Tests of Conditional Asset Pricing Models in the Brazilian Stock Market*

Marco Bonomo

Graduate School of Economics

Getulio Vargas Foundation

René Garcia

Département de sciences économiques, CIRANO and C.R.D.E.

Université de Montréal

First version: March 1997

This version: July 1999

Keywords: Conditional CAPM, Conditional APT, Efficiency of Markets, Time-Varying Risk and Returns.

*Address for correspondence: Département de Sciences Économiques et C.R.D.E., Université de Montréal, C.P. 6128, Succ. A, Montréal, Québec, H3C 3J7, Canada. Financial assistance by the PARADI Research Program funded by the Canadian International Development Agency (CIDA) is gratefully acknowledged. The first author would also like to thank Pronex for financial support and CNPq (National Research Council of Brazil) for financial assistance. Invaluable research assistance has been provided by Stéphanie Lluís.

BB 64662-G

BIBLIOTECA MARIO HENRIQUE SIMONSEN FUNDAÇÃO GETÚLIO VARGAS
<u>F2534/99</u> 10.08.99

D

211,00

Acervo. 52153

ID: 89213

Abstract

In this paper, we test a version of the conditional CAPM with respect to a local market portfolio, proxied by the Brazilian stock index during the period 1976-1992. We also test a conditional APT model by using the difference between the 30-day rate (Cdb) and the overnight rate as a second factor in addition to the market portfolio in order to capture the large inflation risk present during this period. The conditional CAPM and APT models are estimated by the Generalized Method of Moments (GMM) and tested on a set of size portfolios created from individual securities exchanged on the Brazilian markets. The inclusion of this second factor proves to be important for the appropriate pricing of the portfolios.

1 Introduction

Can we explain the evolution of cross-sectional expected stock returns in an emerging market like Brazil when we know that it has lived through a lot of macroeconomic turmoil in the last two decades? Behind such a question lies the widespread belief among international analysts that market efficiency is unlikely to prevail under these conditions. In particular, one would like to know if the models currently used to study expected returns in the United States and other developed markets perform well in emerging markets. Are the empirical test results obtained in the well-studied US markets transferable to an emerging market like Brazil?

In this paper, we use firm-level Brazilian stock return data to test unconditional and conditional versions of the CAPM and the APT, which are the two models that have been used extensively to explain cross-sectional expected returns in developed markets. Surprisingly enough, we show that both types of models explain well the expected returns of three size portfolios formed with individual stocks. For the conditional APT, we use a model with two factors, the market portfolio and an inflation risk factor, which are proxied respectively by the excess returns on the IBOVESPA stock index and the excess return of a 30-day bond over the overnight rate. Of course, inflation risk comes as natural factor for an economy like Brazil, where two-digit monthly inflation rates prevailed until recently.

In choosing a Brazilian stock index as the market portfolio, we assume implicitly that the Brazilian market is segmented. Another approach would be to adopt the perspective of an international investor diversifying his portfolio worldwide. If enough investors diversify internationally their portfolios, the market will move towards integration, and expected returns in Brazil will be well described by the country's world risk exposure, the covariance of the Brazilian stock returns with the world market portfolio. This is the view taken by Harvey (1995) in a recent study on emerging markets. The author tests a dynamic factor asset pricing model in which the risk loadings are measured with respect to the world market return in excess of a risk-free asset return. The results for Brazil show that the beta with the world market return is not significantly different from zero and that the unexpected part of the world risk premium is related to local market information such as the local dividend yield or a local interest rate. This suggests that the Brazilian market is segmented from the world market.

Contrary to the US market, the risk-free rate cannot be a nominal one-month Treasury bill rate. Indeed, the inflation surprises in one month can be quite substantial and create a sizable gap between ex-ante and ex-post real rates. This justifies the choice of an overnight interbank rate as the risk-free interest rate. Since there is considerable inflation risk in a predetermined monthly nominal rate, we chose a 30-day certificate of deposit rate to capture such a risk¹. This does not seem like a long maturity, but it is the longest maturity bond that was traded in the domestic market during the 1976-1992 period that we consider in our study. By choosing this asset as a risk factor instead of a direct measure of the inflation surprise, we avoid two important difficulties: specifying an econometric model for the inflation process affected by many stabilization plans and estimating the inflation risk premium in a conditional setting.

We extend and test the conditional CAPM proposed by Bodurtha and Mark (1991). The beta of a portfolio of assets is defined as the conditional covariance between the forecast error in the portfolio return and the forecast error of the market return divided by the conditional variance of the forecast error in the market return. Both components of the conditional beta are assumed to follow an ARCH process, a concept of conditional heteroskedasticity introduced by Engle (1982). Expected returns for the market also follow an autoregressive structure. This modeling choice can be rationalized in two ways. First, looking at the sample statistics in Table 1, one sees considerable autocorrelation in the squared market returns series, indicating the presence of ARCH effects. Second, the use of autoregressive processes might provide estimates that are more robust to structural change. Ghysels (1998) and Garcia and Ghysels (1998) show that models similar to Harvey (1991, 1995) or Ferson and Korajczyk (1995), where the returns are projected on a set of variables belonging to the information set such as a term spread, a risk spread, or a dividend yield, suffer from instability in the projection coefficients and therefore lead to systematic mispricing of the risk factors. By using an autoregressive structure, we hope to better forecast the returns and their variances and covariances.²

¹Of course it captures also the risk of a change in the short real rate. We believe that this risk is of second order with respect to the inflation risk.

²Our approach assumes a fixed regime of segmentation throughout the period. The concept of time-varying integration proposed by Bekaert and Harvey (1995) could address the problem of projection coefficient instability.

We estimate our models by the generalized method of moments (GMM) and use diagnostic tests to verify ex-post if the residuals are orthogonal to various variables in the information set of the agents not used in the estimation. In particular, we verify whether the residuals are orthogonal to a January dummy to account for a possible January effect put forward in the US market studies, a dividend yield and lags of the risk-free asset.

