma_{AA})$ may be equal to $(r_{BA}, \alpha_{BA}, \sigma_{BA})$ with certainty or may change randomly. The conditional distribution of $(r_{AA}, \alpha_{AA}, \sigma_{AA})$ does not change with t. The change on these parameter are interpreted as the long term impact of the announcement (e.g. change in risk, post-earning announcement phenomena, etc.) or some change in the macro-economic policy (changes in the risk-free rate). The immediate impact is felt as ajump in price S_{T_A}:

$$S_{T_A} = S\left(T_A -\right)\zeta \tag{4.5}$$

where ζ has a lognormal distribution realized[6] at T_A and $S\left(A_T -\right) = \lim_{u \to T_A -} S(u)$ is the left limit. Note that S_t and $S(t)$ have the same meaning. The investor knows the jump's conditional distribution, the distribution of $(r_{AA}, \alpha_{AA}, \sigma_{AA})$ and the time T_A since from beginning.

The investor chooses a consumption rate $c(t) \geq 0$ which is drawn from the bank account without any cost. At any time the investor can decide to transfer money between bank account and the stock incurring into a transaction fixed cost $k > 0$ (drawn from the bank account). In this context the investor will only change his portfolio finitely many times in any finite time interval. The consumption rate $c(t)$ is a regular stochastic control and the trade decision implies an impulse control $v = (\tau_1, \tau_2, ...; \xi_1, \xi_2, ...)$ where $0 \leq \tau_1 < \tau_2 < ...$ are stopping times giving (the times in which the investor decides to change his portfolio) and $\{\xi_j \in \mathbb{R}; j = 1, 2, ...\}$ give the sizes of the transactions at these times.

When the control $w = (c, v)$ is applied the former stochastic differential equations turns into

$$dX_t^w = (rX_t - c(t))\, dt \quad \text{for} \quad \tau_i \leq t < \tau_{i+1} \tag{4.6}$$

$$dS_t^w = \alpha S_t dt + \sigma S_t dW_t + \Delta S_{T_A} \chi_{\{t=T_A\}} \text{ for } \quad \tau_i \leq t < \tau_{i+1} \tag{4.7}$$

[6] Moreover assume for $u < T_A$ and all bounded and measurable function h the following equality holds:

$$E^u \left[h(\zeta, \alpha_{AA}, \sigma_{AA})\right] = E\left[h(\zeta, \alpha_{AA}, \sigma_{AA})\right].$$

between the times in which no transaction happens and

$$X(\tau_{i+1}) = X(\tau_{i+1}-) - k - \xi_{i+1}, \tag{4.8}$$

$$Y(\tau_{i+1}) = Y(\tau_{i+1}-) + \xi_{i+1}, \tag{4.9}$$

when a transaction occurs with the convention that a positive ξ_{i+1} is money being taken from the bank account to buy stocks and $(\tau_{i+1}-)$ is the left limit.

The investor seeks to maximize the expected utility over all admissible[7] controls $w = (c, v)$, where $v = (\tau_1, \tau_2, ...; \xi_1, \xi_2, ...)$. It is not possible to borrow money or short the stock

$$X_t^w \geq 0, \tag{4.10}$$

$$S_t^w \geq 0. \tag{4.11}$$

The utility has a constant risk aversion[8] $\gamma \in (-1, 0)$ and discounts time with rate ρ . For a given w we have the utility

$$J^w(u, x, y) = E_u \left[\int_u^\infty e^{-\rho t} \frac{c^{1-\gamma}}{1-\gamma} dt \right] \tag{4.12}$$

where E_s is the expectation conditional to the information available in s. This gives the value function

$$V(u, x, y) = \sup_{w \in W} J^w(u, x, y). \tag{4.13}$$

Note that $J^w(u, x, y)$ and $V(u, x, y)$ depends upon the realization of $(r_{AA}, \alpha_{AA}, \sigma_{AA})$ for $u \geq T_A$. A more complete (and correct) notation would be $J^w(u, x, y, r, \alpha, \sigma)$ considering (r, α, σ) as three additional dimension in the process but it is omitted for the sake of brevity.

4.2.2 Solution's Characterization After T_A

The agent is forward looking and the price processes' distribution depends only upon the actual prices. Past prices have no impact on future outcomes' distribution. It implies that we can look for the solution after T_A without worrying about the solution before it. We discuss the solution after T_A in this subsection. Recall that the problem after the announcement is the same as in Chancelier *et al.* (2002) and Oksendal and Sulem (2007) and we follow here their work.

In order obtain sufficient conditions for the solution Brekke and Oksendal (1997) make use of some concepts such as the process's generator, the intervention operator and the continuation region. The generator A^c (the superscript c emphasizes the control's role) of the process

$$Z^w = (u, X^w(u), S^w(u)) \tag{4.14}$$

is defined as (when there are no transactions)

$$(A^c f)(u, x, y) = \frac{\partial f}{\partial u} + (rx - c) \frac{\partial f}{\partial x} + \alpha \frac{\partial f}{\partial y} + \frac{1}{2} \sigma^2 y^2 \frac{\partial^2 f}{\partial y^2}, \tag{4.15}$$

[7] A stochastic control is admissible if the Stochastic Differential Equation for $Z_t = (X_t, S_t)$ has a unique and strong solution for all given initial condition and $E_u \left[\int_u^\infty \max(-e^{-\rho(u+t)} \frac{c^{1-\gamma}}{1-\gamma}, 0) \right] < \infty$ and the explosion time τ^* is infinite:

$$\tau^* = \lim_{R \to \infty} (\inf \{t > u || Z_t| \geq R\}) = \infty \text{ a.s.}$$

and

$$\lim_{j \to \infty} \tau_j = \tau_S$$

where τ_S is

$$\tau_S = \inf \{t > 0 | Z(t) \in S\}$$

and the solvency region $S = \Re_+^2$.

[8] This range guarantees that $J^w \geq 0$.

where small y is used instead of small s to express a specific value of the stock. The sufficiency condition for the value function using the generator is better expressed through the definition of the operator Λ

$$\Lambda h(u,x,y) = \sup_{c\geq 0}\left\{A^c h(u,x,y) + e^{-\rho u}\frac{c^{1-\gamma}}{1-\gamma}\right\}. \tag{4.16}$$

The Intervention Operator M is defined as

$$Mh(u,x,y) = \sup_{\xi\in W_\xi}\left\{h(u,x-k-\xi,y+\xi); \xi \text{ belongs to an admissible control}\right\}, \tag{4.17}$$

where h is a locally bounded and the continuation region is defined as

$$\mathbf{C} = \{(u,x,y); V > MV\}. \tag{4.18}$$

The value function V shall satisfies the quasi-variational Hamilton-Jacobi-Bellman inequality (QVHJBI):

$$\max\{\Lambda V, MV - V\} = 0, \tag{4.19}$$

i.e., the value function satisfies $\Lambda V = 0$ in the continuation region $\mathbf{C}$ and $MV = V$ outside the continuation region. Moreover $\Lambda V \leq 0$ outside the continuation region. The optimal controls $w^* = (c^*, v^*)$ are

$$c^*(u,x,y) = \left(\frac{\partial V}{\partial x}\right)^{-\frac{1}{\gamma}} \quad \text{for } x > 0, \tag{4.20}$$

$$c^*(u,x,y) = 0 \qquad \text{otherwise}, \tag{4.21}$$

and the times of the trade are defined inductively as:

$$\tau^*_{j+1} = \inf\left\{t > \tau^*_j; \left(X^{(w^*)}(t), Y^{(w^*)}(t)\right) \notin \mathbf{C}\right\} \tag{4.22}$$

with the convention that $\tau^*_0 = 0$ (note that τ^*_0 does not belong to the control $v^* = (\tau^*_1, \tau^*_2, ...; \xi^*_1, \xi^*_2, ...)$ and there is no ξ^*_0) and the size of transaction at such dates are the argument that maximizes equation (4.17):

$$\xi^*_j = \arg\max_{\xi\in W_\xi}\left\{V(u, X^{(w^*)}(\tau_j-) - \xi - k, Y^{(w^*)}(\tau_j-) + \xi); \xi \text{ belongs to an admissible control}\right\}. \tag{4.23}$$

Brekke and Oksendal (1997) show that the equations (4.19)-(4.23) are sufficient condition for the value function (along with some technical conditions).

Unfortunately the solution may not be smooth enough to satisfy it. In order to overcome this difficulty Oksendal and Sulem (2007) show that the value function is the unique viscosity solution of the equation (4.19). Moreover Chancelier *et al.* (2007) develop a numerical algorithm converging to this viscosity solution and we describe it in the next section.

4.2.3 Solution's Characterization Before T_A

After solving $V(t,x,y)$ for $t \geq T_A$ we proceed to discuss the solution before the announcement. We are considering that the processes are right continuous implying that $V(T_A,x,y)$ is the value function after the jump. But we don't know if V is continuous in t. Even if continuous, we don't know if V is sufficient smooth in order to apply the differential operator A^c in QVHJBI at T_A (equation (4.19)).

In order to overcome this difficulty, we will consider a different, but hopefully equivalent, problem. This new problem implies a new boundary condition at T_A. We were not able to prove it but we rely on the following conjecture:

Conjecture 12 *For $u \leq T_A$, consider the alternative problem:*

$$\widehat{V}(u,x,y) = \sup_{w \in W} \widehat{J}^w(u,x,y) \tag{4.24}$$

where

$$\widehat{J}^w(u,x,y) = E_u\left[\int_u^{T_A} e^{-\rho t}\frac{c^{1-\gamma}}{1-\gamma}dt + g\left(X_{T_A}, S_{T_A}\right)\right] \tag{4.25}$$

and

$$g\left(x,y\right) = E\left[V(T_A,x,y)\right]. \tag{4.26}$$

In this case we have that

$$\widehat{V}(u,x,y) = V(u,x,y) \quad \text{for } u < T_A \tag{4.27}$$

If the above conjecture is true the same reasoning and results used in the previous sub-section is valid here. More precisely, if we know $V(T_A,x,y)$, we can find the value function for $t < T_A$ using the QVHJBI and the a new boundary condition:

$$\widehat{V}(T_A,x,y) = E\left[V(T_A,x,y)\right] \tag{4.28}$$

where the QVHJBI is

$$\max\left\{\Lambda\widehat{V}, M\widehat{V} - \widehat{V}\right\} = 0. \tag{4.29}$$

4.3 Numerical Method

This section presents the algorithm's overview along with some assumptions about the solution around the scheduled announcement and a brief discussion about numerical error. See Appendix 4A for a detailed description on the algorithm implementation.

The QVHJBI is a partial differential equation along with a free boundary problem, i.e., the boundary condition is part of the problem. The main idea of Chancelier *et al.* (2007) is to define an equivalent fixed point problem: given the value function V it is possible to obtain the no-transaction region $\mathbf{C}$, consumption rate function $c^*(u,x,y)$ and optimal transaction ξ^*; by other side, given the $\mathbf{C}$, $c^*(u,x,y)$ and ξ^* the value function V is obtained. More precisely, the algorithm break the non-linear Partial Differential Equation $\Lambda V = 0$ into a optimization problem

$$c^* = \arg\max\left\{A^c V + e^{-\rho u}\frac{c^{1-\gamma}}{1-\gamma}\right\}, \tag{4.30}$$

obtaining the Partial Differential Equation

$$A^{(c^*)}V + e^{-\rho u}\frac{(c^*)^{1-\gamma}}{1-\gamma} = 0, \tag{4.31}$$

and then it considers the continuation region as

$$\mathbf{C} = \left\{(u,x,y); V(u,x,y) > MV(u,x,y\right\}. \tag{4.32}$$

where

$$MV(u,x,) = V(u, x - \xi^* - k, y + \xi^*). \tag{4.33}$$

It is possible to define a iterative procedure converging to the true solution beginning by an (almost) arbitrary initial function. The method begins by: (1) defining a grid for (u,x,y); (2) discretizing the linear operator A^c in this grid, (3) discretizing the non-linear intervention operator M forcing $(x - k - \xi, y + \xi)$ to belong the grid; and (4) defining an initial tentative value function V_0 in the grid. Then one should proceed iteratively. The step n depends upon the function V_{n-1} and are divided in two sub-steps:

$$\text{sub-step 1: Given } V_{n-1} \text{ obtain } \xi_n,\ c_n,\ \mathbf{C}_n \tag{4.34}$$

$$\text{sub-step 2: find } V_n \text{ using } \xi_n,\ c_n,\ \mathbf{C}_n. \tag{4.35}$$

Given some conditions (satisfied by the above problem) V_n gets closer to V as n increases.

The above problem is defined for a grid in $(x, y) \in \Re^2_+$ but it is an infinite set. Some localization procedure is necessary because the computer´s memory is finite. This work follows the suggestion of Oksendal and Sulem (2007) and Chancelier *et al.* (2002) and define a box of size L: $D_L = [0, L] \times [0, L]$ confining (x, y) to that box and defining the Neumann boundary conditions:

$$\frac{\partial V}{\partial x}(L, y) = \frac{\partial V}{\partial y}(x, L) = 0 \qquad \text{for } x, y \in [0, L). \tag{4.36}$$

The time dimension is also infinite but the solution for $t \geq T_A$ has the format:

$$V(t, x, y) = e^{-\rho t}\varphi(x, y) \tag{4.37}$$

and then it is possible to focus on $\varphi(x, y)$. After obtaining $\varphi(x, y)$, it is possible to find the boundary condition implied by the conjecture above and apply the method for $t < T_A$ with $t \in [0, T_A)$.

