Ensaio Econômico

EPGE

Escola

Brasileira de

Economia e

Finanças

Nº 822

ISSN 0104-8910

Optimal Notional Defined Contributions

Carlos E. da Costa, Marcelo R. Santos

Março de 2021

Optimal Notional Defined Contributions/
Carlos E. da Costa, Marcelo R. Santos - Rio de Janeiro:
FGV EPGE, 2021

50p. - (Ensaaios Econômicos; 822)

Inclui bibliografia.

CDD-330

Optimal Notional Defined Contributions*

Carlos E. da Costa

FGV EPGE

carlos.eugenio@fgv.br

Marcelo R. Santos

INSPER

marcelors2@insper.edu.br

March 17, 2021

Abstract

Notional Defined Contributions (NDC) systems mimic the incentive structure of fully funded social security while preserving the Pay-as-you-go nature of most current systems. We study size-preserving social reforms which replace the current US system with alternative NDCs with many alternative contribution rules and deficit/GDP ratios. If one retains the current mandatory age-independent contribution rules we find change to an NDC to *reduce* welfare: the sacrifices in distributive and insurance properties are not compensated by the efficiency gains. NDCs are, however, flexible enough to allow for alternative contribution rules which increase welfare while preserving actuarial fairness. Contributions ought to be age-dependent and concentrated later on a worker's career. The incentive structure induces an increase in capital accumulation that results, through general equilibrium effects, in welfare gains which are larger for low productivity workers, despite the increase in income inequality as captured by the Gini coefficient. *Keywords:* Notional Defined Contributions; Age-dependent wedges *J.E.L. codes:* E6; H3; J2.

*We thank Alvaro Aguirre, Sofía Bauducco, Richard Blundell, Ernesto Pastén, William Peterman, Eduardo Zilberman and seminar participants in Banco Central de Chile and the 2017 Lubramacro Meeting for their comments. da Costa gratefully acknowledges financial support from CNPq Proc. 301140/2017-0. This study was financed in part by the Coordenação de Aperfeiçoamento de Pessoal de Nível Superior - Brasil (CAPES) - Finance Code 001. Santos gratefully acknowledges financial support from CNPq Proc. 311437/2014-1. All errors are our sole responsibility.

1 Introduction

NOTIONAL Defined Contribution (NDC) Systems were first introduced in Sweden, as an attempt to deal with the pervasive fiscal imbalances that most social security systems display.¹ Along with its alleged ease of adjustment, NDCs are also praised for their 'actuarial fairness': the share of each persons' total benefits is proportional to his or her total contributions. Taken from a different perspective actuarial fairness simply means that, by closely connecting contribution to benefits, NDCs sacrifice intragenerational risk sharing and redistribution for better labor market incentives.

In this paper we compare from a Utilitarian perspective the welfare generate under the current U.S. Social Security System with that attained under an NDC of the same size. As we shall try to convey, the concerns voiced against reducing the distributive aspects of social security is, up to a point, well founded. In all reforms that we consider, Gini coefficients increase and insurance statistics worsen. Somewhat paradoxically, for all reform that *do* produce welfare gains, it is always the low productivity agents that benefit the most.

In a Notional Defined Contribution system pension contributions are tracked in accounts which earn a rate of return set by the government instead of being the product of investment returns in the markets. When a person reaches pension age, accumulated contributions and notional returns are converted to an annuity. It is crucial to note that the return that accrues to NDC's is purely notional. No investment is actually made and benefits, while determined by the annuitization of each individual's accumulated value in the notional account at the time of retirement, are still paid out of current contributions: it is a Pay as you go (PAYG) system.² NDCs therefore preserve a PAYG nature while mimicking, from an individual perspective, a defined contribution plan. As one can note from the description above, a pure budget-balanced NDC generates no intra-generational redistribution, whereas inter-generational redistribution depends on its rate of return, which is chosen by the government.³

¹ At times they are also called Non-financial Defined Contribution scheme, e.g., [Holzmann and Palmer \(2006\)](#), preserving the acronym. As of 2006, Poland, Latvia, Sweden and Italy were some of the countries adopting some version of an NDC.