In our two-factor conditional model, we find that the average beta with respect to the market portfolio is increasing with portfolio size. This is in contrast with the evidence on US markets where betas are decreasing with firm size (see Fama and French (1992)). The average 30-day bond betas are negative for all portfolios. Since bond excess return are negatively correlated with inflation surprises, portfolio performance is positively affected by inflation innovations. Our results show that large firms offer the best insurance against inflation. The importance of the inflation factor is illustrated by the difference in predicted average returns for large firms between the two unconditional models we estimate. The unconditional CAPM model underpredicts average excess returns for the large-firm portfolio. If there is a size effect in Brazil it is a large-firm effect, contrary to what is observed on the US stock market. This effect disappears when the excess bond return is added as a second factor in the unconditional APT. The predicted excess average returns for the large firms is now 6.2%, very close to the observed 6.9%. The conditional two-factor model also predicts well the average portfolio returns while tracking better the dynamics of the risk measures and of the expected returns.

The rest of the paper is organized as follows. Section 2 presents the conditional CAPM model, its econometric specification, and the estimation results. Section 3 mirrors section 2 for the APT model and ends with diagnostic tests of the model specification. Section 4 concludes.

2 The Conditional CAPM

2.1 The Model

The conditional CAPM can be stated as follows:

$$E[r_i(t) | \Omega_t] = \beta_{it} E[r_M(t) | \Omega_t] \quad (1)$$

where $r_i(t)$ is the one-period return on portfolio i in excess of the risk-free asset return, $r_M(t)$ the excess return on the market portfolio, and β_{it} is given by the following expression:

$$\beta_{it} = \frac{Cov[r_i(t), r_M(t) | \Omega_t]}{Var[r_M(t) | \Omega_t]} \quad (2)$$

In this version of the CAPM, all moments are made conditional to the information available at time t represented by the information set Ω_t . Many asset pricing studies on the US stock markets (Ferson and Harvey (1991), for example) have shown that allowing the moments to vary with time is essential, since there is evidence that both the beta, the ratio of the covariance to the variance, and the price of risk $E[r_M(t) | \Omega_t]$ are time-varying. This is even more essential in emerging markets, where important and rapid changes in economic and political conditions can translate into considerable variations in the factor loadings. In order to specify model (1) for estimation, we decompose the returns into a forecastable part and an unforecastable part, namely:

$$r_i(t) = E[r_i(t) | \Omega_t] + u_i(t) \quad (3)$$

$$r_M(t) = E[r_M(t) | \Omega_t] + u_M(t) \quad (4)$$

where $u_i(t)$ and $u_M(t)$ are forecast errors orthogonal to the information in Ω_t . Equation (1) can therefore be rewritten as follows:

$$E[r_i(t) | \Omega_t] = \frac{E[u_i(t) u_M(t) | \Omega_t]}{E[u_M(t)^2 | \Omega_t]} E[r_M(t) | \Omega_t] \quad (5)$$

To obtain a set of moment conditions suitable for GMM estimation, we need to specify parametric models for the expectations on the right hand side of (5). As mentioned in the introduction, following Bodurtha and Mark (1991), we choose to specify autoregressive processes for each of the expectations:

$$E[u_M(t)^2 | \Omega_t] = \delta_{0_M} + \sum_{j=1}^{k_{\sigma_M}} \delta_{j_M} u_M(t-j)^2 \quad (6)$$

$$E[u_i(t) u_M(t) | \Omega_t] = \delta_{0_i} + \sum_{j=1}^{k_i} \delta_{ji} u_i(t-j) u_M(t-j) \quad (7)$$

$$E[r_M(t) | \Omega_t] = \alpha_{0_M} + \sum_{j=1}^{k_M} \alpha_{j_M} r_M(t-j) \quad (8)$$

The number of lags k_{σ_M}, k_i, k_M to be included in each of the equations above remains an empirical issue, given the constraint imposed by the number of available observations. The final form of the moment conditions that will be used for GMM estimation can therefore be written as follows:

$$\begin{aligned} r_M(t) &= \alpha_{0_M} + \sum_{j=1}^{k_M} \alpha_{j_M} r_M(t-j) + u_M(t) \\ u_M(t)^2 &= \delta_{0_M} + \sum_{j=1}^{k_{\sigma_M}} \delta_{j_M} u_M(t-j)^2 + v_M(t) \\ u_i(t) u_M(t) &= \delta_{0_i} + \sum_{j=1}^{k_i} \delta_{ji} u_i(t-j) u_M(t-j) + v_{iM}(t), i = 1, \dots, N \\ r_i(t) &= \frac{\delta_{0_i} + \sum_{j=1}^{k_i} \delta_{ji} u_i(t-j) u_M(t-j)}{\delta_{0_M} + \sum_{j=1}^{k_{\sigma_M}} \delta_{j_M} u_M(t-j)^2} [\alpha_{0_M} + \sum_{j=1}^{k_M} \alpha_{j_M} r_M(t-j)] + u_i(t), i = 1, \dots, N \end{aligned} \quad (9)$$

where $v_M(t)$ and $v_{iM}(t)$ are the conditional forecast errors corresponding to the second-moment conditions.

2.2 Data

The IFC Emerging Markets Data Base of the World Bank provides data on stock prices and other financial variables for both the stock index and individual stocks in a series of developing and newly industrialized countries. For the sample of individual securities provided for Brazil, we selected a total of 25 common shares (see list of securities in Appendix 1) which were listed on the IBOVESPA stock exchange from 1976:1 to 1992:12. To test the model of section 2.1 we could theoretically use the returns on the individual securities, but for estimation purposes we have to limit the number of parameters. We follow common practice in grouping the securities into portfolios and testing the model on a small set of portfolios. Given the limited number of available firms, we decided to form three size portfolios, where size refers to the capitalization value of the firms. We create these portfolios by first value-ranking the returns of the individual equities in each month and then by separating the returns into three size (value) quantiles. Within each quantile, we weight each return by the capitalization value of the firm relative to the total capitalization value of the firms in the quantile. The returns are computed in the local currency in excess of the overnight rate.

Table 1 provides some sample statistics on both the returns and the excess return series for the Brazilian stock index. Although the mean of the raw local currency return series is quite high (159% in annualized returns) because of the high inflation that occurred during our sample period, the mean excess return is much smaller (29%) but still high compared to the returns posted in average by developed markets. The squared series show a very strong autocorrelation, a usual feature in financial time series. All series depart also strongly from normality, as indicated mainly by the excess kurtosis statistic.