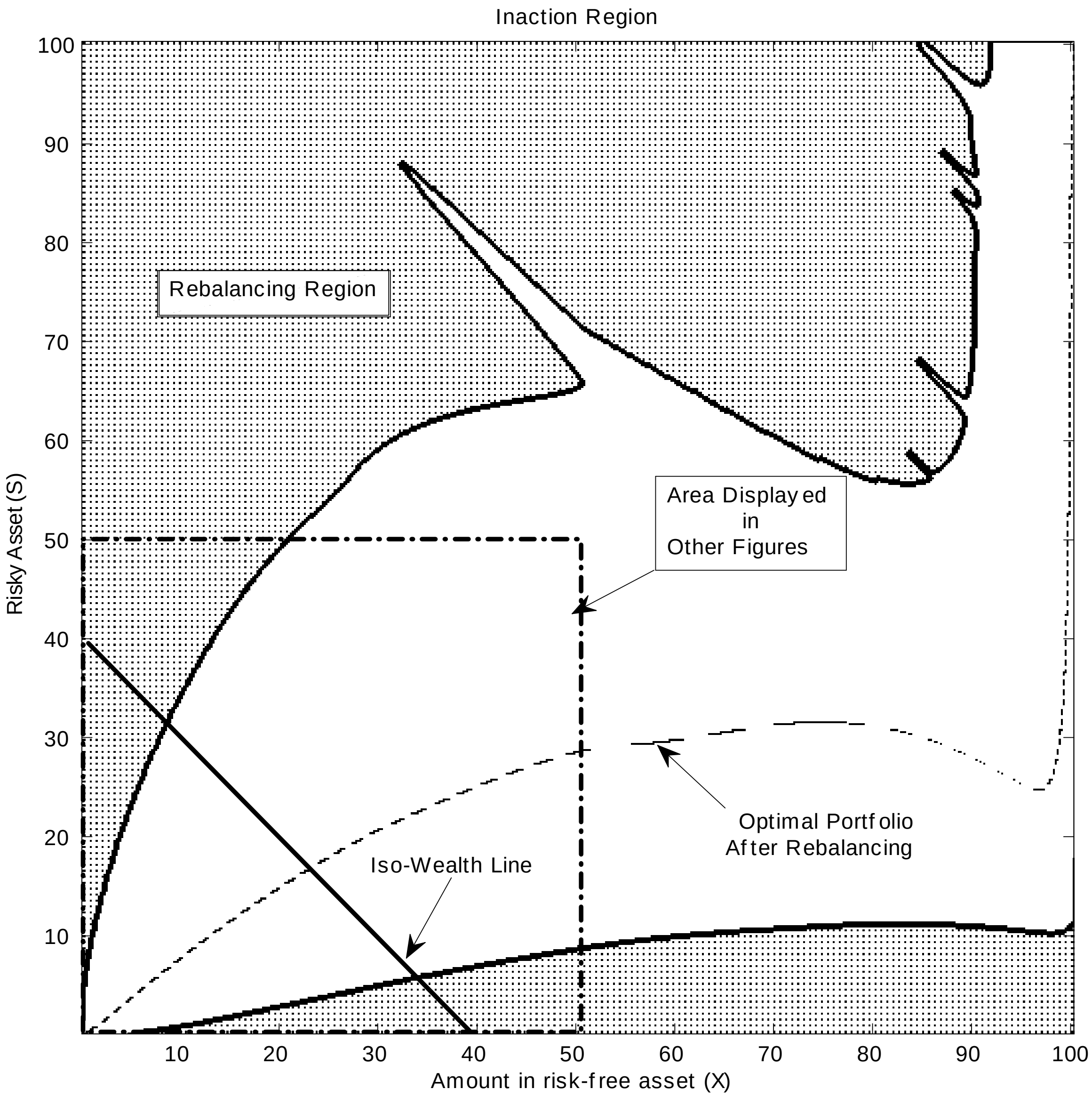

Figure 4.1: Inaction region after the announcement. It remains the same for all time after the news. The parameters are in table 4.1 (case 3). Each point inside the figure represents a possible portfolio where the $x-$axis is the amount of money invested in risk-free asset (bank account) and $y-$axis is the amount in risky asset. The white region is the inaction region and whenever the portfolio is in the dashed area (Rebalancing Region) it is optimal to pay the fixed cost and rebalance. Note that the line with the same wealth (iso-wealth line) is a diagonal one with 45^o. When rebalancing, the investor can choose any portfolio in the iso-wealth line associated with his/her wealth minus the fixed cost. The dashed line is the optimal portfolio after rebalancing. After rebalancing the best the investor can do is to choose the portfolio where the iso-wealth line crosses the dashed line. The simulation are performed in a square box of side size $L = 100$ with Neumann boundary condition at the upper and rightmost side. This is the only figure displaying the whole box. Others figures only display the smaller square box of side size $L/2 = 50$. Note the distortion in the inaction region for $y > 50$. A similar figure is reported by Chancelier *et al.* (2002) using another method.

Table 4.1: Simulation parameters

Parameter	Case 1	Case 2	Case 3	Case 4
α	0.12	-	-	-
σ	0.4	-	-	-
ρ	0.06	-	-	-
r	0.05	0.06	0.07	0.08
γ	0.4	-	-	-
F	0.4	-	-	-
L	100	-	-	-
$\delta_x = \delta_y$	0.2	-	-	-
Tolerance	10^{-9}	c_max	10000	

4.4 Numerical Results

4.4.1 After Announcement

Figure 4.1 represents one possible inaction region[9] after the announcement and it is similar to one depicted in Chancelier *et al.* (2002) using a different method. It is obtained from the case 3 of table 4.1. Each point inside the figure represents a possible portfolio where the $x-$axis is the amount of money invested in risk-free asset (bank account) and $y-$axis is the amount in risky asset. The white region is the inaction region and whenever the portfolio is in the shadowed area (Rebalancing Region) it is optimal to pay the fixed cost and rebalance. Note that the line with the same wealth (iso-wealth line) is a diagonal one with 45^o. When rebalancing, the investor can choose any portfolio in the iso-wealth line associated with his/her wealth minus the fixed cost. The dashed line is the optimal portfolio after rebalancing and the best the investor can do when trading is to choose the portfolio where the iso-wealth line crosses the dashed line.

This picture display all the area used for simulation, i.e., the square box of side size L. Note the inaction region's weird shape when close to (L, L). This distortion is a consequence of Neumann Boundary condition (equation (4.36)) but is attenuated for the region close to the origin. For this reason, almost all other figures in this work only display the internal square box of size $L/2$.

The inaction region remains the same after T_A and this figures enables us to analyze the portfolio evolution when the investor is rational. For instance, suppose the investor begin with the portfolio $(X_{T_A} = 15, Y_{T_A} = 10)$ at time $T\ _A$. Note that this portfolio is inside the inaction region. The investor will withdraw the money from the bank account in order to consume at a rate c^* and let the portfolio evolve. When it hits the boundary of the inaction region (with $X_t + Y_t = 40.40$ for instance) the investor should pay the fixed cost (0.40 in the numerical result of figure 4.1) in order to rebalance ending with the portfolio that crosses dashed line with the iso-wealth line.

Figure 4.2 depicts the value function, the consumption rate (in the box with side size $L = 100$) and the consumption rate (in the internal box with side size $L/2 = 50$). The parameters are the same as used in figure 4.1. Note the consumption rate peaks for the greatest possible wealth. This is again consequence of the Neumann boundary condition.

Figure 4.3 shows the no-transaction region for the same parameter but the risk-free rate. It varies from $r = 0.05$ to $r = 0.08$ and parameters used are in table 4.1. Note that the optimal portfolio line becomes less inclined for higher risk-free rate.

[9] We applied one image filter in order to have a better image. However the change is minor and it has no effect on interpretations. See appendix 4B for details.

Table 4.2: Announcement effect: Jump in the risky price and/or change in the risk-free rate r

Parameter	Jump Type 1	Jump Type 2	Jump Type 3
μ_ζ	0.10	0	-0.02
σ_ζ	0.01	0	0.2
$E[\zeta]$	1.105	1	1
ζ Std. Dev.	0.011	0	0.202
Change in r	No	Yes	No

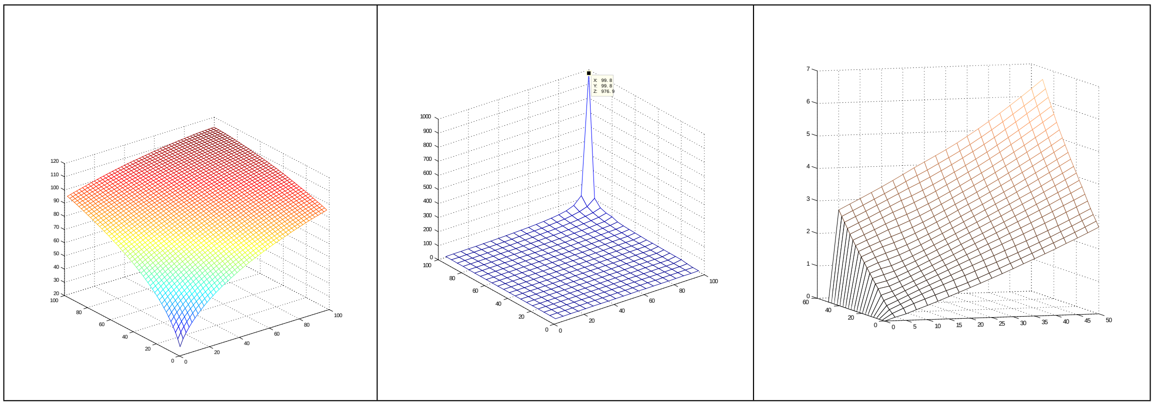

Figure 4.2: From left to right: value function (without the time discount, see equation (4.37)), consumption rate c and zoom in the consumption rate c. These are quantities for $t \geq T_A$. These quantities remain the same through time. The parameters are in table 4.1, case 3. The axis spanning from the origin to the left is the amount invested in the risky asset (S) and the axis spanning from the origin to the right is the amount invested in risk-free asset (X). Note that the consumption is zero for $X = 0$.

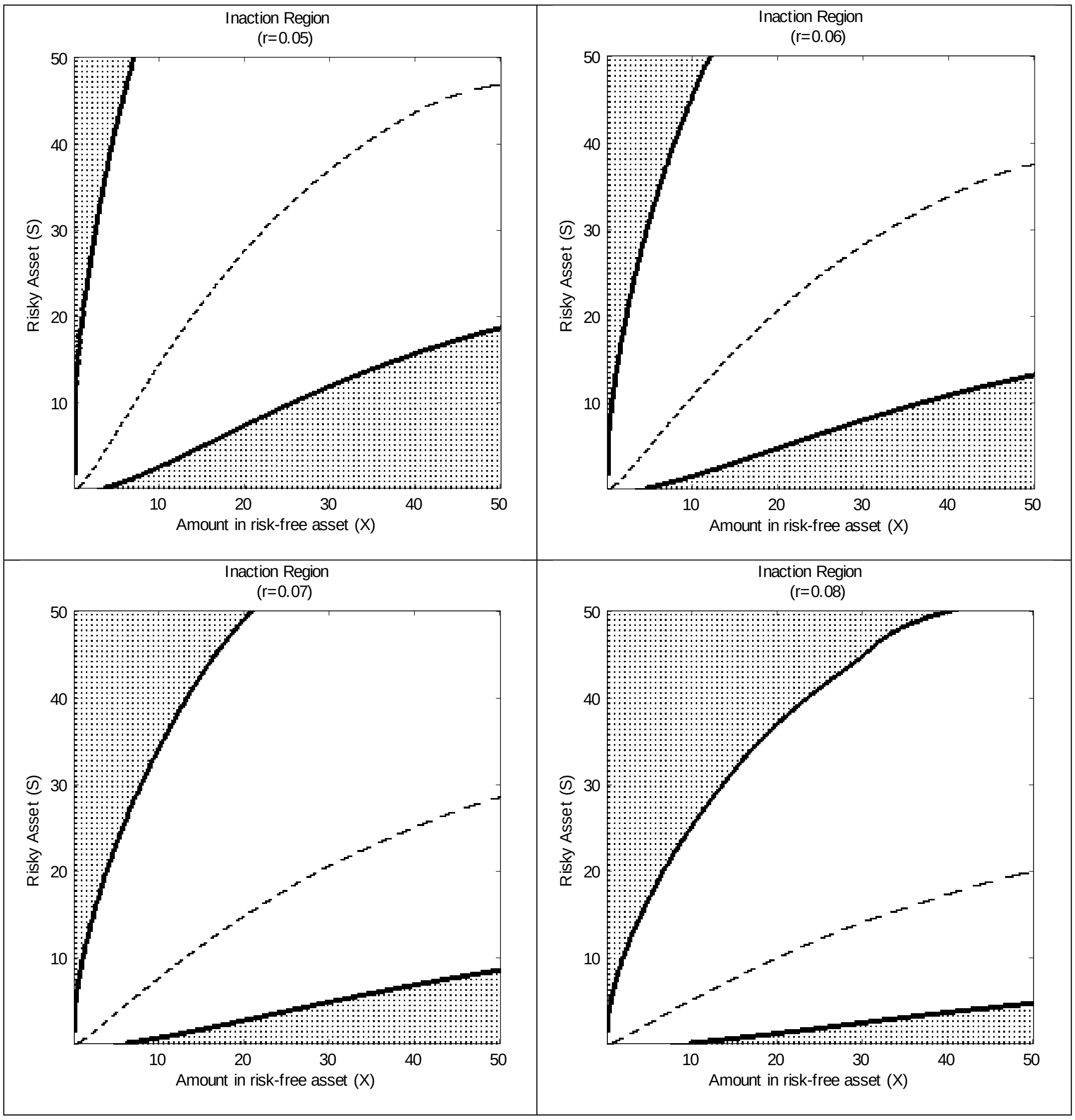

Figure 4.3: Inaction region after the announcement for four different risk-free rate. The paramter are in table 4.1. The inaction region remains the same for all $t \geq T_A$.

