Corporate Capital Structure Revisited: Incomplete Markets and Default

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Abstract

After Modigliani and Miller (1958) presented their capital structure irrelevance proposition, analysis of corporate financing choices involving debt and equity instruments have generally followed two trends in the literature, where models either incorporate informational asymmetries or introduce tax benefits in order to explain optimal capital structure determination (Myers, 2002). None of these features is present in this paper, which develops an asset pricing model with the purpose of providing a positive theory of corporate capital structure by replicating main aspects of standard contractual practice observed in real markets. Alternatively, the imperfect market structure of the economy is tailored to match what is most common in corporate reality. Allowance for default on corporate debt with an associated penalty of seizure of firm’s future cash flows by creditors is introduced, for instance. In this context, a qualitative assessment of financial managers’ decisions is carried out through numerical procedures.
1 Introduction

The corporate finance literature has usually obtained its results from frameworks other than those derived from the asset pricing theory. Nevertheless, Lucas’ (1978) seminal contribution was a remarkable advance towards a theoretical assessment of the firm reality. This fact did not restrain the rich asset pricing theory developed since then to focus almost exclusively on individual consumers trading in securities markets. One possible reason for such a course of events is the difficulty in characterizing financing and investment decisions of a corporate unit when an agent utility function and its endowment and asset ownerships play the central role. Another is the relative difficulty in dealing with informational asymmetries in such frameworks. This paper leaves aside the latter and focuses on financing decisions involving the choice of debt and equity instruments in the absence of moral hazard or adverse selection. Moreover, investment decisions are taken as given. The purpose is to evaluate capital structure choices free of agency problems, by replicating imperfect financial contracts observed in real markets. Pursuing this goal, use of recent developments in asset pricing models, such as trading restrictions and allowance for default on liabilities, permit us to characterize a corporate unit within a GEI framework.

Formally, from Lucas’ model the term Lucas tree was later derived, which, in the original paper, had a referent associated with its approximation to reality that corresponds to a firm’s cash flows. Indeed, the author named the mappings from tree nodes to the real line a firm. The main purpose of this paper is to improve on this description by adding to it still another mathematical object – namely, a corporate bond in the form of an endogenous variable that is chosen by the agent holding the largest number of shares of the firm. What underlies this approach is the idea that one should seek for logical structures that have as correspondents the financial statements. Lucas’ model has a referent on the statement of cash flows, cash flow itself, everything else being static. The present paper delivers a logical structure (incorporating incomplete markets and default for this reason) that has a referent which contemplates diversity on the right-hand side of the balance sheet statement as well - more specifically, it makes the distinction between liabilities and equity.

Since virtually all firms might have their financial and economic activities satisfactorily recorded in financial statements obeying the accounting principles, we would benefit from ultimately reaching a logical structure that refers to all relevant financial statement items. Certainly, there are many aspects of the firm objective reality which do not have immediate correspondence on pure accounting procedures; much of corporate finance literature builds extensively on these (usually relying on informational frictions). Nonetheless, a better match between models of the firm and accounting practice is desirable.

In turn, capital structure is a theme in corporate finance literature that has been subject of extensive academic debate since Modigliani and Miller (1958) established their famous irrelevance proposition. Because this result is not verified empirically, much work has been done since then to approximate the theory to reality. Miller (1988) himself stressed the importance of the conditions under which the theory fails.
to hold (obviously none is present in Lucas, 1978, what further justifies his correspondence between the Lucas tree and the firm), and, indeed, work subsequent to the original contribution builds on failure of some of the premises adopted. Therein, theories of agency costs have played a major role with its results derived from informational asymmetries. This theoretical branch points to causal relations strictly linked to financial patterns affecting cash flow generation. But the financial choice per se has no direct impact on firm value, only through the cash flow channel. Another branch consists in the so-called trade-off theories (Myers, 2002). On the one side tax benefits make debt desirable and on the other financial distress costs associated with increased fixed payment commitments prevent firm’s decision makers from choosing an excessive amount of it. Both forces provide mathematical determination of an optimal capital structure. Nonetheless, Miller (1977) undermined the trade-off explanation showing conditions under which a irrelevance result still holds at the individual firm level. From a normative standpoint, a complete markets structure is unrealistic and once we have incomplete markets there is trading in all assets that are not redundant. Why we see the emergence in the real markets corporations have access to of highly predominant usage of equity and bond instruments is a question this paper leaves aside. The reader is referred to Krueger and Lustig (2006), where equilibrium in an economy with a complete set of Arrow securities against aggregate risk is replicated in an economy with only idiosyncratic risk where agents trade only a discount bond and a stock. In the following it is adopted a positive viewpoint, in which common contractual practice is replicated and its results and implications are analyzed.