Table 2 reports some basic statistics for the three portfolios. Portfolios 1, 2, and 3 represent the small, medium, and large firms respectively. As can be seen on Table 2, the mean of the portfolios increases with size. This feature is clearly opposite to what is observed in the US markets, where the returns are negatively correlated with size. In addition, the variance of the portfolios decreases with size. These two features taken together would clearly not be compatible with an asset pricing model where risk would be measured by the variance of an asset or a portfolio, since a higher risk should lead to a higher return. It could however be compatible with the CAPM model if the small firm returns covary less with the market than the returns of the large firms.

As we will see below it seems to be the case.

Figures 1 and 2 show the graphs of the market excess return series and the three portfolio excess returns series respectively. All graphs show the presence of strong autoregressive conditional heteroskedasticity. In our estimation, we consider specifications to account for this feature of the data.

2.3 Estimation Results

To carry out the estimation, we use the Generalized Method of Moments (GMM) proposed by Hansen (1982) based on the Newey-West method (Newey and West, 1987) for estimating the weighting matrix. To apply the GMM method, the projection variables used for the first and second conditional moments of the returns are lagged values of the dependent variable in each equation. Another common method is to use variables such as a lagged risk or term spread or a dividend yield variable. All variables are good candidates since they are in the information set but autoregressions for both the first and second moments might provide estimates that are more robust to structural change.

To start, we estimate the simplest model, a constant beta CAPM, where all parameters in (9), except $\alpha_0, \delta_{0_M}, \delta_{0_i} (i = 1, 2, 3)$ are constrained to be zero. Estimation results are reported in Table 3. The covariance parameter of the large-firm portfolio is about two times as large as the covariance parameter for the portfolios of medium and small size firms. Since the variance of the market is constant, the calculated betas reflect exactly the same pattern. Therefore, the higher mean return of portfolio 3 can be rationalized by its substantially higher β : 1.6, compared to values close to one for the two other portfolios. However, the predicted average return for this portfolio (5.05%) falls short of the actual mean return of 6.9%. If there is a size effect in the Brazilian market, it is the reverse of the small size effect put forward by studies on the cross-sectional expected returns in the US markets (see, for example, Fama and French (1992)). The average returns for the small firms are overvalued by the model, while the large-firm returns are undervalued.

According to the t-statistics and the test of orthogonality conditions³,

³The J-statistic tests for the overidentifying restrictions imposed by the model. It is computed as T times the value of the minimized value of the objective function, $b = \arg \min \left[\frac{1}{T} \sum_{t=1}^T f_t(\beta) \right]' \Sigma_T^{-1} \left[\frac{1}{T} \sum_{t=1}^T f_t(\beta) \right]$

the model seems to be well supported by the data. A word of caution is in order. Since the number of moment conditions is large with respect to the number of observations, the conventional asymptotic inference might lose its validity⁴.

Next, we introduce lagged terms in the mean, variance and covariance equations to estimate a conditional form of the CAPM. The specification chosen allows for ARCH effects in the market variance (a feature strongly present in the data) and in the portfolio covariances with the market. It is rich enough to test for restricted versions of interest, such as a constant beta model, a constant market price of risk or a constant conditional market variance. We have limited the autoregressions to a maximum of two lags in each of the equations. Overall, there are 14 parameters for 24 equations, which implies a $\chi^2(10)$ distribution for the J-test statistic. This specification is similar to the specification used in Bodurtha and Mark (1991). The parameter estimates for the Conditional CAPM with ARCH effects are shown in Table 4. Although all parameter estimates seem to be significantly different from zero, except in the conditional mean equation, the warning about the high number of moment conditions should be kept in mind. The mean betas have all increased with respect to the unconditional CAPM and are now in line with the ranking of mean excess returns of the three portfolios. This conditional model, which is aimed at capturing better the dynamics of the risk measures and the expected returns, tends to underpredict the average returns of the three portfolios⁵.

To test for the restricted versions of the model mentioned above, we perform Wald tests. For the constant beta model, we test whether all parameters but the constants in each equation are equal to zero. The Wald statistic in

where $\Sigma_T^{-1} = \frac{1}{T} \sum_{t=1}^T f_t(b) f_t(b)'$. The vector $f_t(\beta)$ is constructed by multiplying the error term in each equation by the vector of instruments. Under the null of a correctly specified model, this statistic is distributed asymptotically as a chi-square with $2(N+1) \times q - (N(k_i + 1) + 2 + k_M + k_{\sigma_M})$ degrees of freedom.

⁴Koenker and Machado (1996) show that for the estimation of a linear model with general heteroskedasticity that $q^{5/2}/n \rightarrow 0$, where q is the number of moment conditions and n the number of observations, is a sufficient condition for the validity of conventional asymptotic inference about the GMM estimator. Indeed, using only 8 moment conditions (with only a constant as instrument) instead of 24, we obtain a p-value of 0.05 for the J statistic and the t-values drop to magnitudes of 3 and 4.

⁵The average expected returns are obtained by multiplying the β_{it} by the time t expected return on the market and taking the average over the sample.

this case will be distributed as a $\chi^2(10)$ variable. For the fixed market price of risk, we test whether the coefficients other than the constants in the market conditional mean and variance equations are zero. This is a $\chi^2(4)$ test. Finally, the test for a constant conditional market variance is a test for the equality to zero of δ_{1M} and δ_{2M} , which is a $\chi^2(2)$ test. All these restricted versions of the model are overwhelmingly rejected (at less than 0.01 in all cases). The fact that the mean betas are all greater than one suggests that an important risk dimension is missing in the model. This seems to be confirmed by the time series behavior of the portfolio betas (shown in figure 3) which appear, if anything, to be more volatile during the first half of the sampling period, while the excess returns are much more volatile during the second half. Since Brazil was affected by a high and variable inflation during the second half of the period under study, we explore in the next section a conditional two-factor model, where the second factor aims at capturing the inflation risk.

3 A Conditional Two-Factor Model

In this section we specify and estimate an extension of the conditional CAPM estimated in section 2, where a second asset (a nominal bond called CD) is added as a second risk factor. Since the nominal return of this asset is fixed for a month, its real return is affected by the unforecastable component of inflation. It will therefore capture an important original risk factor in a high inflation economy, which would not be reflected fully in the market portfolio because its return is not fixed. This second factor contains also a real interest rate risk, associated with an unexpected change in monetary policy, but we believe that the variability of inflation is such that most of the nominal interest rate risk is due to inflation risk.