4.4.2 Numerical Results Before the Announcement

Jump with Low Variance and No Change in Parameters

Figure 4.4 shows the optimal no-transaction region evolution. It applies to a investor who believes that the scheduled announcement has a surprising positive content. It displays the optimal inaction region for four different times: long before, little before, just before and after the scheduled announcement. Each point represents a possible portfolio where the x-axis is the amount of money invested in the risk-free asset and the y-axis is the amount in risky asset. The no-transaction region is in white and whenever the portfolio hits the shadowed area the investor should pay the fixed cost and rebalance it. The dashed lines are the optimal portfolios after the investor pay the fixed cost. For each level of wealth (amount invested in risk-free asset plus the amount in risky asset) there is only one optimal portfolio. The parameters of this simulation are in table 4.1 (case 3) and in table 4.2 (jump parameters 1). We follow the convention to show the region inside the box $[0, L/2] \times [0, L/2]$ where $L = 100$ and to apply some image filters explained in appendix 4B.

Recall that there is only one news. In this case, the inaction region remains fixed after the announcement. By other side, the inaction region changes with time before it. Simulations suggest that the inaction region far before the announcement is similar to the no-transaction region after it (compare the top-left subfigure with bottom-right one). As the time gets closer to the event, the inaction region gets bigger. The top-right subfigure suggests it can tend to almost all space. Finally it changes abruptly just before the announcement (the bottom left subfigure) and change again after the disclosure of the information (the bottom-right subfigure).

To be more precise, we obtain the no-transaction region for $T_A - \delta_t$, $T_A - 2\delta_t, \ldots$, where T_A is the announcement time and δ_t is the minimum time distance we use (it is set for finite difference approximation). The inaction region between $T_A - n\delta_t$ and $T_A - (n-1)\delta_t$ for $n \geq 3$ is very similar indicating a smooth transition. This is not the case for the transition between $T_A - 2\delta_t$ and $T_A - \delta_t$. The region at time $T_A - 2\delta_t$ is similar to the top-right subfigure while the region at $T_A - \delta_t$ is reported in the bottom-left subfigure. We verify this behavior for different values of δ_t.

The optimal inaction region just before the news has others interesting features because this is a preparation for the jump. The jump has a positive average (10.5%) with very small standard deviations (1.1%). This is an extreme case but we use it to make the incentives clear. A closer look to the bottom-left figure suggests that it is optimal to invest all the money in the risky asset if the investor has sufficient wealth. Moreover, if the portfolio is in the shadowed area just before the announcement, the investor will probably pay the cost twice. This is because he/she will rebalance just before and (probably) do it again just after the jump. By the other side, putting all the money in the risky asset isn't optimal if the expected gain is less than the fixed cost. This will be the case if the wealth isn't high enough or if the amount in risk-free asset is less than a certain threshold ($4.40 in the present case). The investor still wants to take advantage of the positive jump in this case but he/she has to worry about the portfolio composition after it[10].

The most striking feature is that the investor prefers to prepare the portfolio for the jump just before it but not earlier. This is so because preparing the portfolio before (but not just before) the announcement has two adverse effects: (1) the portfolio will not be the optimal one at the announcement (almost surely) and (2) the investor may need to rebalance it just before the jump paying the fixed cost again.

Random Change in Risk-Free Rate and No Jump in Prices

Figure 4.5 presents the results when there is a change in risk-free rate at the scheduled announcement but no jump in prices. We assume that the investor knows all details of the model. The risk-free rate before the news is 7% and after the news may be 6%,7% or 8% with the same probability, i.e.,

$$Prob\,[r_{AA} = 0.06] = Prob\,[r_{AA} = 0.07] = Prob\,[r_{AA} = 0.08] = 1/3. \tag{4.38}$$

The other parameters are the same as in case 3 of table 4.1 and doesn't change with time. After the news the inaction region doesn't change and it may be the same as in the upper-right or in

[10] In this simulation we found that we may interpret the jump being 10% with certainty. Note that the fixed cost is \$ 0.40. Analyzing the bottom-left subfigure carefully we found that if the investor has more than \$ 4.40 invested in risk-free asset and more than \$ 4.00 in risk asset, he/she will rebalance the portfolio putting all the money in the risky asset. The portfolio (\$4.40; \$4.00) is a nice threshold. Suppose the total wealth before the announcement is \$ 8.40. After rebalancing the first time, the total wealth becomes \$8.00. The jump makes it \$8.80 (approximately). Then the investor rebalances it ending with \$ 8.40 after pay again the cost. In the end, the investor has the same wealth but the portfolio is optimally rebalanced. Moreover, note that it is never optimal to rebalance the portfolio if there is less than \$ 4.40 in the riskless asset. This is because the gain to put this money in the risky asset doesn't compensate paying the fixed cost twice.

the bottom two subfigures of figure 4.3 depending upon the realized risk-free rate. Note that the optimal portfolio after rebalancing is different after information release for each risk-free rate. On the other hand, the figure 4.5 depicts the inaction region long before, little before and just before the scheduled announcement.

The inaction region and the optimal rebalancing portfolio long before are similar to the inaction region and rebalancing portfolio after the jump with the same parameters, i.e., the subfigure in the top left are similar to figure inner square of figure 4.1 (or, equivalently, to the bottom-left subfigure in figure 4.3). The inaction region gets bigger as the time approaches to the announcement. In the present case, the simulations suggest that the inaction region evolves continuously until it encompasses all the space we considered.

This is the interesting result for this case: the investor avoids transacting just before (or a little before) the announcement if there is a fixed cost, there is no jump in prices and the optimal portfolio depends upon the information. If there is no jump in prices, the difference in (X_{T_A-dt}, S_{T_A-dt}) and (X_{T_A}, S_{T_A}) is infinitesimal. Then there is a small risk (if any) in waiting the time interval dt. By other side, at T_A the investor can choose the portfolio better informed than choosing in $T_A - dt$. Note that the information (the value of risk-free rate) matters for the choice of optimal rebalancing portfolio. Finally, it is better to rebalance just once than rebalance twice because of fixed cost.

Jump with Appreciable Variance and No Change in Parameters

Figure 4.6 presents the inaction region's evolution when there is no change in parameters but there is a jump in the risky asset price with zero mean and variance approximately 20%. The parameters are in table 4.1 (case 3) and table 4.2 (column 3). The organization of figure 4.6 is similar to figure 4.4.

Again, the top-left subfigure (representing a time long before the news) is similar to the bottom-right figure (the inaction region after T_A). Note that the bottom-right subfigure in figure 4.6 is the same as in figure 4.4. The difference is in the top-right and bottom-left subfigures. The transition from inaction region long before to the one little before is smooth as the first case. What changes in the present case is that the transition between the top-right to the bottom-left subfigure seems to be smooth also. At least for our numerical accuracy and in the region depict in those figures. Nonetheless for $x > L/2$ (not in depicted in figure 4.6) the simulations suggest a sudden although small change in the inaction region. We think it might be due to the localization procedure as for this region the estimate of the jump effect is more biased.

It is ambiguous whether the transaction chance is higher or less than usual for the time a little before or just before the announcement. It is because the rebalancing region for $y > x$ gets bigger but the rebalancing region for $y < x$ disappear. Nonetheless there is still a higher chance to transact after the news because the risky price jumps and the inaction region changes.

The investor behavior may be explained by his/her small risk-aversion. As the jump mean is zero, a risk-averse agent would prefer to have less on risky asset. This explain the vanishing rebalancing region for $y < x$ and the bigger region for $y > x$. On the other hand the rebalancing region isn't so big for $y > x$ and the optimal portfolio after rebalancing isn't so different when comparing to optimal portfolio without the announcement. This is explained by the small risk-aversion as the agent is not willing to pay the fixed cost in order to have a more "protected"portfolio against the jump.

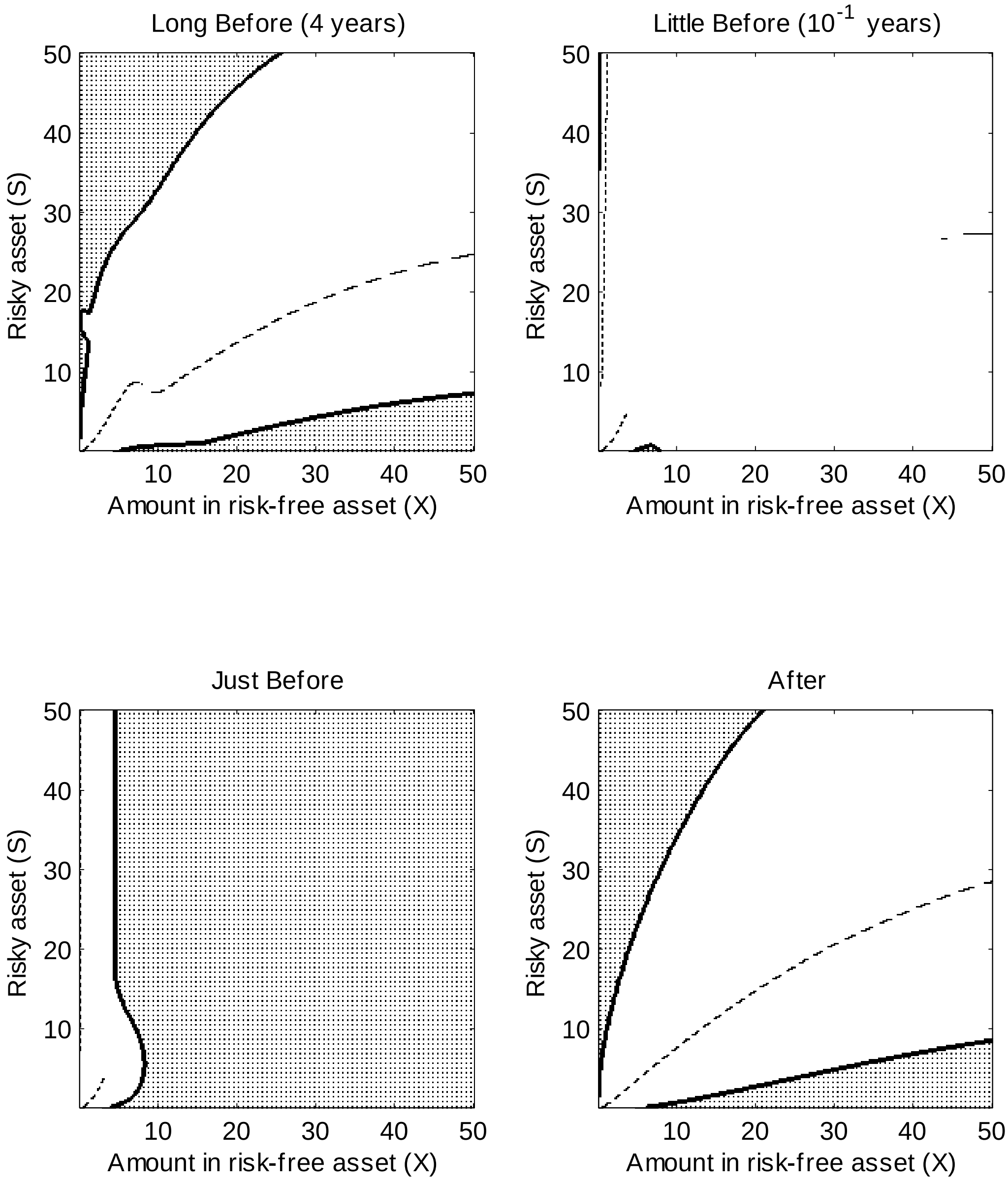

Figure 4.4: No-transaction region's evolution in the presence of scheduled announcement. The white area is the non-transaction region. The dashed line is the optimal portfolio for a given wealth, i.e., it is the optimal portfolio when the investor pays the fixed cost and rebalances it. The figure displays the no-transaction region for four different times: long before, little before, just before and after the scheduled announcement. The numeric results suggests a sudden change from little before to the just before inaction region. It implies that there is a low chance to rebalance the portfolio a little before but a high chance of it just before and just after the announcement consistent with the empirical volume trading behavior reported by Chae (2005). The parameter of this simulation are in table 4.1 (case 3) and in table 4.2 (column 1). The localization is the square box of side $L = 100$ but the present figure is displaying the smaller square box of size $L/2$ as discussed in figure 4.1.