² It is therefore equivalent to moving to a fully funded system, where contributions are invested in the public debt, a zero net supply asset.

³ Since it is the government which sets the rate of return, the political process involved in adjusting it to demographic and/or economic changes is, allegedly, greatly facilitated. This latter feature explains much of NDC's appeal.

We consider an overlapping generations economy of heterogeneous agents who live a meaningful life-cycle facing idiosyncratic productivity and mortality risk. The model is calibrated to approximate the current US tax and social security policies. The policy exercises consist in replacing the current US system with alternative NDC systems of equal size, as measured by total entitlements, but with varying contribution structures and rate of return on notional accounts.

In our first reform proposal, the contribution structure is similar to the current one: a mandatory age-independent rate, adjusted to keep the system deficit at the current level. We find that this reform reduces welfare. The loss in insurance and redistribution overwhelms the efficiency gains. Interestingly, when we evaluate a system which relies on age-dependent mandatory contributions we find that it is capable of generating efficiency gains which are large enough to compensate for its negative distributive impact. Welfare gains are far from trivial — consumption equivalent variation of almost 5% — occur in a system with contributions only in the last decade before retirement. To keep tax revenues constant given the changes induced by the reform, we allow the labor income tax schedule to adjust. More importantly, while almost all distribution/insurance statistics worsen, it is the low productivity workers that benefit the most. In this sense, these reforms would be accepted under an ex-ante Rawlsian criterion. The bulk of welfare gains arise due to general equilibrium effects – wage increases – induced by the change in the incentives to accumulate assets under the age-dependent NDC.

In our baseline exercise we hold the social security deficit fixed as a fraction of GDP and consider these two alternative contribution rules. We also consider a reform in which we leave the labor income tax schedule fixed, but allow the social security deficit to vary, provided that the consolidated public sector deficit remains constant. We find that it is not optimal to allow the deficit to increase, thus suggesting that social security contributions are a better instrument for raising revenues to finance social security benefits than the *current* labor income tax schedule. We cannot rule out the possibility that an optimized schedule would lead this reform to be better. As we have mentioned, the entitlements as a percentage of GDP are fixed.

Because a large fraction of welfare gains result from wage increases, we experiment with a reform in which the government absorbs all increases in the demand for assets by increasing the public debt. Welfare gain is lower and now accrues to high productivity agents. Moreover, servicing the additional debt would require reducing current government expenditures by more than 3% of GDP.

Finally we consider a radical reform in which mandatory contributions are replaced by incentives to voluntarily save through NDC's. This reform rids the social security system from all potential labor supply distortions. We find that it is the high productivity agents who end up using this options the most. Their social security contributions crowd out the contribution of low productivity agents that end up saving through capital accumulation. Welfare gains are significantly lower than what is attained with a mandatory age-dependent system, and are more evenly distributed across productivity types.

The rest of the paper is organized as follows. After a brief literature review, in Section 2 the economy is presented. Section 3 then offers the heuristics for our main exercises and results. The calibration is explained in Section 4 and the results for the baseline exercise in Section 5. Section 6 consider alternative reforms and alternative specifications to check for robustness. In Section 7 we ask whether a system purely based on voluntary contribution might not do better by eliminating most labor supply distortions. Finally, concluding remarks are found in Section 7.

Closely Related Literature

Social Security schemes can be classified according to: i) their degree of funding, with PAYG and fully funded representing the extremes of a spectrum that exists in practice, and; ii) the relationship between contributions and benefits, with defined contributions and defined benefits at the two extremes.⁴ The nature of funding is crucial for capital accumulation, an aspect of the system with enormous long run consequences. This explains why most work done thus far have focused on the funding aspect.⁵ We, in contrast, focus on the connection between benefits and contributions.