3.1 The Model

We assume the following conditional two-factor model for excess returns:

$$E[r_i(t) | \Omega_t] = \beta_{iM}(t)E[r_M(t) | \Omega_t] + \beta_{iF}(t)E[r_F(t) | \Omega_t] \quad (10)$$

where r_F is the excess return of the thirty-day CD over the overnight rate. We assume that the factors are conditionally uncorrelated and obtain the following expressions for the conditional betas:⁶

$$\begin{aligned}\beta_{iM}(t) &= \frac{Cov_t[r_i(t), r_M(t) | \Omega_t]}{Var_t[r_M(t) | \Omega_t]} \\ \beta_{iF}(t) &= \frac{Cov_t[r_i(t), r_F(t) | \Omega_t]}{Var_t[r_F(t) | \Omega_t]}\end{aligned}$$

While keeping the decomposition of the market portfolio return in (4), we also breakdown the return of the CD into a forecastable and an unforecastable term:

$$r_F(t) = E[r_F(t) | \Omega_t] + u_F(t) \quad (11)$$

Then, the betas can be rewritten as follows:

$$\begin{aligned}\beta_{iM}(t) &= \frac{E[u_i(t)u_M(t) | \Omega_t]}{E[u_M(t)^2 | \Omega_t]} \\ \beta_{iF}(t) &= \frac{E[u_i(t)u_F(t) | \Omega_t]}{E[u_F(t)^2 | \Omega_t]}\end{aligned}$$

We maintain the autoregressive specification of section 2 for the conditional expectation of the market return, for the conditional variance of the forecast error and for the conditional covariance between the forecast errors in predicting the market portfolio return and each individual asset return. Similarly, we also specify an autoregressive process for the additional variables that appear as a consequence of the addition of a second factor:

⁶This assumption simplifies the expressions and reduces the number of parameters to be estimated but also seems to be supported by the data. The unconditional correlation between the factors found in the data is low (-0.08), and the projections of the cross-products of the error terms u_{Mt} and u_{Ft} appear to be orthogonal to various variables in the information set.

$$\begin{aligned}
E[r_F(t)|\Omega_t] &= \alpha_{0M} + \sum_{j=1}^{k_F} \alpha_{jM} r_F(t-j) \\
E[u_F(t)^2|\Omega_t] &= \delta_{0F} + \sum_{j=1}^{k_{\sigma_F}} \delta_{jM} u_F(t-j)^2 \\
E[u_i(t)u_F(t)|\Omega_t] &= \delta_{iF} + \sum_{j=1}^{k_{iF}} \delta_{ji} u_i(t-j) u_F(t-j)
\end{aligned}$$

We finally obtain the following moment conditions for the GMM estimation:

$$\begin{aligned}
r_M(t) &= \alpha_{0M} + \sum_{j=1}^{k_M} \alpha_{jM} r_M(t-j) + u_M(t) \\
u_M(t)^2 &= \delta_{0M} + \sum_{j=1}^{k_{\sigma_M}} \delta_{jM} u_M(t-j)^2 + v_M(t) \\
r_F(t) &= \alpha_{0F} + \sum_{j=1}^{k_F} \alpha_{jF} r_F(t-j) + u_F(t) \\
u_F(t)^2 &= \delta_{0F} + \sum_{j=1}^{k_{\sigma_F}} \delta_{jF} u_F(t-j)^2 + v_F(t) \\
u_i(t) u_M(t) &= \delta_{0iM} + \sum_{j=1}^{k_{iM}} \delta_{jiM} u_i(t-j) u_M(t-j) + v_{iM}(t), i = 1, \dots, N \\
u_i(t) u_F(t) &= \delta_{0iF} + \sum_{j=1}^{k_{iF}} \delta_{jiF} u_i(t-j) u_F(t-j) + v_{iF}(t), i = 1, \dots, N \\
r_i(t) &= \frac{\left(\delta_{0iM} + \sum_{j=1}^{k_{iM}} \delta_{jiM} u_i(t-j) u_M(t-j) \right)}{\left(\delta_{0M} + \sum_{j=1}^{k_{\sigma_M}} \delta_{jM} u_M(t-j)^2 \right)} \left[\alpha_{0M} + \sum_{j=1}^{k_M} \alpha_{jM} r_M(t-j) \right] \\
&\quad + \frac{\left(\delta_{0iF} + \sum_{j=1}^{k_{iF}} \delta_{jiF} u_i(t-j) u_F(t-j) \right)}{\left(\delta_{0F} + \sum_{j=1}^{k_{\sigma_F}} \delta_{jF} u_F(t-j)^2 \right)} \left[\alpha_{0F} + \sum_{j=1}^{k_F} \alpha_{jF} r_F(t-j) \right] + u_i(t), i = 1, \dots, N
\end{aligned} \tag{12}$$

3.2 Estimation Results and Comparison

As before, we first estimate a model with constant factor loadings where all parameters, except the ones with subscript zero, are constrained to be zero. Table 5 reports the results. The magnitude of the covariance parameters is small in absolute value, due to the small variance of the CD excess return, but large in relative terms: the CD betas of the second and third portfolios, reported at the bottom of Table 5, are -5 and -30, respectively. Their negative

values and the pattern followed by their magnitudes, which increases with the capitalization value, indicate that the high-value portfolios offer the best hedge against the inflation risk. It should be noticed that, by adding a second factor, all the market portfolio betas become slightly lower than one. The portfolio average returns predicted by this model become remarkably close to the actual mean returns.

Table 6 reports the estimation results of the conditional two-factor model with ARCH effects. The results show that ARCH effects play an important role. First, it should be noticed at the bottom of the table that the average market portfolio betas change in an important way: they are now all less than one and they increase with the size of the portfolio. The market beta for the large firm portfolio is almost twice as big as the one for the small firm portfolio. The average returns predicted by this model have improved compared to the CAPM with ARCH effects. The time-varying betas are plotted in Figures 4 and 5. The betas of the three portfolios with respect to the market portfolio become much more volatile after 1987. This coincides with a period of higher and more volatile inflation, suggesting that the ARCH effects are important mainly because of the volatility of inflation. This is in contrast with the variability of the betas produced by the CAPM model (see figure 3), where no clear pattern emerges. Because of the lack of reliability of the J-statistic with these many moment conditions, we perform in the next section various diagnostic tests on the residuals to assess the adequacy of the model.