Furthermore, finance researchers adopt one of two behavioral premises when analyzing corporations: either agents are utility maximizers who are concerned with the stream of cash flows generated by their decisions or they seek to maximize firm value. For instance, the approach in Modigliani and Miller (1958), in a partial equilibrium and perfect markets context, consists in firm value maximization. Under their premises, this is coherent because one has a well-defined market discount rate for every uncertain cash flow stream that may arise. However, under the hypothesis of incomplete markets agents’ marginal utilities are not necessarily the same in equilibrium and there may not be well-defined market discount rates. Therefore, it is adopted the utility-based approach and argued that it is fairly reasonable to assume that standard corporate behavior can be described by a board of directors making choices of capital structure (and others) based on their preferences. In the model presented, the leading shareholder, i.e., the agent holding the largest number of firm shares, has the right to make the capital structure choice.

Summarizing, there is an entrepreneur who is a utility maximizer, owns a firm and issues equity and debt in order to diversify her portfolio. The mix of both instruments issued is determined by her preferences over future consumption, according to the stream of wealth it generates - as long as she does not lose control of the firm to another agent. Numerical methods are employed so her choices can be analyzed under different scenarios in which an overall assessment of capital structure choice responses to varying circumstances (such as firm size and risk pattern of cash flows) is performed. Within the mathematical structure developed it is obtained the
description of a corporate unit in a general equilibrium model.

Section 2 presents the dynamic general equilibrium framework developed, which is the basis of the numerical exercise, along with the explanation of how imperfect financial contracts observed in reality are characterized. Section 3 describes the parameterization chosen for the numerical exercise. Section 4 shows the results. Finally, Section 5 concludes.

2 Economic Model and Contractual Depiction

The framework presented below is built in order to provide a qualitative analysis of capital structure choices of an entrepreneur who is initially the sole owner of a firm. This section gives the description of the physical environment and markets used to theorize about financing decisions.

It is considered an exchange economy that extends over a finite horizon and faces uncertainty. There are three agents - referred to as entrepreneur, bond market and equity market - indexed (respectively) by $i \in I = \{en, bm, em\}$. They are assumed to have rational expectations about the outcomes of the economic variables. Each period is denoted $t \in T = \{0, ..., T\}$. In addition, there are three Lucas trees, $l \in L = \{e, b, f\}$, and the dividends they yield amount to the entire wealth generation of the economy. One Lucas tree is interpreted as total free cash flow (free cash flow is defined as cash flow minus debt payments) distribution from outstanding equity of the aggregate of all other firms ($e$), another is interpreted as interest and principal payments on outstanding bonds of the aggregate of all other firms ($b$), while the last consists in the cash flows generated by fixed investment decisions made by the entrepreneur ($f$). Agent $i$’s ownership share in tree $l$ in period $t$ is denoted $\sigma_{l, i, t}$. Henceforth, agent $i$, $i \in \{en, em\}$, such that $\sigma_{f, en, t} = \max(\sigma_{f, en, t}, \sigma_{f, em, t})$, is referred to, at time $t$, as the leading shareholder. By convention, if $\sigma_{f, en, t} = \sigma_{f, em, t}$ the entrepreneur is the leading shareholder.

In the first period we have $\sigma_{f, en, -1} = 1$, $\sigma_{b, bm, -1} = 1$, and $\sigma_{e, em, -1} = 1$, where the last subscript denotes initial values at date $0$. Hence, equity market and bond market agents are interpreted as representative utility maximizers with whom trading occurs in a stock exchange and in a corporate bond public market, respectively. After date $0$, the realization of an exogenous state variable, $s_t \in S$, the latter being a finite set, determines the amount of the economy’s single good yielded by each tree, $d_{l, t} : S^t \rightarrow R_{++}$ - where $S^t$ denotes the Cartesian product of $S$ by itself $t$ times. Associated with the sequences $\{s_t\}_{t=1}^T$ are state probabilities which all agents foresee and regard as truthful. Initial dividend yield by each tree is $d_{l, 0} \in R_{++}$. Each agent has preferences over consumption of the existing good in a given period, $c_{i, t} \in R_+$, numerically represented by a time-separable expected utility function:
\[
U_i(c_i) = E \left\{ \sum_{t=0}^{T} \beta_t^t u_i(c_{i,t}) \right\},
\]