In a pure pay-as-you-go (PAYG) system, where benefits are paid out of current contributions, one may understand the difference between the implicit rate or return on social security contributions and the rate of return on capital through what [Diamond and Orszag \(2004\)](#) call a 'legacy debt': the transfers made for the first generation which contributed zero to the system but still received benefits. The 'legacy debt' reduces

⁴Under a defined benefits system, the agent's benefits may be determined as a function of his or her wage, but are to a great extent independent on his or her contributions. The feature that induces people to link i) and ii) is the fact that in a defined benefits system it is the sponsor's contribution that guarantees the system's balance. Our focus is on the fact that under a defined contributions system, a much tighter connection exists between contributions and benefits.

⁵Almost as important is the fact that the early literature almost invariably adopted a representative agent framework thus precluding the examination of the equity/efficiency trade-off.

the value of contributions, thus introducing distortions that go beyond what a system of compulsory savings generates. The reform we consider retains the PAYG funding which sets aside the treatment of the 'legacy debt'. According to [Geanakoplos et al. \(1999\)](#), transition costs associated with this debt would, for example, completely eliminate any potential gain in the rate of return from privatization. Despite retaining the PAYG structure of the U.S. current system the reform tightens the connection between contributions and benefits. Indeed, while the separation of aggregate and individual savings is a defining feature of a PAYG system, the connection between benefits and contributions is to a certain degree an independent feature of particular social security designs.⁶

With regards to the relationship between contributions and benefits it is also worth recalling the two extreme forms of social security pensions: Bismark and Beveridge systems. In a Bismark system, social security benefits are financed by social security contributions and benefits are proportional to contributions. In a Beveridge system, social security benefits are independent of past choices and generally financed through general taxes. The crucial distinction between these two systems is, therefore, the amount of intra-generational distribution that takes place through social security. In their purest form, no redistribution occurs in a Bismark system and substantial amount occurs in a Beveridge system.⁷ Notional Defined Contribution (NDC) system provide a pure form of a Bismark social security system.⁸

We explore in our policy experiments the potential role of social security design in generating dynamic incentives that have been shown to be quantitatively important in recent mechanism design based studies, and that have the potential to add policy elements that have been elsewhere shown to help the implementation of second best al-

⁶In practice, of course, the two characteristics are not orthogonal. In funded systems, it is typically the case that each agent's contributions becomes a capital investment. Note, however, that it is the fact that investments which are connected to an agent's contributions become his or her benefit funding that matters for incentives. Indeed while one of the key elements of the reform considered here, to increase the connection between benefits and contributions, is also typically attained by privatization – e.g., [Nishiyama and Smetters \(2007\)](#) – it is perfectly possible to have a fully funded privatized system in which, one's contribution is not directly related to one's benefits do to extensive mandatory cross-subsidies.

⁷As for intergenerational transfers, different parameter choices can lead to more or less transfers under either system.

⁸A characteristic of a PAYG system that is preserved in all our analysis, the separation between total private savings and aggregate savings would also be accomplished if the system were replaced by a fully funded one where retirement assets were used to increase the public debt. Only in Section 6.5 we allow the size of the disconnect to vary since we actually increase the size of overall (implicit plus explicit) debt. As [Auerbach and Lee \(2009\)](#) summarize it, an NDC system mimics the philosophy of a fully funded private account system but uses a pay-as-you-go financing structure.

locations.⁹ Consider age-dependent taxes. For an economy with a constant dependency ratio, the rate of return on the notional accounts which keeps the deficit constant is the growth rate of the economy, see [Murphy and Welch \(1998\)](#). In this case, if the economy is dynamically efficient, one's contribution exceeds the present value of one's benefits. The discrepancy between the value of contributions and the present value of benefits generates labor wedges which increase with distance from retirement.¹⁰ Also important is the fact that the non-linearity in both the contribution and the benefits schedule observed in the US Social Security System implies that one's replacement ratio depends on one's average earnings. Wedges therefore depend not on a single period's earnings but on one's entire history of earnings.¹¹

NDC systems got some track on the 1990's in the realm of a social security reform movement that arose in response to the long term fiscal problems facing pay as you go systems around the world. Most of the literature that ensued focused on how flexible the system was to changes in demographics, expected growth rate, etc. This is understandable. From a purely conceptual view, a defined benefit system could be designed to produce the same outcome. Yet, in practice, the connection between contribution and benefits in most systems found in the world is usually very weak, which is in contrast to what we propose here.