3.3 Diagnostic Tests

We have already mentioned that the J-statistic could be misleading because of the large number of moment conditions used for estimation compared with the available number of observations. Yet, many more orthogonality conditions could be used for estimation that would be consistent with the implications of the asset pricing models we are testing. Newey (1985) proposed a test (called CS test) of orthogonality conditions not used in estimation but implied by the model. Intuitively, this test verifies whether the residuals in the various equations used for estimation are orthogonal to other variables in the information set that have not been used as instruments. It can therefore be seen as a diagnostic test of the specification maintained as the null hypothesis.

The CS statistic is computed as follows:

$$CS = T [L_T g_T(b_T)]' Q_T^{-1} [L_T g_T(b_T)], \quad (13)$$

where:

$$g_T(b_T) = [g_{1T}(b_T)' g_{2T}(b_T)']'$$

with: $g_{iT}(b_T) = \frac{1}{T} \sum_{t=1}^T f_{iT}(b_T), i = 1, 2$.

The f_{1T} and f_{2T} are respectively the orthogonality conditions used and not used in estimation. The vector $f_{2T}(\cdot)$ of dimension s is formed by multiplying the residual by variables (say p) in the information set that have not been used in estimation. The rest of the variables defining the statistic CS are as follows:

$$\begin{aligned} L_T &= [0_{s \times (p+s)} : I_s], \quad S_{ij,T} = \frac{1}{T} \sum_{t=1}^T f_{it}(b_T) f_{jt}(b_T)', (i = 1, 2; j = 1, 2), \\ B_T &= (H'_{1,T} S_{11,T}^{-1} H_{1,T}) H'_{2,T}, \quad H_{i,T} = \frac{1}{T} \sum_{t=1}^T \frac{\partial}{\partial b} f_{iT}(b_T), (i = 1, 2), \\ Q_T &= S_{22,T} - S_{21,T} S_{11,T}^{-1} H_{1,T} B_T - B_T' H'_{1,T} S_{11,T}^{-1} S_{12} + H_{2,T} B_T, \end{aligned} \quad (14)$$

and b_T is the minimizer of $g_{1T}(\beta)' S_{11,T}^{-1} g_{1T}(\beta)$. The results of the diagnostic tests are reported in Table 7.

There is always some arbitrariness in choosing the information variables that should be orthogonal to the residuals, but since we chose an autoregressive specification it seems natural to test if we put enough lags. Therefore, we test first if the residuals are orthogonal to six of their own lags.

All residuals related to the market portfolio conditions appear to be serially uncorrelated. Not too surprisingly, this is not the case however for the residuals corresponding to the inflation conditions. There is strong evidence of remaining serial correlation in these residuals. Given the high persistence and complex dynamics (both in mean and variance) of the rates of inflation that Brazil experienced during this period, especially during the second part of the sample, long lags would be necessary to make these residuals uncorrelated.

Next, we test whether residuals from each equation are orthogonal to lagged returns: we use the corresponding portfolio excess returns for v_{iM}, v_{iF}

and u_i , the market portfolio excess returns for u_M and v_M , and the CD excess returns for u_F and v_F . The null hypothesis of orthogonality cannot be rejected, except for the u_F residual. Again, this is an indication that more lags are necessary in the mean equation for the CD excess returns to account for persistent inflation.

The other way to build conditional asset pricing models has been to use variables that are deemed to help predict excess stock returns and returns volatility, as in Harvey (1995) for example. Based on data availability, we select three of these variables, a January dummy (to test if there is a January effect in Brazil), the risk-free rate (in our case the overnight rate), and the dividend yield. Given that we chose to test an autoregressive conditional asset pricing model, it is a good way to test if we omitted some important economic variables in our information set. Overall, the test results show little evidence that we left some important information aside. The main failure comes from residuals in the CD variance equation, which confirms the results obtained with the other orthogonality tests.

4 Conclusion

In this paper, we test various conditional asset pricing models for the Brazilian stock market. Our best specification involves a two-factor model, where the equilibrium returns are determined by their covariances with the market portfolio and with a factor capturing inflation risk. The time series obtained for the portfolio betas seem to characterize well the evolution of risk during the estimation period and the predicted mean returns are in line with the actual. Without the inflation risk factor, the average returns of the large firms are underestimated, which suggests the presence of a large-firm effect, the opposite of what has been put forward in the US markets.

Our diagnostic tests tend to support the specification chosen, apart from the equations modeling the mean and the variance of the CD excess returns. The highly unstable behavior of the inflation rate during the second part of the sample, as well as its high persistence, makes it difficult to come up with an effective parsimonious model. The introduction of the “inflation” factor is however essential for the portfolio pricing. A more careful modeling of the CD equations, which for example would take into account the stabilization plans introduced during the period, might improve the results further. Cati,

Garcia, and Perron (1999) propose a time-series model for inflation accounting for various changes in regime brought about by the various stabilization plans introduced during the sampling period. Incorporating such changes in regime in risk factors could be a useful extension to our models.

References

- [1] Bekaert, G. and C.R. Harvey (1995), "Time-Varying World Market Integration," *Journal of Finance*, 50, 2, 403-44.
- [2] Bodurtha, J.M. and Nelson C. Mark (1991), "Testing the CAPM with Time-varying Risks and Returns," *Journal of Finance*, 46, 4, 1485-1505.
- [3] Engle, R. F.(1982), "Autoregressive Conditional Heteroskedasticity with Estimates of the Variance of United Kingdom Inflation," *Econometrica* 50, 987-1007.
- [4] Fama, E. F. and K. R. French (1992), "The Cross-section of Expected stock returns", *Journal of Finance*, 47, 427-465.
- [5] Ferson, W. E. and C. R. Harvey (1991), "The Variation of Economic Risk Premiums," *Journal of Political Economy* 99, 385-415.
- [6] Ferson, W. E. and R. A. Korajczyk (1995), "Do arbitrage pricing models explain the predictability of stock returns?", *Journal of Business*, 309-350.
- [7] Garcia R. and E. Ghysels (1998), "Structural Change and Asset Pricing in Emerging Markets," *Journal of International Money and Finance*, 17, 455-473.
- [8] Ghysels, E. (1998), "On Stable Factor Structures in the Pricing of Risk," *Journal of Finance*, 53, 549-573.
- [9] Hansen, L.P., (1982), "Large Sample Properties of Generalized Method of Moments Estimators," *Econometrica* 50, 1029-1054.
- [10] Harvey, C.R. (1991), "The World Price of Covariance Risk," *Journal of Finance* XLVI, 111-157.
- [11] Harvey, C.R. (1995), "Predictable Risk and Returns in Emerging Markets," *Review of Financial Studies*, 773-816.