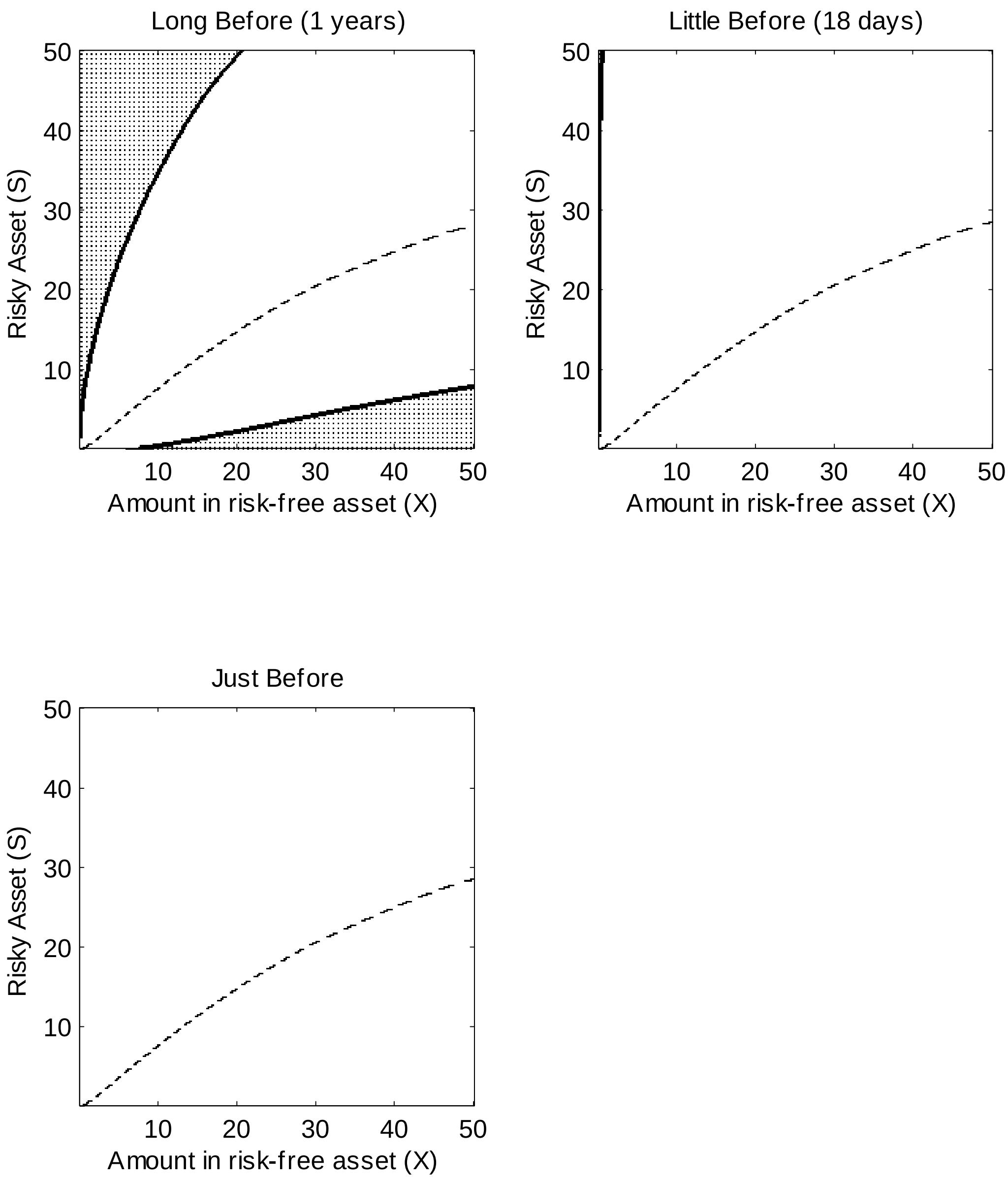

Figure 4.5: Inaction region‘s time evolution. There is no jump at prices but there is a change in the risk-free rate at announcement. Before the announcement the parameters are in table 1 (case 3). After the announcement the parameters may be as in table 4.1 cases 2,3 or 4 with same probability. There is no bottom-right subfigure because it has 3 possibilities. These possibilites are depicts in figure 4.3 (subfigure in the top right and the two in the bottom) . The chance to rebalance decreases a little before and just before the announcement. This chance is greater than normal after the news because the rebalancing region is bigger after the announcement. The shadowed area is the rebalancing region and the dashed line is the optimal portfolio after rebalancing for a given wealth. The localization is the square box of side $L = 100$ but the present figure is displaying the smaller square box of size $L/2$ as discussed in figure 4.1.

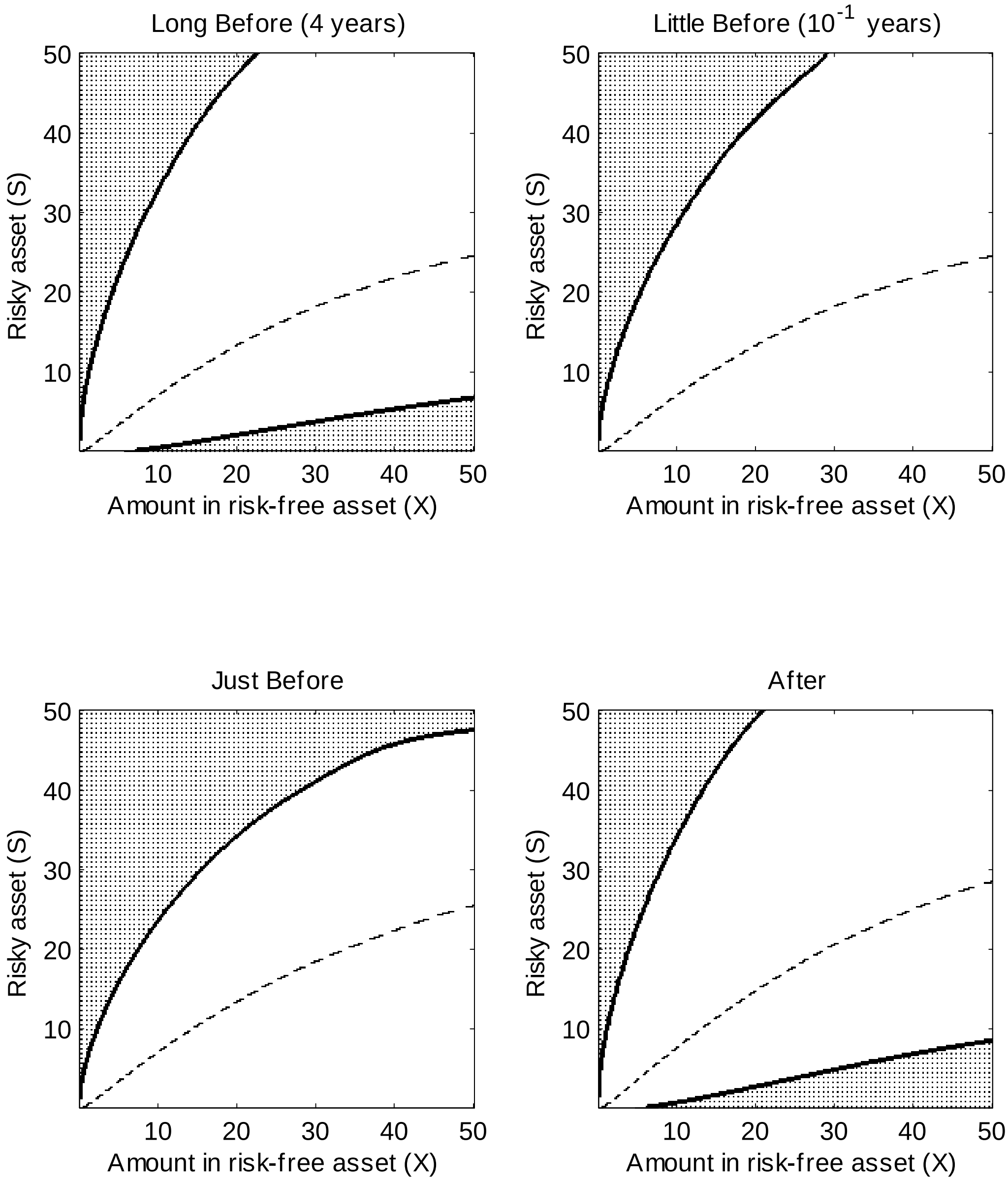

Figure 4.6: Inaction region's time evolution. There is a jump in the risky price with zero mean and variance appoximately 20%. The numerical results suggest that the transition between a little before to just before inaction region is smooth. It is not clear whether the chance of rebalancing increases or decreases a little before and just before the announcement. After the news there is a higher chance to rebalance because of the jump and because of a different inaction region. The parameters are in table 4.1 (case 3) and table 4.2 (column 3). The localization is the square box of side $L = 100$ but the present figure is displaying the smaller square box of size $L/2$ as discussed in figure 4.1.

4.5 Discussion

There are several theoretical results about the volume behavior around announcements. Nonetheless, to the best of our knowledge, the predictions are that or it raises or it falls before the news but does not have the interesting pattern found in Chae (2005). We want to argue here that transaction cost may be an important piece in explaining such behavior.

When there is no announcement the trades happens because the investor aims to a more balanced portfolio and because he needs to transfer from risky asset to the bank account in order to consume. In the presence of news, modeled as a jump in prices (or a random change in parameters) at a fixed date, the investor wants to prepare the portfolio for this event also. Usually, the optimal portfolio prior the news is different from the portfolio without news. The interesting analysis is how to prepare the portfolio for the announcement and when there is enough incentive to pay the fixed cost.

In some cases, the best thing to do is just to wait for the announcement. When close to announcement those investors do nothing until the announcement. This is usually true when the process parameters change randomly but there is no jump in prices.

In other cases it is worth to pay the fixed cost in order to prepare for the news. If the investor pays this cost a little before the news he/she may end without the best portfolio at the time of announcement. Then it is optimal to pay it and change the portfolio just before the announcement. This is true when the investor believes that a jump in price with a definite direction may take place.

The incentive to halt trading a little before and to transact just before may explain the volume behavior before the news and this happens because of the fixed cost. In both cases the volume increases after the announcement. Indeed, for any state dependent model, a jump in variables increases the chance of some action to be taken because the state variables may have a positive probability to fall outside the inaction region.

4.6 Conclusion

We analyze here an optimal portfolio problem in the presence of fixed cost and a scheduled announcement through a numerical method developed in Chancelier *et al.* (2007). We model the prices around announcements according to empirical findings. The results suggest that transaction cost may be an important feature in order to explain the volume behavior around scheduled announcement. In particular, it may be important to explain the fall in volume between 10 to 3 days before the news, followed by a rise in volume beginning 2 or 1 day before the news as is found in Chae (2005).

It would be interesting to analyze different types of cost for the same price process. For instance, what happens if the cost is proportional? And if there is no cost at all? On the other hand, note that the present work takes as given the price process and derives the optimal policies. It is an optimal portfolio analysis and it would be interesting to study a general equilibrium. Lo *et al.* (2004) introduces a general equilibrium model in continuous time modeling financial markets when there is a fixed cost. A promising research venue would be incorporating jump innovations at a fixed time to the this model.

4.A Numerical Method

In this appendix we describe the numerical algorithm in details. The algorithm's properties are developed in Chancelier *et al.* (2007) and are described in Oksendal and Sulem (2005, Chapter 9) as well. We give also some details for the operator's matrix implementation well suited to a Matlab implementation. First we describe the time invariant case (consistent with $t \geq T_A$) and then we incorporate the time variation.

The reader interested in the algorithm as a whole but not in implementation details may want to skip the subsection "Operators as Matrices". In this subsection we describe exactly the differential operator approximated by a difference operator and construct it in a suitable way using matrices. This procedure transforms the method into a sequence of system of linear equations. Moreover our discretization guarantees that this sequence converges to the solution.

4.A.1 Algorithm Overview

Assume that the value function after T_A may be written as

$$V(t,x,y) = e^{-\rho t}\varphi(x,y) \quad \text{for } t \geq T_A, \tag{4.39}$$

which implies

$$\left(A^c V\right)(u,x,y) = A^c e^{-\rho t}\varphi(x,y) = e^{-\rho t}\left(A^c\varphi(x,y) - \rho\varphi(x,y)\right). \tag{4.40}$$

Chancelier et al. (2002) shows that it is indeed the case (our problem is the same as their after $t \geq T_A$). QVHJBI reads now

$$\max\{\Lambda V, MV - V\} = 0 \tag{4.41}$$

$$\max\left\{\sup_{c\geq 0}\left\{A^c\varphi - \rho\varphi + \frac{c^{1-\gamma}}{1-\gamma}\right\}, M\varphi - \varphi\right\} = 0 \quad \text{for } t \geq T_A, \tag{4.42}$$

where

$$M\varphi = \sup_{\xi\in W_\xi(x,y)}\{\varphi(x-k-\xi, y+\xi)\} \tag{4.43}$$

and $W_\xi(x,y)$ is the set of admissible ξ given the state (x,y).