2 The Environment

At each point in time, the economy is inhabited by multiple cohorts of individuals of different ages. Each cohort is comprised of a continuum of individuals who live for a finite, random number of periods.

⁹Representative of these second best studies are [Weinzierl \(2011\)](#); [Farhi and Werning \(2013\)](#); [Golosov et al. \(2016\)](#); [Stantcheva \(2017\)](#).

¹⁰Although age-dependence has been shown to be a desirable feature of tax systems, e.g., [Erosa and Gervais \(2002\)](#); [Blomquist and Micheletto \(2008\)](#), most recent works have shown that marginal rates ought to *increase* in age, e.g., [Petersman \(2016\)](#); [Karabarbounis \(2016\)](#); [da Costa and Santos \(2017\)](#).

¹¹Income averaging is a feature of tax systems famously advocated by [Vickrey \(1947\)](#) in his [Agenda for Progressive Taxation](#). Although the current system does introduce some measure of income averaging this is not an intended feature designed to optimally interact with the income tax schedule. Recent work by [Kapička \(2020\)](#) have assessed the potential welfare gains from history dependent taxes.

2.1 Demography

Each period, j , a new generation is born. Uncertainty regarding the time of death for each person is captured by the fact that each individual faces a probability ψ_{t+1} of surviving to the age $t + 1$ conditional on being alive at age t . Hence, an individual born in period j is alive in period $j + t$ with probability $\prod_{k=1}^t \psi_k$. We also assume that there is $T > 0$ such that $\psi_{T+1} = 0$.

We assume independence of individual death shocks and appeal to the law of large numbers to map the survival probability into the time invariant age profile of the population denoted $\{\mu_t\}_{t=1}^T$. Letting g_n denote the population growth rate, the fraction of agents t years old in the population is found using the following law of motion

$$\mu_t = \frac{\psi_t}{1 + g_n} \mu_{t-1},$$

with $\mu_t \geq 0$, $\sum_{t=1}^T \mu_t = 1$. Our focus is on working lives, hence an agents life starts at the age 20 and assume that no agent lives after age 90.

For most of our analysis we focus on steady-state allocations. Since it greatly simplifies the notation, we drop all time indices, j , from aggregate variables and use t to represent age.

2.2 Technology

Technology is standard. The production side of the economy aggregates and the technology for producing the consumption good is summarized by a Cobb-Douglass production function with constant returns to scale. That is,

$$Y = BK^\alpha N^{1-\alpha},$$

where K is aggregate capital, N is aggregate efficient units of labor, and B is a scale parameter.

Every period, the standing representative firm solves the static optimization problem,

$$\max_{K,N} \{BK^\alpha N^{1-\alpha} - \delta K - wN - rK\},$$

where r is the rental rate of physical capital and w is the rental rate of efficiency units

of labor, i.e. the wage rate. Note that the rental rate of capital is assumed to be net of depreciation costs which are born directly by the firm.

The first order conditions for the firm's profit maximization problem are,

$$(1 - \alpha)BK^\alpha N^{-\alpha} = w, \quad (1)$$

and

$$\alpha BK^{\alpha-1} N^{-\alpha} - \delta = r. \quad (2)$$

2.3 Households

Preferences Individuals derive utility from consumption, c , and leisure, l .