- [12] Koenker, R. and J. A. Machado (1996), "GMM Inference when the Number of Moment Conditions is Large," Discussion Paper, University of Illinois.
- [13] Newey, W. K. and K. D. West (1987), "A Simple, Positive Semi-Definite, Heteroskedasticity-Consistent Covariance Matrix," *Econometrica*, 55, 703-708.
- [14] Cati, R. C., M. G. P. Garcia and P. Perron (1999), "Unit Roots in the Presence of Abrupt Governmental Interventions with an Application to Brazilian Data," *Journal of Applied Econometrics*, 14, 27-56.

Appendix 1

List of Securities Used to Form the Size Portfolios

Acesita-ON
Alpargatas-ON
Belgo-Mineira-ON
Brahma-ON
Brahma-OP
Brasil-ON (190.1)
Brasil-ON (190.2)
Brasmotor-PN
Docas-OP
Ford-OP
Klabin-ON
Light-OP
Lojas Am.-OP
Mannesmann-ON
Moinho Sant-ON
Moinho Sant-OP
Paranapanema-ON
Paul F. Luz-OP
Petrobras-ON
Pirelli-ON
Samitri-ON
Souza-Cruz -ON
Val R. Doce -ON
Vidr S. Marina -OP
Whit Martins -ON

Table 1
Sample moments for the Brazilian Stockmarket Return Series

	Local Currency Return Series		Local Currency Excess Return Series	
	x	x^2	x	x^2
Mean	159.46	83.52	28.82	50.14
Std. Dev.	79.15	51.06	70.49	37.63
Skewness	1.21	3.63	0.167	6.39
Exc. Kurt.	1.77	15.30	4.70	50.86
ρ_1	0.156	0.159	0.001	-0.015
ρ_2	0.227	0.292	0.019	0.122
ρ_3	0.146	0.136	-0.062	0.061
ρ_4	0.169	0.189	-0.019	0.182
ρ_5	0.107	0.072	-0.015	0.044
Box-Ljung Statistic	60.00	61.95	15.03	46.99
P -Value	3.6d-09	1.6d-09	0.13	9.5d-07

Table 2
Sample moments for the Three Size Portfolios
(Monthly Returns)

	Portfolio 1	Portforlio 2	Portfolio 3
Mean	2.36	3.04	6.86
Variance	11.53	5.49	5.29
Skewness	2.44	0.93	1.46
Exc. Kurt.	13.07	6.38	3.76

Table 3
CAPM with constant beta

<i>Parameters of the Market Portfolio</i>				
		α_{1M}		
Mean		0.0309 (5.45)		
		δ_{0M}		
Variance		0.0308 (18.26)		
<i>Parameters of the Portfolio Covariances</i>				
Portfolios		1	2	3
δ_{0i}		0.0298 (15.18)	0.0293 (16.78)	0.0503 (17.63)
β_i		0.9675	0.9513	1.6331
ER_i		2.99	2.94	5.05
Test of	J	12.6455		
Orthogonality	d.f.	19		
Conditions	P-value	0.8562		

Notes: t-statistics are in parentheses. The system of equations (9), where all parameters, except α_{0M} , δ_{0M} and $\delta_{0i}(i = 1, 2, 3)$, are constrained to be zero, is estimated by GMM with the following instruments: constant and 2 lags of market return (R_{Mt}) for market and portfolio returns; constant and 2 lags of portfolio covariances with market return ($u_{it}u_{Mt}$) for portfolio covariances; constant and 2 lags of market return variance (u_{Mt}^2) for market variance. ER stands for expected returns.

Table 4
Conditional CAPM with ARCH
Conditional Variance and Covariances

<i>Parameters of the Market Portfolio</i>			
	α_{0M}	α_{1i}	
Conditional Mean	0.0137 (1.40)	0.0206 (0.64)	-
	δ_{0M}	δ_{1M}	δ_{2M}
Conditional Variance	0.0033 (1.67)	0.4230 (15.63)	0.4760 (12.64)
<i>Parameters of the Portfolio Conditional Covariances</i>			
Portfolios	1	2	3
δ_{0i}	0.0087 (1.00)	0.3179 (3.91)	0.2656 (3.60)
δ_{1i}	0.0127 (2.38)	0.2080 (4.02)	0.2853 (6.79)
δ_{2i}	0.0485 (5.18)	-0.2300 (-4.08)	-0.2547 (-5.83)
β_i (average)	1.10	1.28	3.23
ER_i	1.52	1.77	4.36
Test of Orthogonality Conditions	J 10.0464	d.f. 10	p-value 0.4364

Notes: t-statistics are in parentheses. The system of equations (9), with $k_M = 1$, $k_{\sigma_M} = 2$, $k_i = 2$ ($i = 1, 2, 3$), is estimated by GMM with the following instruments: constant and 2 lags of market return (R_{Mt}) for market and portfolio returns; constant and 2 lags of portfolio covariances with market return ($u_{it}u_{Mt}$) for portfolio covariances; constant and 2 lags of market return variance (u_{Mt}^2) for market variance. ER stands for expected returns.