In some cases the value function aren't sufficiently smooth and the differential operator may not make sense in some regions. In order to overcome this difficult it is possible to invoke the concept of viscosity solution of the QVHJBI. There is no need to discuss such type of solution here. It is only necessary to know that the value function is a viscosity solution of the QVHJBI and that the viscosity solution is unique for the case considered here. Moreover, the numerical method described in the present appendix converges to the viscosity solution of the above problem as the grid becomes more refined. In other words, the solution to the appropriate discrete version of the above problem converges to the value function even if it is not continuous.

We want to discretize appropriately the QVHJBI. First, note that the positive octant is contained in the definition of the problem. The computer can't represent it fully and some strategy is necessary to overcome this difficulty. We accomplish this by truncating the problem considering a box $D = [0,L]\times[0,L]$, i.e.:

$$D = \left\{(x,y)\in\Re^2 | 0 \leq x \leq L, 0 \leq y \leq L\right\}$$

and assuming zero Newmann condition at boundaries

$$\frac{\partial V}{\partial x}(L,y) = \frac{\partial V}{\partial y}(x,L) = 0 \qquad \text{for } x,y\in[0,L). \tag{4.44}$$

This introduces an error but it should be small for (x,y) far from the upper or rightmost boundary .

We approximate the function as belonging to the finite difference grid $D_h = (ih, jh)$ where $i \in \{0,...,N\}$ and $j \in \{0,...,N\}$ assuming that $N = L/h$ is an integer. A limited consumption rate $0 \leq c \leq c_{\max}$ is also necessary condition for the algorithm but $c_{\max}$ shall be big enough. The discrete version of QVHJBI is written as

$$\max\left\{\sup_{0\leq c\leq c_{\max}}\left\{A_h^c\varphi_h - \rho\varphi_h + \frac{c^{1-\gamma}}{1-\gamma}\right\}, M_h\varphi_h - \varphi_h\right\} = 0 \tag{4.45}$$

where A_h^c and M_h are the discrete version of generator and of the intervention operator. Note that the solution depends upon the grid and to stress this fact we write φ_h as depending h. We can define a more appropriate and equivalent problem[11]:

$$\varphi_h(x,y) = \max\left\{\sup_{0\le c\le c_{\max}}\left\{\frac{(\Delta A_h^c + I_h)\,\varphi_h(x,y) + \Delta\frac{c^{1-\gamma}}{1-\gamma}}{1+\rho\Delta}\right\}, \sup_z B^z\varphi_h(x,y)\right\}, \tag{4.46}$$

where $x = ih$, $y = ih$ and $(ih, jh) \in D_h$, I_h is the identity operator, Δ is any constant satisfying

$$0 < \Delta \le \min_i \frac{1}{|\left(A_h^c - \rho I_h\right)_{ii} + \rho|} \tag{4.47}$$

where $\left(A_h^c - \rho I_h\right)_{ii}$ is the diagonal elements of the matrix (define later) representing the operator $A_h^c - \rho I_h$ and $\sup_z B^z$ substitutes the discrete version of the intervention operator:

$$\sup_z B^z\varphi_h(x,y) = M_h\varphi_h(x,y). \tag{4.48}$$

The constant Δ plays an important role in order to obtain the convergence of the method. The operator B^z is just a suitable way to define the discrete version of M_h. More precisely, the operator B^z evaluates a function after paying the fixed cost $k = k_h h$ and transferring zh from the bank account to the risky asset:

$$B^z\varphi_h(ih,jh) = \varphi_h\left(ih - k_h h - zh, jh + zh\right) \quad \text{if } (ih - k_h h - zh, jh + zh) \in D_h. \tag{4.49}$$

Nonetheless, if the transference zh forces (x, y) outside D_h then $B^z\varphi_h$ assumes the minimum value of φ_h at the origin (that is zero in our case):

$$B^z\varphi_h(ih,jh) = \varphi_h(0,0) = 0 \quad \text{if } (ih - k_h h - zh, jh + zh) \notin D_h. \tag{4.50}$$

Note that B^z depends upon h and $k_h h$ but we omit it for notational simplicity. It will be useful to define $z(i, j)$ as the policy function where the optimal one is defined as

$$\begin{aligned} z^*(i,j) &= \arg\max_z B^z\varphi_h(ih,jh) \quad \text{if exist } z \text{ such that } (ih - k_h h - zh, jh + zh) \in D_h \\ z^*(i,j) &= -(N+1)^2 \qquad \text{otherwise.} \end{aligned} \tag{4.51}$$

If there is no possible transference then we put the policy as $-(N + 1)^2$ to indicate that no answer is possible and that $B^{z^*}\varphi_h(ih, jh) = \varphi_h(0, 0)$.

The Fixed Point Problem

We want to be able to compute the solution in the equation (4.46). Two features make this problem intricate: the definition of the rebalancing region (or its complement, the continuation region) and the optimization problems. Given the optimal rebalancing region $T^* \subset D_h$, the

[11] Note that

$$\sup_{0\le c\le c_{\max}}\left\{A_h^c\varphi_h - \rho\varphi_h + \frac{c^{1-\gamma}}{1-\gamma}\right\} \le 0$$

if, and only if,

$$\sup_{0\le c\le c_{\max}}\left\{A_h^c\varphi_h - \rho\varphi_h + \frac{c^{1-\gamma}}{1-\gamma}\right\}\frac{\Delta}{1+\rho\Delta} + \varphi_h \le \varphi_h,$$

for $\Delta > 0$. Then we can exchange the above relations inside the maximization operator obtaining the suitable problem.

optimal policy function $z^*(i,j)$ and the optimal consumption rate $c^*(i,j)$ we can write equation (4.46) as

$$\begin{aligned} \varphi_h(ih,jh) &= \frac{\left(\Delta A_h^{c^*}+I_h\right)\varphi_h(ih,jh)+\Delta\frac{c^*(i,j)^{1-\gamma}}{1-\gamma}}{1+\rho\Delta} \quad \text{for } (ih,jh)\notin T^* \\ \varphi_h(ih,jh) &= B^{z^*(i,j)}\varphi_h(ih,jh) \qquad\qquad\qquad\qquad \text{for } (ih,jh)\in T^*. \end{aligned} \tag{4.52}$$

Note that the operator $O_{T,c,z}$, the set T^*, the function $z^*(i,j)$ and $c^*(i,j)$ should depend on h but we omit it for notational sake. We sum up the above equation with

$$\varphi_h(ih,jh) = O_{T^*,c^*,z^*}\left[\varphi_h(ih,jh)\right] \tag{4.53}$$

where

$$\begin{aligned} O_{T,c,z}\left[v(ih,jh)\right] &= \frac{\left(\Delta A_h^{c}+I_h\right)v(ih,jh)+\Delta\frac{c(i,j)^{1-\gamma}}{1-\gamma}}{1+\rho\Delta} \quad \text{for } (ih,jh)\notin T \\ O_{T,c,z}\left[v(ih,jh)\right] &= B^{z(i,j)}v(ih,jh) \qquad\qquad\qquad \text{for } (ih,jh)\in T. \end{aligned} \tag{4.54}$$

It is possible to write the function $v(ih,jh)$ as a vector and O_{T^*,c^*,z^*} as a matrix and now the equation (4.53) is solvable using standard methods in linear algebra. Unfortunately, we don't know the optimal rebalancing region T^* and optimal policies c^*, z^*.

On the other hand, once we know the value function $V(\cdot)=e^{-\rho t}\varphi_h$ we may recover T^*, c^*, z^* by

$$(T^*,c^*,z^*) = \arg\max_{T\subset D_h;0\le c\le c_{\max};z} O_{T,c,z}\left[\varphi_h(ih,jh)\right], \tag{4.55}$$

or more explicitly

$$c^*(ih,jh) = \arg\max_{0\le c\le c_{\max}}\left\{\frac{\left(\Delta A_h^{c}+I_h\right)\varphi_h(ih,jh)+\Delta\frac{c(i,j)^{1-\gamma}}{1-\gamma}}{1+\rho\Delta}\right\}, \tag{4.56}$$

$$\begin{aligned} z^*(ih,jh) &= \arg\max_z B^z\varphi_h(ih,jh) \;\; \text{if exist } z \text{ such that } (ih-k_hh-zh,jh+zh)\in D_h, \\ z^*(ih,jh) &= -(N+1)^2 \qquad\qquad \text{otherwise.} \end{aligned} \tag{4.57}$$

$$T^* = \left\{(ih,jh)\in D_h; B^{z^*(i,j)}\varphi_h(ih,jh) > \frac{\left(\Delta A_h^{c}+I_h\right)\varphi_h(ih,jh)+\Delta\frac{c^*(i,j)^{1-\gamma}}{1-\gamma}}{1+\rho\Delta}\right\}. \tag{4.58}$$

Chancelier *et al.* (2007) shows a equivalence between the QVHJBI (equation (4.46)) and the fixed point problem

$$\varphi_h(ih,jh) = O_{T^*,c^*,z^*}\left[\varphi_h(ih,jh)\right] \tag{4.59}$$

and defines the following policy iteration algorithm (or Howard algorithm) to find φ_h: Let v^0 be a given function in D_h and for $n\ge 0$ do the iterations

- (step n, sub-step 1) Given v^n find $(T_{n+1},c_{n+1},z_{n+1})$ such that

$$(T_{n+1},c_{n+1},z_{n+1}) \in \arg\max\max_{T\subset D_h;0\le c\le c_{\max};z} O_{T,c,z}\left[v^n(ih,jh)\right]. \tag{4.60}$$

- (step n, sub-step 2) Compute v^{n+1} as the solution of

$$v^{n+1} = O_{T_{n+1},c_{n+1},z_{n+1}}\left[v^{n+1}\right]. \tag{4.61}$$

Under some technical conditions, the sequence $\{v^n\}$ and $\{(T_{n+1},c_{n+1},z_{n+1})\}$ converges to φ_h and (T^*,c^*,z^*) respectively.

Remark 2 *We omit several technical conditions in the above presentation. They hold for the problem we are dealing with and we refer to Oksendal and Sulem (2007) and Chancelier et al. (2002) in order to account for them.*

4.A.2 Operators Approximation on the Grid

The present section details the sub-step 1 and 2 described above. We use the upwind scheme for the discretization of A^c in the grid. In this case, the operator A_h^c is written as a matrix where the off-diagonal elements are positive. Such scheme implies that the solution of the difference equation converges to the viscosity solution of the differential partial equation as $h \to 0$. Moreover, this scheme along with the condition for the constant Δ implies the convergence to the solution of the combined stochastic control and impulse control.

The approximation of A^c on the grid D_h (for $t \geq T_A$) is

$$A_h^c v = rx\partial_x^{h+} v + \alpha y \partial_y^{h+} v + \frac{1}{2}\sigma^2 y^2 \partial_{yy}^{2,h} v - c\partial_x^{h-} v, \tag{4.62}$$

where

$$\partial_x^{h\pm} v(x,y) = \pm \frac{v(x \pm h, y) - v(x,y)}{h}, \tag{4.63}$$

$$\partial_y^{h\pm} v(x,y) = \pm \frac{v(x, y \pm h) - v(x,y)}{h}, \tag{4.64}$$

$$\partial_{yy}^{2,h} v(x,y) = \frac{v(x,y+h) - 2v(x,y) + v(x,y-h)}{h^2}, \tag{4.65}$$

Note that we omit the time derivative. In the present context it isn't necessary as φ_h doesn't depend upon time.

One way to incorporate the Neumann boundary condition (equation (4.44)) is to define the points outside the grid $(L, Nh+h)$ and $(Nh+h, L)$ as

$$v\left(L, Nh+h\right) = v\left(L, Nh-h\right) \tag{4.66}$$

and

$$v\left(Nh+h, L\right) = v\left(Nh-h, L\right). \tag{4.67}$$

and then for the derivatives at the uppermost and rightmost grid boundaries we have

$$\partial_x^{h+} v(Nh, jh) = \frac{v(Nh+h, jh) - v(Nh, jh)}{h} = \frac{v(Nh-h, jh) - v(Nh, jh)}{h}, \tag{4.68}$$

$$\partial_y^{h+} v(ih, Nh) = \frac{v(ih, Nh+h) - v(ih, Nh)}{h} = \frac{v(ih, Nh-h) - v(ih, Nh)}{h} \tag{4.69}$$

and

$$\partial_{yy}^{2,h} v(ih, Nh) = \frac{v(ih, Nh+h) - 2v(ih, Nh) + v(ih, Nh-h)}{h^2} = \frac{-2v(ih, Nh) + 2v(ih, Nh-h)}{h^2}. \tag{4.70}$$

For the points belonging to the horizontal and vertical axis ($(0,y)$ and $(x,0)$ where $x, y \geq 0$), A_h^c simplifies to

$$(0, jh)_{j\in[1,N]}: \quad c = 0 \text{ (the only admissible } c\text{) and } A_h^c v = \alpha y \partial_y^{h+} v + \frac{1}{2}\sigma^2 y^2 \partial_{yy}^{2,h} v \tag{4.71}$$

$$(ih, 0)_{i\in[1,N]}: \text{The expression simplifies to } A_h^c v = rx\partial_x^{h+} v - c\partial_x^{h-} v, \tag{4.72}$$

$$(0,0): \text{The expression simplifies to } A_h^c v = 0 \text{ and we have } V = 0. \tag{4.73}$$

Operators as Matrix

The functions will be written as vectors and the convention is that the vectors are written as a column matrix. We change the points labels in the grid using only one integer instead two. Then instead of using two integer to define a point (as in the (i, j)) we will use one integer through the transform defined below.