Preferences defined over random paths of (c_t, l_t) are represented by the time-separable von-Neumann Morgenstern utility function,

$$\mathbb{E} \left[\sum_{t=1}^T \beta^{t-1} \left(\prod_{k=1}^t \psi_k \right) U_t(c_t, l_t) \right], \quad (3)$$

where β is the subjective discount factor, and $\mathbb{E}$ is the expectation operator conditional on information at birth.

We allow preferences over consumption-leisure bundles to vary with age by indexing the flow utility by t . Specifically, flow utility is of the form

$$U_t(c_t, l_t) = \frac{(c_t^{1-\rho_t} l_t^{\rho_t})^{1-\gamma} - 1}{1-\gamma} - \iota_t \chi_t, \quad (4)$$

for $\rho_t \in (0, 1) \forall t, \gamma > 0, \gamma \neq 1$.

The fixed cost of work, χ_t , is expressed in utility terms and depends on age. It is introduced to generate an extensive margin decision through a nonconvexity in the utility function. This parameter is important for the model to match the employment rate over the lifecycle. ι_t is an indicator function that takes value 0 if the individual is nonemployed and 1 otherwise.

The fact that we allow the marginal rate of substitution between leisure and consumption to vary with age gives us more degrees of freedom to try to match the behavior of hours along the life-cycle. As ρ decreases, agents become more willing to forego leisure to obtain more consumption. Since lower ρ implies higher (in absolute terms)

Frisch elasticity of leisure, the two effects compound to generate more variation in the Frisch elasticity of labor supply.

Another issue raised by our choice of time-varying preferences is that, for a given n , the marginal utility of consumption varies with age. A perfectly smooth profile would no longer be optimal even if hours were constant. This choice of preferences demands more caution in measuring, reporting and interpreting welfare gains.

Independently of the use of time-varying parameters, under these preferences, the Frisch elasticity of labor supply decreases with hours worked. Indeed, if ϵ^f denotes the Frisch elasticity of labor supply, then,

$$\epsilon_t^f = \frac{(1 - \gamma)(1 - \rho_t) - 1}{\gamma} \frac{1 - n_t}{n_t}.$$

Even if $\rho_t = \rho$ for all t , ϵ_t^f will vary with n_t due to the ratio $(1 - n_t)/n_t$.

Wealth in the Utility Function In order for the model to be able to reproduce the observed wealth distribution observed, we assume that individuals derive utility from leaving a bequest, a , to their children

$$\nu(a) = \eta_1 \left(1 + \frac{a}{\eta_2} \right)^{(1-\gamma)}. \quad (5)$$

The idea of including a utility for wealth as a motive for accumulation is not new but has been regaining traction in recent times. We motivate it through a utility from bequeathing which, first introduced by [Andreoni \(1989\)](#), has been called a 'warm glow' or 'joy-of-giving' motive. It creates non-homotheticities that have been proven essential to generate a wealth distribution that is closer to what one observed in the data thanks to its functional form, described in (5), first proposed by [De Nardi \(2004\)](#).

Labor Supply and Retirement Every period, individuals choose labor supply, consumption, and asset accumulation to maximize their objective (3) subject to a budget constraint which we shall explain momentarily.

Each person has a unit time endowment per period which can be directly consumed in the form of leisure, l , or used in market related activities. An agent's period-by-period time constraint is, therefore, $l_t + n_t = 1$.

An individual of age t who works for n hours supplies to the market a total of $n_t s_t e^{(u+z_t)}$ efficiency units. Each unit is paid a rental rate, w . The variable $u \sim \mathcal{N}(0, \sigma_u^2)$ is a permanent component of an individual's skills. It is realized at birth and retained throughout one's life. In contrast, z follows an AR(1) process, $z_t = \varphi_z z_{t-1} + \varepsilon_t$, with innovations $\varepsilon_t \sim N(0, \sigma_\varepsilon^2)$.