Table 5
Two-Factor Model with Constant Factor Loadings:
Market Portfolio and CD

<i>Parameters of the first factor (Market Portfolio)</i>				
		α_{0M}		
Mean		0.0214	-	
		(4.85)		
		δ_{0M}		
Variance		0.0409		
		(78.25)		
<i>Parameters of the second factor (CD)</i>				
		α_{0F}		
Mean		-0.0014		
		(-2.22)		
		δ_{0F}		
Variance		0.00002		
		(3.86)		
<i>Parameters of the Portfolio Covariances</i>				
Factors	Portfolios	1	2	3
Market Portfolio	δ_{0M}	0.0386 (23.43)	0.0337 (26.77)	0.0382 (38.92)
CD	δ_{0iF}	1.38e-05 (0.20)	-0.0001 (-1.82)	-0.0006 (-3.47)
	β_{iM}	0.94	0.82	0.93
	β_{iF}	0.69	-5.00	-30.00
	ER_i	1.99	2.45	6.19
Test of Orthogonality Conditions	J d.f. P-value	13.2089 35 0.9997		

Notes for Table 5: t-statistics are in parentheses. The system of equations (12), where all parameters except α_{0M} , δ_{0M} , α_{0F} , δ_{0F} and δ_{0iM} and δ_{0iF} , are constrained to be zero, is estimated by GMM with the following instruments: constant and 2 lags of market return (R_{Mt}) for market return; constant and 2 lags of CD return (R_{Ft}) for CD return; constant and 2 lags of portfolio covariances with market return ($u_{it}u_{Mt}$) for portfolio covariances with market return; constant and 2 lags of portfolio covariances with CD return ($u_{it}u_{Ft}$) for portfolio covariances with CD

return; constant and 2 lags of market return variance (u_{Mt}^2) for market variance; constant and 2 lags of CD return variance (u_{iFt}^2) for CD variance; constant, 2 lags of market return (R_{Mt}), 2 lags of CD return (R_{Ft}) for portfolio returns. ER stands for expected returns.

Table 6
Two-Factor Model with ARCH
Conditional Variances and Covariances

<i>Parameters of the first factor (Market Portfolio)</i>				
	α_{0M}	α_{1M}		
Conditional Mean	0.0160 (1.52)	-0.0654 (-2.18)		
	δ_{0M}	δ_{1M}		
Conditional Variance	0.0320 (8.15)	0.1205 (4.97)		
<i>Parameters of the second factor (CD)</i>				
	α_{0F}	α_{1F}		
Conditional Mean	-0.0015 (-1.88)	0.0166 (1.80)		
	δ_{0F}	δ_{1F}		
Conditional Variance	4.1e-06 (1.22)	0.0005 (1.25)		
<i>Parameters of the Portfolio Conditional Covariances</i>				
Factors	Portfolios	1	2	3
Market Portfolio	δ_{0iM}	0.0218 (2.25)	0.0293 (7.51)	0.0304 (9.50)
	δ_{1iM}	0.2630 (3.78)	-0.0516 (-2.59)	0.1204 (7.21)
	δ_{2iM}	-0.1975 (-4.57)	0.0353 (1.66)	0.0019 (0.28)
CD	δ_{0iF}	-1.0e-05 (-0.17)	-1.7e-05 (-0.80)	-0.0001 (-1.63)
	δ_{1iF}	0.0595 (2.18)	0.0045 (1.61)	-0.0005 (-0.22)
	δ_{2iF}	-0.0827 (-1.71)	-0.0486 (2.43)	-0.0046 (1.39)
	β_{iMi} (average)	0.58	0.82	0.95
	β_{iF} (average)	-0.96	-1.17	-27.62
	ER_i	1.30	1.95	5.45
Test of Orthogonality Conditions	J 11.0059	d.f 19		p-value 0.9236

Notes: t-statistics are in parentheses. The system of equations (12), with k_M , k_{σ_M} , k_F , and k_{σ_M} equal to 1, and k_{iM} and k_{iF} equal to 2, is estimated by GMM with the following instruments: constant and 2 lags of market return (R_{Mt}) for market return; constant and 2 lags of CD return (R_{Ft}) for CD return; constant and 2 lags of portfolio covariances with market return ($u_{it}u_{Mt}$) for portfolio covariances with market return; constant and 2 lags of portfolio covariances with CD return ($u_{it}u_{Ft}$) for portfolio covariances with CD return; constant and 2 lags of market return variance (u_{Mt}^2) for market variance; constant and 2 lags of CD return variance (u_{Ft}^2) for CD variance; constant, 2 lags of market return (R_{Mt}), 2 lags of CD return (R_{Ft}) for portfolio returns. ER stands for expected returns.

Table 7
Diagnostic Test on Residuals

	6 Lags Residuals	6 Lags Re	January Dummy	Risk Free rate	Dividend Yield
ν_{1M}	2.6530	9.8864	1.4334	3.4801	4.5456
ν_{2M}	12.2732	8.1239	2.9404	9.0804	2.2916
ν_{3M}	12.4749	10.2905	1.6851	9.9797	1.3459
ν_{1F}	191.8595	6.8963	0.8751	12.3604	8.8475
ν_{2F}	171.4355	7.9183	0.8240	14.1972	2.0839
ν_{3F}	26.4639	5.9086	0.8084	5.6555	2.2142
u_M	2.0472	2.3051	1.3603	1.6926	2.6261
u_F	50.5790	465.5397	0.4684	10.7349	2.1296
u_1	0.1439	0.0283	3.85E-05	2.79E-16	0.0030
u_2	0.0004	7.55E-05	0.0076	1.03E-15	0.0033
u_3	1.23E-05	0.0001	0.0411	2.61E-14	30.0435
ν_M	4.9013	10.9817	1.8765	14.2590	1.7425
ν_F	560.4755	5.0561	2.8866	47.1011	64.1722
df	6	6	1	3	3
$\chi^2_{5\%}$	12.5916	12.5916	3.8415	7.8147	7.8147
$\chi^2_{1\%}$	16.8119	16.8119	6.6439	11.3449	11.3449