We begin by defining the differences operators in one dimension. Then we transform it in two-dimensional operator by using the Kronecker multiplication in a suitable way.

Difference Operators in 1 Dimension Let D^{1d} be a one-dimensional grid with $N+1$ points with $x = hi$ for $i = 0, ..., N$. The positive first difference operator $\Delta_{N+1}^{+}v(ih) = (v(ih+h) - (ih))/h$ where $N+1$ is the number of points may be written as a matrix. For instance for $N+1 = 4$ we have

$$\Delta_4^+ = \frac{1}{h}\begin{pmatrix} -1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & -1 \end{pmatrix}. \tag{4.74}$$

By the same token, the second difference operator $K_{N+1}v(ih) = [v(ih+h) - 2v(ih) + v(ih-h)]/h^2$ has a matrix representation and for $N+1 = 4$ it is

$$K_4 = \frac{1}{h^2}\begin{pmatrix} -2 & 1 & 0 & 0 \\ 1 & -2 & 1 & 0 \\ 0 & 1 & -2 & 1 \\ 0 & 0 & 1 & -2 \end{pmatrix}. \tag{4.75}$$

Note that the first or last (or both) row is different from the others. It is because we are not considering any boundary condition yet. We will incorporate the boundary condition later by summing another matrix.

A simple way to construct those matrixes is a sum of identity and shifted identity matrix I_{N+1} and I_{N+1}^{s}. The superscript s is the number of rows shift to the right if $s > 0$ or left if $s < 0$. For instance

$$I_{N+1}^{0} = I_{N+1},$$

$$I_4^1 = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix} \tag{4.76}$$

and

$$I_4^{-2} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}. \tag{4.77}$$

Now we can construct the positive, negative and centered first difference, as

$$\Delta_{N+1}^{+} = \frac{1}{h}\left(I_{N+1}^{1} - I_{N+1}\right), \tag{4.78}$$

$$\Delta_{N+1}^{-} = \frac{1}{h}\left(I_{N+1} - I_{N+1}^{-1}\right), \tag{4.79}$$

$$\Delta_{N+1} = \frac{1}{2h}\left(I_{N+1}^{1} - I_{N+1}^{-1}\right), \tag{4.80}$$

and the second difference operator as

$$K_{N+1} = \frac{1}{h^2}\left(I^{1}_{N+1} - 2I_{N+1} + I^{-1}_{N+1}\right). \tag{4.81}$$

We shall account for the Neumann boundary condition, equations (4.68) and (4.69), adding a suitable matrix. Again equations (4.68) and (4.69) reads

$$\partial_x^{h+} v(ih, Nh) = \frac{-2v(ih, Nh) + 2v(ih, Nh - h)}{h^2}, \tag{4.82}$$

$$\partial_x^{h+} v(Nh, jh) = \frac{v(Nh - h, jh) - v(Nh, jh)}{h}. \tag{4.83}$$

In the matrixes Δ^+_{N+1} and K_{N+1} we don't have the term $v(ih, Nh+h)$ as it doesn't belong to D^{1d}. To incorporate the Neumann boundary condition define $\Delta^+_{C,N+1}$ and $K_{C,N+1}$ as

$$\left(\Delta^+_{C,N+1}\right)_{N+1,N} = \left(K_{C,N+1}\right)_{N+1,N} = 1, \tag{4.84}$$

$$\left(\Delta^+_{C,N+1}\right)_{i,j} = \left(K_{C,N+1}\right)_{i,j} = 0 \quad \text{if } i \neq N+1 \text{ or } j \neq N\ . \tag{4.85}$$

And finally we have the operators accounting for Neumann boundary as

$$\Delta^+_{N+1} + \Delta^+_{C,N+1}, \tag{4.86}$$

$$K_{N+1} + K_{C,N+1}. \tag{4.87}$$

For instance

$$\Delta^+_{C,4} = \frac{1}{h}\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}, \tag{4.88}$$

$$K_{C,4} = \frac{1}{h^2}\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

$$\Delta^+_4 + \Delta^+_{C,4} = \frac{1}{h}\begin{pmatrix} -1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 1 \\ 0 & 0 & 1 & -1 \end{pmatrix}, \tag{4.89}$$

$$K_4 + K_{C,4} = \frac{1}{h^2}\begin{pmatrix} -2 & 1 & 0 & 0 \\ 0 & -2 & 1 & 0 \\ 0 & 1 & -2 & 1 \\ 0 & 0 & 2 & -2 \end{pmatrix}. \tag{4.90}$$

Difference Operators in 2 Dimensions We are concerned with two-dimensional operators.

This is achieved by a simple Kronecker product if we define properly the grid labeling. As in the main text, let the bidimentional grid be defined as $D_h = (ih, jh)$ where $i \in \{0, ..., N\}$ and $j \in \{0, ..., N\}$ assuming that $N = L/h$ is an integer. Instead of using the two integer (i, j) we define $p = 1 + i + (N+1)j$ and the point labeled with p refers to the point (ih, jh). The figure below shows this convention for p with $N = 3$. For instance, when $i = 2$ and $j = 1$ we have

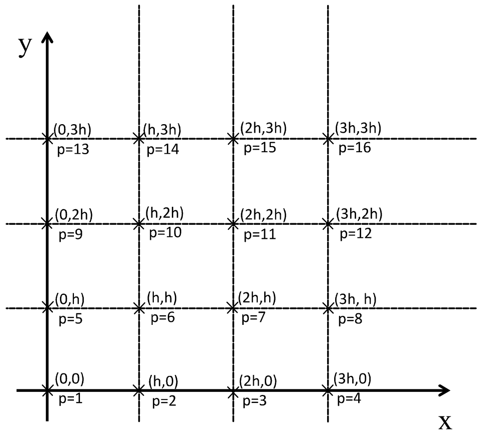

Figure 4.7: Appropriate point labeling in two dimension.

$p = 7$ and $\varphi_h(2h, h) = (\varphi_h)_{p=7}$. A function in such grid is represented as a $(N+1)^2-$dimensional vector and the operator as square matrix.

In such grid the first and second difference operator fox $x-$axis are

$$\Delta 2D^{x,\pm}_{N+1} = I_{N+1} \otimes \Delta^{\pm}_{N+1}, \tag{4.91}$$

$$K2D^{x}_{N+1} = I_{N+1} \otimes K_{N+1} \tag{4.92}$$

and for the $y-$axis

$$\Delta 2D^{y,\pm}_{N+1} = \Delta^{\pm}_{N+1} \otimes I_{N+1}, \tag{4.93}$$

$$K2D^{y}_{N+1} = K_{N+1} \otimes I_{N+1}, \tag{4.94}$$

Where $A \otimes B$ is the Kronecker multiplication of two matrixes. Finally, in order to account for the Neumann boundary condition we simply add the correction term in the unidimensional matrix:

$$\Delta 2D^{x,+}_{N+1} + correction = I_{N+1} \otimes (\Delta^{+}_{N+1} + \Delta^{+}_{C,N+1}), \tag{4.95}$$

$$K2D^{x}_{N+1} + correction = I \otimes (K_{N+1} + K_{C,N+1}), \tag{4.96}$$

$$\Delta 2D^{y,\pm}_{N+1} + correction = (\Delta^{+}_{N+1} + \Delta^{+}_{C,N+1}) \otimes I_{N+1}, \tag{4.97}$$

$$K2D^{y}_{N+1} + correction = (K_{N+1} + K_{C,N+1}) \otimes I_{N+1}. \tag{4.98}$$

Matrix Representation for the Generator We need the operator $x\partial^{h+}_x$ and not just the derivative ∂^{h+}_x. Again, we first construct the unidimensional operator and then apply the Kronecker in a convenient way. Let $diag(v)$ put the values of the vector on a diagonal matrix. Then if $H^v = diag(v)$ we have

$$(H^v)_{ii} = v_i \tag{4.99}$$

$$(H^v)_{ij} = 0 \ \text{ for } i \neq j. \tag{4.100}$$

For instance, let $n_{h,N} = (0, h, 2h, 3h, ..., Nh)'$, then

$$diag(n_{h,N}) = \begin{pmatrix} 0 & 0 & \dots & 0 \\ 0 & h & & 0 \\ \vdots & & \ddots & \vdots \\ 0 & 0 & \dots & Nh \end{pmatrix}. \tag{4.101}$$

Let $n_{h,N}^2 = (0, h^2, 4h^2, 9h^2, ..., N^2h^2)$ and denote by $(x\partial)_{N+1}$ and $\left(x^2\partial^2\right)_{N+1}$ the matrix representing the unidimensional first derivative multiplied by x. We have

$$\left(x\partial^{\pm}\right)_{N+1} = diag(n_{h,N+1}) * \Delta_{N+1}^{\pm}, \tag{4.102}$$

$$\left(x^2\partial^2\right)_{N+1} = diag(n_{h,N+1}^2) * K_{N+1}, \tag{4.103}$$

where " $*$ " denotes the usual matrix multiplication. As example consider $N+1=4$. Then

$$(x\partial)_4^h = \frac{1}{2h}\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 3 \end{pmatrix}\begin{pmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & -1 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{pmatrix} = \frac{1}{2h}\begin{pmatrix} 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & -2 & 0 & 2 \\ 0 & 0 & -3 & 0 \end{pmatrix}, \tag{4.104}$$

$$\left(x^2\partial^2\right)_4^h = \frac{1}{h^2}\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 9 \end{pmatrix}\begin{pmatrix} -2 & 1 & 0 & 0 \\ 1 & -2 & 1 & 0 \\ 0 & 1 & -2 & 1 \\ 0 & 0 & 1 & -2 \end{pmatrix} = \frac{1}{h^2}\begin{pmatrix} 0 & 0 & 0 & 0 \\ 1 & -2 & 1 & 0 \\ 0 & 4 & -8 & 4 \\ 0 & 0 & 9 & -18 \end{pmatrix}. \tag{4.105}$$

The bidimensional operator composing A_h^c are

$$\left(x\partial_x^{h+}\right)_{N+1}^h = I_{N+1} \otimes \left(x\partial^+\right)_{N+1}, \tag{4.106}$$

$$\left(y\partial_y^{h+}\right)_{N+1}^h = \left(x\partial^+\right)_{N+1} \otimes I_{N+1}, \tag{4.107}$$

$$\left(y^2\partial_{yy}^{2,h+}\right)_{N+1}^h = \left(x^2\partial^2\right)_{N+1} \otimes I_{N+1}, \tag{4.108}$$

$$\left(c\partial_x^{h-}\right)_{N+1}^h = diag(c_{N+1}) * \Delta 2D_{N+1}^{x,-}, \tag{4.109}$$

and finally

$$(A_h^c)_{N+1} = r\left(x\partial_x^{h+}\right)_{N+1}^h + \alpha\left(y\partial_y^{h+}\right)_{N+1}^h + \frac{\sigma^2}{2}\left(y^2\partial_{yy}^{2,h+}\right)_{N+1}^h - \left(c\partial_x^{h-}\right)_{N+1}^h + \text{Neumann corrections}, \tag{4.110}$$

where c_{N+1} is a representation of consumption rate $c(ih, jh)$ such that the p-th element of c_{N+1} is

$$(c_{N+1})_p = c(ih, jh), \tag{4.111}$$

where (as before)

$$p = 1 + i + (N+1)j, \tag{4.112}$$

$(A_h^c)_{N+1}$ is the matrix representation of the generator A_h^c and the "corrections"accounts for the Neumann boundary condition. Note that the equations (4.71)-(4.73) automatically hold in the above construction of $(A_h^c)_{N+1}$ once one sets[12] $c(0, jh) = 0$.

[12] This prevents the borrowing of money by the investor, i.e., it prevents the portfolio goes outside D with negative x.

Defining the Optimal $c_{n+1}, z_{n+1}, T_{n+1}$

The value for $c_{n+1}(x,y)$ is

$$c_{n+1}(x,y) = \left(\frac{\partial v^n}{\partial x}\right)^{\frac{1}{\gamma}} \tag{4.113}$$

or

$$\begin{aligned} c_{n+1}(ih,jh) &= \min\left\{c_{\max}; \left(\frac{v^n(ih,jh) - v^n(ih-h,jh)}{h}\right)^{\frac{1}{\gamma}}\right\}, & (4.114)\\ &= \min\left\{c_{\max}; \left[\left(\Delta 2 D^{x,-}_{N+1} v^n\right)_p\right]^{\frac{1}{\gamma}}\right\}. & (4.115)\end{aligned}$$

Note that we used the negative first difference. This is necessary in order to have all off diagonal elements positive in A^c_h.