where \( c_i = \{c_{i,t}\}_{t=0}^{T}, \beta_t \in (0, 1), u_i : \mathbb{R}_+ \rightarrow \mathbb{R} \) is continuous, concave and strictly increasing. In every period (but the two last) a financial market opens for trading on shares of the Lucas trees. Moreover, a financial market opens in every period (but the two last) for trading on a firm bond, a one-period security on which the leading shareholder may only hold a short position. This asset is a promise to deliver a homogeneous quantity of the existing good across state realizations, equal to one unit. The consumption good price is normalized to one, while shares on the Lucas trees and the bond asset are transacted at prices \( p_t \in \mathbb{R}_+ \) and \( q_t \in \mathbb{R}_+ \), respectively. The leading shareholder chooses the amount of corporate debt to be held by the firm through the bond asset, \( b_{f,t} \in [-\infty, 0] \). Correspondingly, the bond market chooses to absorb firm’s corporate debt through the variable \( b_{bm,t} \in [0, +\infty] \). The leading shareholder is also responsible for decisions on the liability proportion to default on the bond asset and in firm’s dividend distribution, \( \gamma_t \in [0, 1] \) and \( \delta_t \in [0, 1] \), respectively. As can be inferred from constraints (4) and (5) below, both default variables are actually predetermined since the amount of previous payment commitments on bonds and the state realization establish whether firm’s cash flow generation is sufficient for meeting debtors’ claims, and, if so, the amount left for shareholders is given. It is zero otherwise - equity dividend income is a residual claim solely related to firm’s cash flows and therefore limited liability is assumed. Initial bond holdings are null. Information about default on bond payment commitments is recorded in \( h_t \in \{0, 1\} \), which is initially set equal to one and is set equal to zero in the period concomitant to this type of default occurrence and henceforth, for any \( \gamma_t > 0 \).

Furthermore, when default on corporate bonds occurs the leading shareholder can issue no further debt. In every period with a clean history of default (\( h_t = 1 \)) her bond sales are restricted to

\[
b_{f,t} \geq K_t = \max_{s_{t+1}} \{d_{f,t+1}(s_{t+1}, s_t, \ldots, s_1)\},
\]

which is in fact an innocuous restriction since on the margin a sale of a unit of the bond asset promises nothing in any event next period. In addition, assets are not traded in the two final periods so that in the last period the firm dividend share she has the right to receive serves as collateral for a possible default in the previous period, which would not be penalized otherwise. It may be interpreted as the leading shareholder’s house together with a violation of limited liability after the firm ends its activities.

Two auxiliary sets are defined, \( L^{en} = \{e, b, f\} \) and \( L^{em} = \{e, f\} \), for purposes clarified below. Leading shareholder’s choices in each time-event (except for the two final dates when there is no trading in bonds and Lucas trees) consist in the set
\{c_{i,t} \in \mathbb{R}_+, \{\sigma_{i,t} \in [0,1]\}_{t \in T}, b_{f,t} \in [-\infty,0], \gamma_t \in [0,1], \delta_t \in [0,1]\}. \text{ They must satisfy the following set of constraints:}

\begin{align*}
c_{i,t} + \sum_{l \in L^i} p_{l,t} \sigma_{i,t} + \sigma_{f,i,t-1} q_l b_{f,t} & \leq (1 - \gamma_t) b_{f,t-1} + \sum_{l \in L^e \setminus \{f\}} \sigma_{i,t-1} (p_{l,t} + d_{i,t}) + \\
& + \sigma_{f,i,t-1} [p_{f,t} + h_{t-1} (1 - \delta_t) d_{f,t}], \quad (2)
\end{align*}

\begin{align*}
b_{f,t} & \geq h_t K_t. \quad (3)
\end{align*}

Therefore, after default occurs the leading shareholder (and the other shareholder as well, as shown below) is no longer entitled to the cash flows the investments are generating. Proceeds from debt issues fully accrue to shareholders immediately, since no further investments are possible. The auxiliary sets $L^i, i \in \{en, em\}$, were introduced in order to prevent trading in Lucas trees between the equity market and the bond market. Conditional on $h_{t-1} = 1$, we have the following additional constraints:

\begin{align*}
[-b_{f,t-1} - d_{f,t}]^+ & = -\gamma_t b_{f,t-1}, \quad (4)
\end{align*}

\begin{align*}
[d_{f,t} + b_{f,t-1}]^+ & = (1 - \delta_t) d_{f,t}. \quad (5)
\end{align*}

The notation $[x]^+$ denotes the positive part of $x$, i.e., $[x]^+ = \max(x, 0)$. The first constraint guarantees that bond’s associated transfers are fully executed only when they do not surpass firm’s cash flow proceeds. Otherwise, default is enforced in the exact amount of the shortfall - this separates the firm as a legal entity from its owners since the leading shareholder can not use her income from other sources to pay firms claims; besides, it is being assumed full enforcement of bondholders’ claims when cash flow generation is sufficient. In addition, in the contractual penalty adopted, law authorities assure creditors the right to receive all future firm’s proceeds (with possible losses) in the event of default on their liabilities - see equation 7, below. The constraint adopted here does not explicitly prevent the entrepreneur from trading her share on the firm after default on corporate debt. However, her share on the associated Lucas tree will be worthless since creditors will be entitled to all of the tree’s yields; moreover, buying new shares will grant her no benefits.