Figure 1
Market Excess Returns in Local Currency

Figure 2
Size Portfolio Excess Returns in Local Currency

Figure 3
Conditional Market Betas for the CAPM Model

Figure 4
Conditional Market Betas for the Two-Factor Model

Figure 5
Conditional CD Betas for the Two-Factor Model

ENSAIOS ECONÔMICOS DA EPGE

- 324. A MISÉRIA DA CRÍTICA HETERODOXA PRIMEIRA PARTE: SOBRE AS CRÍTICAS – Marcos de Barros Lisboa – Maio 1998 – 44 pág.
- 325. A MISÉRIA DA CRÍTICA HETERODOXA SEGUNDA PARTE: MÉTODO E EQUILÍBRIO NA TRADIÇÃO NEOCLÁSSICA – Marcos de Barros Lisboa – Maio 1998 – 44 pág.
- 326. CURRENCY ACCOUNTING IN THE CENTRAL BANK BALANCE SHEET – Antonio Carlos Porto Gonçalves – Maio 1998 – 16 pág.
- 327. A INDETERMINAÇÃO DE SENIOR: UM PROGRAMA DE PESQUISA – Antonio Maria da Silveira – Junho 1998 – 22 pág.
- 328. ALGUNS ASPECTOS MACRO E MICROECONÔMICOS DA ECONOMIA BRASILEIRA PÓS-REAL – Rubens Penha Cysne – Junho 1998 – 35 pág.
- 329. NOTAS DE AULA: CÁLCULO DE PREÇO DE OPÇÃO DE COMPRA PARA O MERCADO BRASILEIRO – Gyorgy Varga – Junho 1998 – 42 pág.
- 330. ANAIS DO IV ENCONTRO NACIONAL SOBRE MERCADOS FINANCEIROS, POLÍTICA MONETÁRIA E POLÍTICA CAMBIAL- PARTE I – POLÍTICA CAMBIAL – Rubens Penha Cysne (editor) – Julho 1998 – 66 pág.
- 331. ANAIS DO IV ENCONTRO NACIONAL SOBRE MERCADOS FINANCEIROS, POLÍTICA MONETÁRIA E POLÍTICA CAMBIAL- PARTE II – REFORMA DO SISTEMA FINANCEIRO – Rubens Penha Cysne (editor) – Julho 1998 – 60 pág.
- 332. ANAIS DO IV ENCONTRO NACIONAL SOBRE MERCADOS FINANCEIROS, POLÍTICA MONETÁRIA E POLÍTICA CAMBIAL – PARTE III – POLÍTICA MONETÁRIA – Rubens Penha Cysne (editor) – Julho 1998 – 55 pág.
- 333. OPTIMAL IV ESTIMATION OF SYSTEMS WITH STOCHASTIC REGRESSORS AND VAR DISTURBANCES WITH APPLICATIONS TO DYNAMIC SYSTEMS – David M. Mandy e Carlos Martins Filho – Agosto 1998 – 33 pág.
- 334. PUBLIC DEBT SUSTAINABILITY AND ENDOGENOUS SEIGNIORAGE IN BRAZIL: TIME-SERIES EVIDENCE FROM 1947-92 (REVISED VERSION) – João Victor Issler e Luiz Renato Lima – Setembro 1998 – 28 pág.
- 335. COMMON CYCLES AND THE IMPORTANCE OF TRANSITORY SHOCKS TO MACROECONOMIC AGGREGATES (REVISED VERSION) – João Victor Issler e Farshid Vahid – Setembro 1998 – 37 pág.
- 336. TIME-SERIES PROPERTIES AND EMPIRICAL EVIDENCE OF GROWTH AND INFRASTRUCTURE (REVISED VERSION) – João Victor Issler e Pedro Cavalcanti Ferreira – Setembro 1998 – 38 pág.
- 337. ECONOMIC LIFE OF EQUIPMENTS AND DEPRECIATION POLICIES – Clovis de Faro – Outubro 1998 – 18 pág.

338. RENDA PERMANENTE E POUPANÇA PRECAUCIONAL: EVIDÊNCIAS EMPÍRICAS PARA O BRASIL NO PASSADO RECENTE (VERSÃO REVISADA) – Eustáquio Reis, João Victor Issler, Fernando Blanco e Leonardo de Carvalho – Outubro 1998 – 48 pág.
339. PUBLIC VERSUS PRIVATE PROVISION OF INFRASTRUCTURE - Pedro Cavalcanti Ferreira - Novembro 1998 - 27 pág.
340. “CAPITAL STRUCTURE CHOICE OF FOREIGN SUBSIDIARIES: EVIDENCE FORM MULTINATIONALS IN BRAZIL”- Walter Novaes e Sergio R.C.Werlang – Dezembro 1998 – 36 pág.
341. THE POLITICAL ECONOMY OF EXCHANGE RATE POLICY IN BRAZIL: 1964-1997 – Marco Bonomo e Cristina Terra – Janeiro 1999 – 46 pág.
342. PROJETOS COM MAIS DE DUAS VARIAÇÕES DE SINAL E O CRITÉRIO DA TAXA INTERNA DE RETORNO – Clovis de Faro e Paula de Faro – Fevereiro 1999 – 31 pág.
343. TRADE BARRIERS AND PRODUCTIVITY GROWTH: CROSS-INDUSTRY EVIDENCE – Pedro Cavalcanti Ferreira e José Luis Rossi – Março 1999 – 26 pág.
344. ARBITRAGE PRICING THEORY (APT) E VARIÁVEIS MACROECONÔMICAS – Um Estudo Empírico sobre o Mercado Acionário Brasileiro – Adriana Schor, Marco Antonio Bonomo e Pedro L. Valls Pereira – Abril de 1999 – 21 pág.
345. UM MODELO DE ACUMULAÇÃO DE CAPITAL FÍSICO E HUMANO: UM DIÁLOGO COM A ECONOMIA DO TRABALHO – Samuel de Abreu Pessoa – Versão Preliminar: Abril de 1999 – 24 pág.
346. INVESTIMENTOS, FONTES DE FINANCIAMENTO E EVOLUÇÃO DO SETOR DE INFRA-ESTRUTURA NO BRASIL: 1950-1996 – Pedro Cavalcanti Ferreira e Thomas Georges Malliagros – Maio de 1999 – 42 pág.
347. ESTIMATING AND FORECASTING THE VOLATILITY OF BRAZILIAN FINANCE SERIES USING ARCH MODELS (Preliminary Version) – João Victor Issler – Junho de 1999 – 52 pág.
348. ENDOGENOUS TIME-DEPENDENT RULES AND INFLATION INERTIA – (Preliminary Version) – Marco Antonio Bonomo e Carlos Viana de Carvalho – Junho de 1999 – 22 pág.
349. OPTIMAL STATE-DEPENDENT RULES, CREDIBILITY, AND INFLATION INERTIA – Heitor Almeida e Marco Antonio Bonomo – Junho de 1999 – 34 pág.
350. TESTS OF CONDITIONAL ASSET PRICING MODELS IN THE BRAZILIAN STOCK MARKET – Marco Bonomo e René Garcia – Julho de 1999 – 35 pág.

N.Cham. P/EPGE EE 350

Autor: BONOMO, Marco Antônio Cesar

Título: Tests of conditional asset pricing models in the Bra

00089213

52153

FGV - BMHS

Nº

000089213