After obtaining the consumption rate it is possible to obtain the optimal rebalancing policy $z_{n+1}(ih,jh)$:

$$\begin{aligned} z_{n+1}(ih,jh) &= \arg\max_z B^z v^n(ih,jh) & (4.116)\\ &= \arg\max_z v^n\left(ih - k_h h - zh, jh + zh\right) & (4.117)\end{aligned}$$

if there is an integer z such that $(ih - k_h h - zh, jh + zh) \in D_h$, otherwise $z_{n+1}(ih,jh)$ is not defined and $B^z v^n(ih,jh) = v^n(0,0) = 0$.

The region T_{n+1} may be stored in computer in convenient way as a vector filled with 0 or 1 such that if $(T_{n+1})_p = 1$ we have the point labeled by p belongs to T_{n+1} and if $(T_{n+1})_p = 0$ we have the opposite: $p \notin T_{n+1}$. Then

$$\begin{aligned} (T_{n+1})_p &= 1 \text{ if } (B^{z_{n+1}} v_n)_p > \frac{\left(\left(\Delta A^c_h + I_h\right) v_n + \Delta \frac{(c_{n+1})^{1-\gamma}}{1-\gamma}\right)_p}{1+\rho\Delta},\\ (T_{n+1})_p &= 0 \quad \text{otherwise.} \end{aligned} \tag{4.118}$$

The Linear Operator $\widehat{O}_{T_{n+1},c_{n+1},z_{n+1}}$

The operator $O_{T_{n+1},c_{n+1},z_{n+1}}$ is:

$$\begin{aligned} \left(O_{T_{n+1},c_{n+1},z_{n+1}}[v]\right)_p &= \frac{\left(\left(\Delta A^c_h + I_h\right) v + \Delta \frac{(c_{n+1})^{1-\gamma}}{1-\gamma}\right)_p}{1+\rho\Delta} \text{ for } p \notin T_{n+1}, & (4.119)\\ \left(O_{T_{n+1},c_{n+1},z_{n+1}}[v]\right)_p &= \left(B^{z_{n+1}} v\right)_p \qquad \text{for} \quad p \in T_{n+1}. \end{aligned}$$

and this is equivalent to

$$O_{T_{n+1},c_{n+1},z_{n+1}}[v] = diag(T_{n+1}) * (B^{z_{n+1}} v) + (I_h - diag(T_{n+1})) * \frac{\left(\left(\Delta A^c_h + I_h\right) v + \Delta \frac{(c_{n+1})^{1-\gamma}}{1-\gamma}\right)}{1+\rho\Delta}. \tag{4.120}$$

In order to write the system of linear equation it is convenient to write the above operator separating the linear operator part from $\frac{(c_{n+1})^{1-\gamma}}{1-\gamma}$:

$$O_{T_{n+1},c_{n+1},z_{n+1}}[v] = \widehat{O}_{T_{n+1},c_{n+1},z_{n+1}}[v] + \widehat{c_{n+1}} \tag{4.121}$$

where $\widehat{O}_{T_{n+1},c_{n+1},z_{n+1}}$ is the matrix

$$\widehat{O}_{T_{n+1},c_{n+1},z_{n+1}} = diag(T_{n+1}) * B^{z_{n+1}} + (I_h - diag(T_{n+1})) * \frac{(\Delta A_h^c + I_h)}{1+\rho\Delta} \tag{4.122}$$

and $\widehat{c_{n+1}}$ is the vector

$$\widehat{c_{n+1}} = (I_h - diag(T_{n+1})) * \frac{\Delta}{1+\rho\Delta}\frac{(c_{n+1})^{1-\gamma}}{1-\gamma}. \tag{4.123}$$

Sub-step 2

The sub-step 2 is the computation of v^{n+1} as the solution of the system of linear equation

$$v^{n+1} = \widehat{O}_{T_{n+1},c_{n+1},z_{n+1}}\left[v^{n+1}\right] + \widehat{c_{n+1}} \tag{4.124}$$

equivalent to

$$\left(\widehat{O}_{T_{n+1},c_{n+1},z_{n+1}} - I_h\right) v^{n+1} = -\widehat{c_{n+1}}. \tag{4.125}$$

This is a vectorial equation and as long as $\left(\widehat{O}_{T_{n+1},c_{n+1},z_{n+1}} - I_h\right)$ isn‘t singular we have a unique solution for v^{n+1}.

This is a vectorial equation of the type $Ax = y$ and the Matlab‘s backslash command $(A\backslash y = x)$ solves this equation.

4.A.3 Modification in the Algorithm for $t < T_A$

Possible Time Discretization Schemes and Our Choice

The main difference for the numerical method before T_A is that we have to include the time derivative explicitly in the generator operator. Recall that we wrote:

$$V(t,x,y) = e^{-\rho t}\varphi(x,y) \quad \text{for } t \geq T_A, \tag{4.126}$$

$$\left(A^c V\right)(u,x,y) = A^c e^{-\rho t}\varphi(x,y) = e^{-\rho t}\left(A^c\varphi(x,y) - \rho\varphi(x,y)\right). \tag{4.127}$$

and the discrete version of A^c as

$$A_h^c v = rx\partial_x^{h+}v + \alpha y\partial_y^{h+}v + \frac{1}{2}\sigma^2 y^2\partial_{yy}^{2,h}v - c\partial_x^{h-}v.$$

We omitted the time derivative in A_h^c because it had no effect in $\varphi(x,y)$

$$\partial_t\varphi(x,y) = 0.$$

Nonetheless we need to include it explicitly in the present context.

Define the time grid as $\{T_A - \delta_t k | k = 0,1,2,...,N_t\}$. It is convenient to use a similar notation for $t < T_A$

$$V(t,x,y) = e^{-\rho t}\phi(t,x,y) \quad \text{for } t < T_A, \tag{4.128}$$

$$\left(A^c V\right)(t,x,y) = A^c e^{-\rho t}\phi(t,x,y) = e^{-\rho t}\left(A^c\phi(t,x,y) - \rho\phi(t,x,y)\right). \tag{4.129}$$

but now we include the time derivative in the discrete version of the generator explicitly:

$$\begin{aligned} A_h^{3d,c}v &= \partial_t^{\delta_t}v + rx\partial_x^{h+}v + \alpha y\partial_y^{h+}v + \frac{1}{2}\sigma^2 y^2\partial_{yy}^{2,h}v - c\partial_x^{h-}v \qquad (4.130)\\ A_h^{3d,c}v &= \partial_t^{\delta_t}v + A_h^c v. \end{aligned}$$

We keep considering $A_h^c v$ as the discrete version of generator without the time derivative for notational convenience. We add a superscript $3d$ to highlight tridimensional nature of the problem.

There are at least three possible schemes for time derivative: explicit, Crank-Nicholson and implicit. We choose the implicit scheme because it is usually well behaved:

$$\begin{aligned} \partial_t^{\delta_t} v &= \frac{v(t+\delta_t, x, y) - v(t, x, y)}{\delta_t} \\ \partial_t^{\delta_t} v &= \frac{v(T_A - (k-1)\delta_t, ih, jh) - v(T_A - k\delta_t, ih, jh)}{\delta_t}. \end{aligned} \tag{4.131}$$

The second main difference is that we have a new boundary condition implied by the conjecture 12:

$$\phi(T_A, x, y) = E\left[e^{-\rho t}\varphi(x + dX_{T_A}, y + dS_{T_A})\right] \tag{4.132}$$

In the main text we have $dX_{T_A} \to 0$ but $dS_{T_A} = y(\zeta - 1)$ where ζ is lognormal. Moreover, V is function of the fixed cost k and the parameters in the process r, α, σ that may change at T_A.

Transforming a 3D Problem into a Sequence of 2D Ones

In the first part of this appendix we transformed the three dimensional problem into a two dimensional one by assuming that $V(t, x, y) = e^{-\rho t}\varphi(x, y)$. This procedure isn't adequate for $t < T_A$ and all matrix operators should, in principle, be transformed to accommodate the time. Nonetheless, the structure of time derivative allows for a different procedure. Instead solving a large linear equation system for a the three dimensional partial differential equation it is possible to solve a sequence of smaller linear equation system for a sequence of two dimensional partial differential equation.

The above transformation is possible because in order to solve for $\phi(T_A - k\delta_t, x, y)$ it is necessary only to know $\phi(T_A - (k-1)\delta_t, x, y)$ and nothing else. Then, given the boundary condition at T_A :

$$\phi(T_A, x, y) = E\left[e^{-\rho t}\varphi(x + dX_{T_A}, y + dS_{T_A})\right] \tag{4.133}$$

the algorithm solves for $\phi(T_A - \delta_t, x, y)$. Then it solves for $\phi(T_A - 2\delta_t, x, y)$ using only the function $\phi(T_A - \delta_t, x, y)$. And then it solves for $\phi(T_A - 3\delta_t, x, y)$ using $\phi(T_A - 2\delta_t, x, y)$ and keep iterating until the desired time.

In order to keep using two dimensional matrix operators, we will use the notation

$$\phi_{T_A - k\delta}(x, y) = \phi(T_A - k\delta_t, x, y) \tag{4.134}$$

considering $\phi_{T_A - k\delta}$ as a two dimensional function: $\phi_{T_A - k\delta} : \Re^2 \to \Re$.

We solve for each k an equation similar to equation (4.125). Nonetheless we shall add the time derivative:

$$\partial_t^{\delta_t} v = \frac{v(T_A - (k-1)\delta_t, ih, jh) - v(T_A - k\delta_t, ih, jh)}{\delta_t}. \tag{4.135}$$

Note that the time derivative is an operator that transforms a function $v : \Re^3 \to \Re$ to a function $\partial_t^{\delta_t} v : \Re^3 \to \Re$. But we want to use the two dimensional function $\phi_{T_A - k\delta_t} : \Re^2 \to \Re$. To this end we use the forward operator L_k :

$$L_k\left[\phi_{T_A - k\delta_t}\right] = \phi_{T_A - (k-1)\delta_t}, \tag{4.136}$$

and use only functions whose domain is $\Re^2$:

$$\partial_t^{\delta_t}\left[\phi_{T_A - k\delta_t}\right](ih, jh) = \frac{\phi_{T_A - (k-1)\delta_t}(ih, jh)}{\delta_t} - \frac{\phi_{T_A - k\delta_t}(ih, jh)}{\delta_t} \tag{4.137}$$

$$\begin{aligned}\partial_t^{\delta_t}\left[\phi_{T_A-k\delta_t}\right] &= \frac{1}{\delta_t}L_k\left[\phi_{T_A-k\delta_t}\right]-I_h\left[\phi_{T_A-k\delta_t}\right] \\ \partial_t^{\delta_t}\left[\phi_{T_A-k\delta_t}\right] &= \frac{1}{\delta_t}\left(L_k-I_h\right)\left[\phi_{T_A-k\delta_t}\right].\end{aligned} \tag{4.138}$$

Remember that the equation (4.125) is a linear equation system:

$$\left(\widehat{O}_{T_{n+1},c_{n+1},z_{n+1}}-I_h\right)v^{n+1}=-\widehat{c_{n+1}}. \tag{4.139}$$

where

$$\widehat{O}_{T_{n+1},c_{n+1},z_{n+1}}=diag(T)*B^{z_{n+1}}+(I-diag(T))*\frac{(\Delta A_h^c+I_h)}{1+\rho\Delta}, \tag{4.140}$$

$$\widehat{c_{n+1}}=(I-diag(T))*\frac{\Delta}{1+\rho\Delta}\frac{(c_{n+1})^{1-\gamma}}{1-\gamma}. \tag{4.141}$$

We keep the two-dimensional structure and the main change is replacing the operator A_h^c with $A_h^{3d,c}$ in a consistent way. We have that

$$\left(\widehat{O^{3d}}_{T_{n+1},c_{n+1},z_{n+1}}-I_h\right)v^{n+1}=-\widehat{c_{n+1}} \tag{4.142}$$

the left hand side is:

$$\begin{aligned}\widehat{O^{3d}}_{T_{n+1},c_{n+1},z_{n+1}} &= diag(T)*B^{z_{n+1}}+(I-diag(T))*\frac{\left(\Delta\left(A_h^c+\partial_t^{\delta_t}\right)+I_h\right)}{1+\rho\Delta} \\ &= diag(T)*B^{z_{n+1}}+(I-diag(T))*\frac{\left(\Delta\left(A_h^c+\frac{1}{\delta_t}\left(L_k-I_h\right)\right)+I_h\right)}{1+\rho\Delta}\end{aligned} \tag{4.143}$$

$$\widehat{O^{3d}}_{T_{n+1},c_{n+1},z_{n+1}}=\widehat{O}_{T_{n+1},c_{n+1},z_{n+1}}+\left(\frac{\Delta}{1+\rho\Delta}\right)\frac{1}{\delta_t}\left(L_k-I_h\right). \tag{4.144}$$

Then we have

$$\widehat{O}_{T_{n+1},c_{n+1},z_{n+1}}v^{n+1}+\left(\frac{\Delta}{1+\rho\Delta}\right)\frac{1}{\delta_t}\left(L_k-I_h\right)v^{n+1}=-\widehat{c_{n+1}}. \tag{4.145}$$

Note that $L_k v^{n+1}=\phi_{T_A-(k-1)\delta_t}$ is given. We put it at the right hand side:

$$\widehat{O}_{T_{n+1},c_{n+1},z_{n+1}}v^{n+1}+\left(\frac{\Delta}{1+\rho\Delta}\right)\frac{-I_h}{\delta_t}v^{n+1}=-\widehat{c_{n+1}}-\left(\frac{\Delta}{1+\rho\Delta}\right)\frac{L_k}{\delta_t} \tag{4.146}$$

$$\left(\widehat{O}_{T_{n+1},c_{n+1},z_{n+1}}-\left(\frac{\Delta}{1+\rho\Delta}\right)\frac{I_h}{\delta_t}\right)v^{n+1}=-\widehat{c_{n+1}}-\left(\frac{\Delta}{1+\rho\Delta}\right)\frac{1}{\delta_t}\phi_{T_A-(k-1)\delta_t}. \tag{4.147}$$

The above equation is a linear equation system with the same dimensionality as (4.125) . Note that we shall do the procedure for each k beginning with $k=1$.