In turn, the other shareholder (either the entrepreneur or the equity market) choices in each time-event (except for the two final dates when there is no trading in bonds and Lucas trees) consist in the set $\{c_{i,t} \in \mathbb{R}_+, \{\sigma_{i,t} \in [0,1]\}_{t \in T}\}$. They must satisfy the following set of constraints:
\[ c_{i,t} + \sum_{l \in L} p_{i,t} \sigma_{l,i,t} + \sigma_{f,i,t-1} q_{t} b_{f,t} \leq \sum_{l \in L \setminus \{f\}} \sigma_{l,i,t-1} (p_{i,t} + d_{l,t}) + \sigma_{f,i,t-1} [p_{f,t} + h_{t-1} (1 - \delta_{t}) d_{f,t}]. \quad (6) \]

Finally, the functions \( \eta_{t} : S^{t} \rightarrow [0, 1] \) capture recovery rates corresponding to institutional efficiency in assuring creditor’s rights, losses due to poor allocation of resources, or both. Bond market choices in each time-event (except for the two final dates when there is no trading in bonds and Lucas trees) consist in the set \( \{c_{bm,t} \in \mathbb{R}_{+}, b_{bm,t} \in [0, +\infty], \sigma_{b,bm,t} \in [0, 1]\} \). They must satisfy the following set of constraints:

\[ c_{bm,t} + p_{b,t} \sigma_{b,bm,t} + q_{t} b_{bm,t} \leq (1 - \gamma_{t}) b_{bm,t-1} + \sigma_{b,bm,t-1} (p_{b,t} + d_{b,t}) + (1 - h_{t-1}) \eta_{t} d_{f,t}, \quad (7) \]

\[ b_{bm,t} \leq -h_{1} K_{t}. \quad (8) \]

In the numerical equilibria computed, all agents optimize and the following market clearing conditions hold:

\[ \sum_{i \in 1} (\sigma_{c,i,t}, \sigma_{b,i,t}, \sigma_{f,i,t}) = (1, 1, 1), t \in T \setminus \{T - 1, T\}, \quad (9) \]

\[ \sum_{i \in 1} c_{i,t} = \sum_{l \in L} d_{l,t}, t \in T, h_{t} = 1 \quad (10) \]

\[ \sum_{i \in 1} c_{i,t} = \sum_{l \in L \setminus \{f\}} d_{l,t} + \eta_{t} d_{f,t}, t \in T, h_{t} = 0 \quad (11) \]

\[ b_{f,t} = b_{bm,t}, t \in T \setminus \{T - 1, T\}, h_{t} = 1. \quad (12) \]

### 3 Parameterization

For the dynamic general equilibrium solution a Matlab code was developed. Equilibria identification is accomplished using a Quasi-Newton method with line-search for different sets of prices. Using cubic-splines the solutions to an enlarged set are approximated with polynomial functions. The information generated by the piecewise
polynomial interpolation made is grouped so that a verification is performed of what set of prices best approximates markets' clearance.

The economy extends over three periods and it is assumed a \((5 \times 5)\) Markov transition matrix with all entries equal to 0.2. All agents have the same preferences, represented by a CRRA utility function. Dividends yielded by the tree representing the firm are considerably smaller than those yielded by the trees initially owned by the agents representing the equity market and the bond market. Therefore, in the exchanges that are computed, asset prices are (almost) not influenced by choices on the firm capital structure.

Equity is a share on the firm tree and, as mentioned above, its dividends are interpreted as the cash flow generated by previous investment made by the entrepreneur. The initial quantity in the baseline case is two thousand times smaller than other tree’s dividends, representing dividend and bond payments from all other existing firms. In addition, below are presented the factors by which the dividend yield from the previous period is multiplied, giving dividend values for each of the five subsequent nodes; these are denominated dividend factors.

Four distinct sets of parameters are analyzed. First, there is case one which is the baseline computation. In case two, the size of the firm’s cash flows is increased by a factor of ten. In case three, cash flows become riskier. Finally, in case four the entrepreneur’s preferences display a reduced degree of risk-aversion. The following set of tables presents the actual values.