Detailing the Procedure Used to Obtain the Boundary Condition

In the section 4.4 we consider the case where the price doesn't jump at T_A but the risk-free rate change randomly between three possible rates with the same probability. In this case we do

$$\begin{aligned}\phi(T_A,x,y) &= E\left[e^{-\rho T_A}\varphi(x+dX_{T_A},y+dS_{T_A})\right] \\ \phi(T_A,ih,jh) &= \frac{1}{3}e^{-\rho T_A}\left(\varphi_{r_1}(x,y)+\varphi_{r_2}(x,y)+\varphi_{r3}(x,y)\right).\end{aligned} \tag{4.148}$$

We consider also the case with a jump in the risky asset but without changes on parameters.

$$\begin{aligned} \phi(T_A, x, y) &= E\left[e^{-\rho T_A}\varphi(x + dX_{T_A}, y + dS_{T_A})\right] \\ \phi(T_A, ih, jh) &\approx \sum_{l=1}^{\infty} e^{-\rho T_A}\varphi(ih, lh)\left[F_{jh}(lh) - F_{jh}([l-1]\,h)\right] \end{aligned} \tag{4.149}$$

where F_y is the cumulative distribution function (CDF) of S_{T_A} given $S_{T_A-} = y$:

$$F_y(a) = Prob\left[S_{T_A} \leq a\right] \tag{4.150}$$

and

$$\left[F_{jh}(lh) - F_y([l-1]\,h)\right] = Prob\left[(l-1)\,h \leq S_{T_A} \leq lh\right]. \tag{4.151}$$

Note that F_y is defined by the distribution of the jump. Note that the sum runs from 0 to ∞ but we don't know the value of $\varphi(x, y)$ for $y > L = Nh$. Truncating the sum in $l = N$ would give a biased expectation estimate. A better approach is to approximate $\varphi(x, L + a)$ with $\varphi(x + a, L)$. When $(x, L+a) \notin \mathbf{C}$ and $(x+a, L) \notin \mathbf{C}$ we have indeed that $\varphi(x, L+a) = \varphi(x+a, L)$. Otherwise, it is a way to have some estimate of $\varphi(x, L + a)$. Then, we do:

$$\phi(T_A, ih, jh) = \sum_{l=1}^{N} e^{-\rho t}\varphi(ih, lh)\left[F_{jh}(lh) - F_{jh}([l-1]\,h)\right] + \sum_{l=i+1}^{N} e^{-\rho t}\varphi(lh, Nh)\left[F_{jh}(lh) - F_{jh}([l-1]\,h)\right]. \tag{4.152}$$

4.B Image Filters

We use 2 image filters. First we zoom in and consider only the square $[0, L/2] \times [0, L/2]$. Then we fill the diagonal lines inside the grey area in the top left subfigure of figure 4.8 and then find the boundaries. In the final figure we fill the rebalancing region with a shadowed pattern and add the optimal portfolio after rebalancing as a dashed line.

Each image has $n \times n$ pixels[13] where each pixel represent a point in the grid. We associate the white in the figure 4.8 with the inaction region and with the number 0. The grey is the rebalancing region and is associated with number 1. This representation is consistent with the representation code used for set T in the appendix 4A. We consider the neighbor of a point (i, j) in the grid as the points $(i + 1, j), (i - 1, j), (i, j + 1)$ and $(i, j - 1)$. We call walls the points belonging to the boundaries of the square box $[0, L] \times [0, L]$. This is the set $\{(0, y)|y \in [0, L]\} \cup \{(x, 0)|x \in [0, L]\} \cup \{(L, y)|y \in [0, L]\} \cup \{(x, L)|x \in [0, L]\}$.

First note that all grey pixels have at least 3 grey neighbors except some points in the boundaries. Moreover, note that all white pixels have less than 3 grey neighbors except the diagonal blue lines inside the grey region (and very few points in the boundaries). The first image filter transformed the white pixels into grey ones whenever 3 or 4 neighbors are grey. This make the white diagonal lines disappear. However a few points are added at the boundary, but it has a minor effect.

After this first filter, we want only to show the boundary. It is easier to define a point inside the transaction region but not in the boundary: it is the grey ones with all neighbors grey. Then we consider the boundary as the grey points not inside the transaction region, i.e., all the grey points with at least one white neighbor.

Finally we add the dashed line representing the optimal portfolio after rebalancing.

[13] In computer graphics, pixel is the smallest element represented in the image.

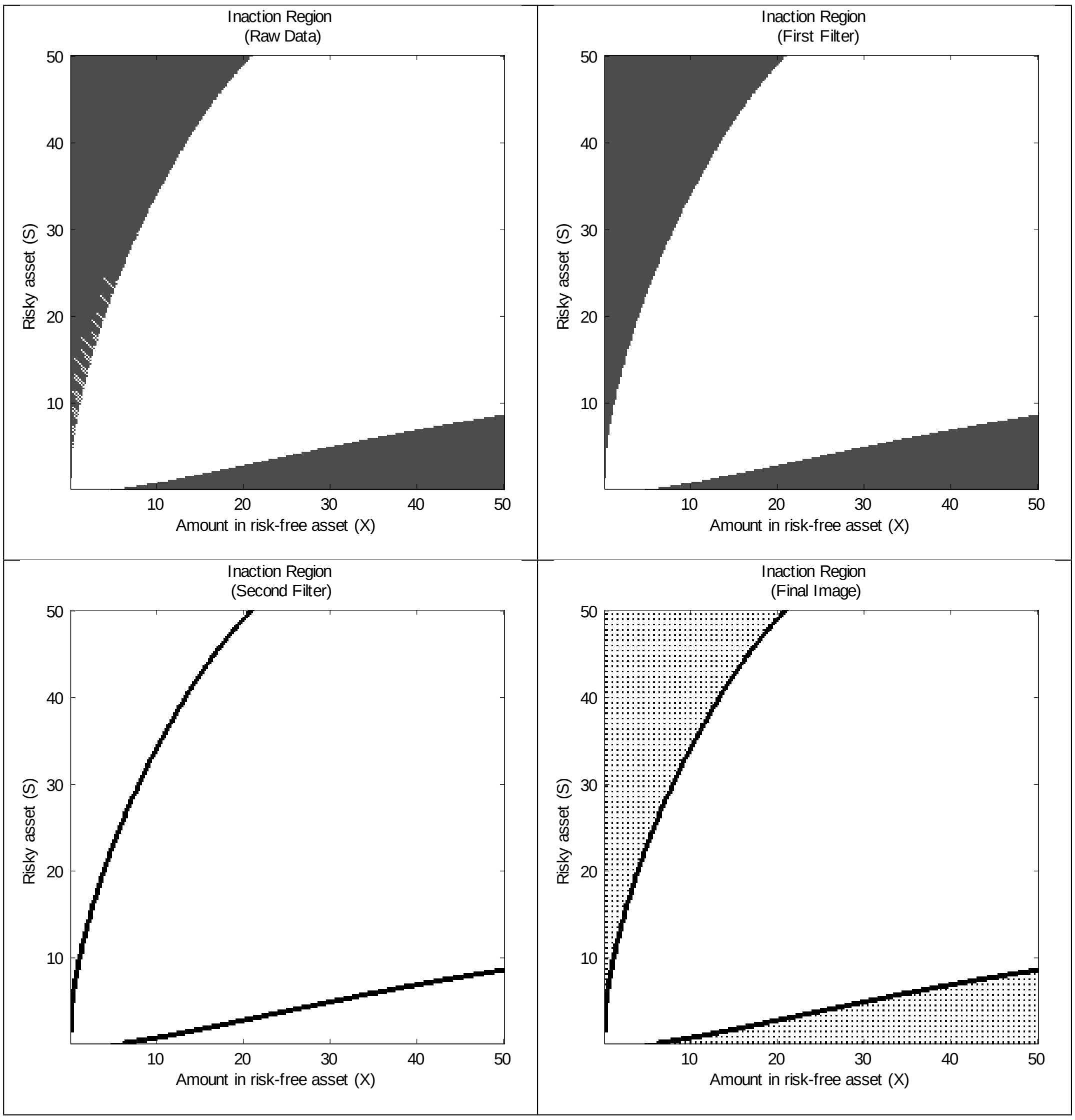

Figure 4.8: Filters being applied to the inaction region raw data after the announcement for parameters in table 4.1, case 3. The white region is the inaction region and the grey one is the rebalancing region. Note the diagonal white lines in the rebalancing region in the top left figure. The first image filter eliminates these lines. Then the boundary is determined as in the bottom left figure and we apply the shadow pattern to the rebalancing region.

Chapter 5

Conclusion

Each article in the present thesis investigates a different topic. The first one has the title "Nonparametric Option Pricing with Generalized Entropic Estimators"and studies a pricing method in incomplete markets. This method is linked to members in the Cressie-Read family function where each member provides one risk-neutral measure. The results are encouraging and suggest a new way to define robust intervals for derivative prices.

The second article is titled "Watching the News: Optimal Stopping Time and Scheduled Announcements"and studies optimal stopping times problems in the presence of a jump at a fixed time. It investigates the general case theoretically and provides a numerical solution for optimal time to sell an asset. The results characterize situations in which it is not optimal to stop just before the news. The results are naturally applied to financial instruments with a structure similar to American options.

The third article is a numerical study on an optimal portfolio problem with fixed cost and scheduled announcement. Again, the scheduled announcement is modeled as a jump at a fixed time. It is called "Dynamic Portfolio Selection with Transactions Costs and Scheduled Announcement". The goal is to investigate the impact of costs on investor's trading activity given the price process. The most interesting result is that the chance to transact may be consistent with the trading volume behavior found by Chae (2005) and barber et al. (2013). In the simulations, it happens when the investor has a strong belief on the jump direction (i.e., if the jump has a high or low average with low variance).

There are several interesting venues for future work. In relation to the first article, the method may be generalized to deal with multiples maturities. This is the case of interest rate derivatives such as caps and floors. It means that the stochastic discount factor (SDF) is a process and new relations may arise. This is so because there might be dependencies on outcomes of different maturities. Some question naturally emerges in this setting. For instance, can the dual problem be interpreted as a dynamic portfolio problem? Or, is this approach better for pricing interest rate derivatives than a simpler one investigated by Chowdhury and Stutzer (1999)? Another promising investigation is to better understand the relationship with information theory. This may provide a more solid foundation for the choice of Cressie-Read family. Finally, it is interesting to assess the empirical performance at firm level.

The second article may provide insights on how investors, households or firms behave at scheduled announcements when facing an optimal stopping time problem. The question is to know if it is optimal to take some action before the news or if it is better to delay such decision. The obvious application is on American option type securities. The results suggest that the exercises are less frequent just before the scheduled announcement and more frequent just after it for some cases. Future empirical work may confirm this prediction on different types of securities ranging from equities options to callable bonds. Other applications may involve search problems, stopping time games or optimal default time. For instance, how default time decisions made by households or firms are related to macro-announcements? Does the number of defaults diminish before some announcement?

Several future works may complement or build on the third article. From the theoretical point of view, the model may incorporate periodical scheduled announcement. This is an approximation for quarterly and/or annual earnings announcements. Another possible venue is to study a general equilibrium model with fixed cost and scheduled announcements. Possibly, it may build on the work of Lo et al. (2004). It is also interesting to find and compare the results with different types of costs or with no cost at all. From the empirical point of view, the third article suggests tests on the relationship between fixed cost and trading volume. Chae (2005) and Barber et al. (2013) found that the trading volume is lower between 2 to 10 days before the news. It is an average of a large sample. If the sample is organized in quintiles of fixed cost, would this result change across the quintiles? Do higher fixed costs imply lower abnormal trading volume before the news? Finally, there is room for technical contribution related to the localization procedure. For instance, a singular control may confine the process to a finite region. These controls imply a boundary condition involving derivatives in some cases. Possibly, these conditions may be rewritten as a Neumann boundary condition. This would provide an interpretation on the localization procedure used.

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