### Table Set - Dividend factors, initial values and risk-aversion

<table>
<thead>
<tr>
<th>Firm dividend factors:</th>
<th>cases 1,2,4</th>
<th>0.50</th>
<th>0.70</th>
<th>1.00</th>
<th>1.30</th>
<th>1.50</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>case 3</td>
<td>0.30</td>
<td>0.60</td>
<td>1.00</td>
<td>1.40</td>
<td>1.70</td>
</tr>
<tr>
<td>Total equity dividend factors:</td>
<td>all cases</td>
<td>0.70</td>
<td>0.85</td>
<td>1.00</td>
<td>1.15</td>
<td>1.30</td>
</tr>
<tr>
<td>Initial dividend values:</td>
<td>cases 1,3,4</td>
<td>10</td>
<td>20,000</td>
<td>20,000</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>case 2</td>
<td>13</td>
<td>20,000</td>
<td>20,000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Constant relative risk aversion:</td>
<td>cases 1,2,3</td>
<td>0.50</td>
<td>0.50</td>
<td>0.50</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>case 4</td>
<td>0.20</td>
<td>0.50</td>
<td>0.50</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### 4 Numerical Results

The numerical results obtained from the set of parameters described in the previous section are presented below in three graphics and analyzed in the following. These
are fairly in accordance with the empirical findings presented in Rajan and Zingales (1995), which indicates a positive correlation between presence of debt in a firm’s capital structure and firm size, and with the consensus outlined in Harris and Raviv (1991), which additionally asserts a negative correlation between financial leverage and cash flow volatility.

As the first graph below illustrates, with higher cash flows the entrepreneur chooses a capital structure with more financial leverage (debt value divided by equity value). Since there may be a commitment to larger fixed payments without incurring in default - that would happen during the realization of certain events if the firm’s cash flow situation was worse -, there is an increase in the value of firm’s bond issues relative to the value of total equity. Second, there is a decline in this ratio when firm’s cash flows are riskier. This is because of default avoidance on future unfavorable uncertainty outcomes. Third, with a smaller degree of risk aversion, the entrepreneur issues less bonds and sells a smaller amount of stocks (the latter can be seen in the second graph, below). This is due to lower diversification needs. In the computations performed these quantities together with equilibrium prices implied a relative fall in leverage.

In turn, the final graph displays the equilibrium prices of both instruments in all four cases. In case 2, where the firm’s cash flows are larger, the equity price naturally rises. When cash-flows are riskier, equity price is similar to the one computed in the benchmark case and this can be explained by the leverage adjustment that occurred. Finally, when the entrepreneur is closer to being risk-neutral, prices remain almost unchanged.

In all cases both instruments are used and the firm’s financial manager and partial owner chooses among the available combinations of assets the one that results in the most preferred consumption stream to her. On the one hand, selling equity provides safety for the risk-averse agent who wishes to pay a fixed fraction of the firm’s cash flow generation at each point in time without any further commitments. On the other, if she is not completely risk-averse she may choose to lever the firm’s earnings by issuing bonds, modifying the payoff she receives from the investment’s fraction she owns. Thus, in a world without a tax authority, on the right side of a balance sheet we simply have a collection of individuals engaging in contractual relations that provide a better allocation of their wealth across time and through states of nature. With imperfect markets, both bond and equity instruments are valuable means for that.
5 Conclusion

A broad variety of methodological frameworks along with equilibrium concepts have disputed the preferences of corporate researchers and suited their different needs according to the focus of their study. Recently, Tirole (2006) replicated an enormous set of results from the literature with models based on the theory of contracts. If we adopt the view that the firm is a legal entity composed of contractual relations among individuals it is easy to see why such frameworks accomplish the task so successfully. Nevertheless, there is a drawback to this approach which lies on the fact that firm’s contractual basis extends over long periods of time, what implies that one foresees a vast set of uncertainty outcomes and this has an impact on the nature of the contracts firmed. Theory of contracts is still inadequate to deal with this sort of complexity while an asset pricing type of framework is better suited for the task.

In fact, from all the hypotheses made in the irrelevance proposition, the one of perfect markets was the major departure taken in this paper. The modeling technology available almost fifty years ago relied heavily on this assumption. In the past years, though, we have seen great progress on the asset pricing literature with growing research on models with incomplete markets, heterogeneous agents, trading constraints and, lately, large use of models which allow for default on liabilities. These features permit us to replicate contractual relations among individuals who engage in economic activities under a corporate legal basis.
References