Whereas u aims at capturing the heterogeneity at birth, the most relevant source of welfare variation, z is the main source of uncertainty affecting choices. The parameter φ_z accommodates the empirically observed persistence of productivity shocks. Finally, s_t is what we call the age-efficiency profile.

Labor productivity shocks are independent across agents. As a consequence, there is no uncertainty regarding the aggregate labor endowment even though there is uncertainty at the individual level. Retirement is mandatory at the age of 65, or $t = 46$.

Asset Accumulation Besides choosing how much leisure to consume, individuals trade a risk free asset which holdings we denoted by a_t . Asset holdings are subject to an exogenous lower bound. More precisely, for our main exercise, we follow [Conesa et al. \(2009\)](#) in assuming that agents are not allowed to contract debt at any age, so that the amount of assets carried over from age t to $t + 1$ is such that $a_{t+1} \geq 0$. Because no agent can hold a negative position in assets at any time, we assume without loss that asset takes the form of capital, $a_t = k_t$, as in [Aiyagari \(1994\)](#).

As we shall make clear, there is exogenous (as well as endogenous) variation in productivity along the life-cycle. Consumption smoothing thus provides a reason for one to accumulate assets. Another aspect of choices is that individuals may resort to self-insurance to protect themselves against the uncertainty on labor income. Savings will be, to some extent, motivated by precautionary reasons. By precluding borrowing we potentially affect the valuation of future social security benefits as compared to what we would observe in a world with perfect capital markets.

Also, households are born with initial wealth endowment a_1 drawn from an endogenous distribution that integrates up to the overall amount of wealth bequeathed in the economy by the deceased households. To account for the intergenerational persistence in wealth inequality, we assume that the draw is correlated with the permanent component of the elderly's skills. In particular, let $\hat{a}_u$ and $\sigma_{\hat{a}_u}^2$ denote the mean and the variance of wealth bequeathed by the deceased households with permanent skill u , respectively. Thus, an agent at the first age with permanent skill u draws her initial wealth from a

log-normal distribution with mean $\hat{a}_u$ and variance $\sigma_{\hat{a}_u}^2$.¹²

Budget Constraints To write each agent's flow budget constraint we need to specify the fiscal policy that is being used by the Government. In our case, it is important to distinguish between the current fiscal policy, needed to calibrate the model, and the ones we evaluate. The current tax system will be the benchmark for our studies.

2.4 The Government

The Benchmark Tax and Social Security Systems The government levies taxes on capital income, consumption, and labor income. We assume that consumption is taxed at a flat rate τ_c and capital income at a flat rate τ_k . The government also runs a social security system with contributions and benefits that are equal in equilibrium.

Labor income taxes, are allowed to be non-linear. In particular, we use the specification $T(y) = y - \xi y^{1-\epsilon}$ first proposed by Musgrave (1959) and since adopted by Feldstein (1969); Bénabou (2000, 2002); Heathcote et al. (2017), to name a few.

Tax revenues are raised to finance an exogenous flow of expenditures, G , and possibly to cover the social security deficit. In addition, government provides individuals a minimum consumption, $\underline{c}$. We assume that transfers, tra , are conditional on individuals available resources. In particular, following Hubbard et al. (1995), we specify:

$$tra = \max\{\underline{c} - [1 + (1 - \tau_k)r]a - \tilde{y} + T(\tilde{y}), 0\} \quad (6)$$

Retirement benefits under the Benchmark In the benchmark economy, retirement benefits are given by

$$b(x) = \begin{cases} \theta_1 x, & \text{if } x \leq y_1 \\ \theta_1 y_1 + \theta_2(x - y_1), & \text{if } y_1 \leq x \leq y_2 \\ \theta_1 y_1 + \theta_2(y_2 - y_1) + \theta_3(x - y_2), & \text{if } y_2 \leq x \leq y_{max} \\ \theta_1 y_1 + \theta_2(y_2 - y_1) + \theta_3(y_{max} - y_2), & \text{if } x \geq y_{max} \end{cases} \quad (7)$$

where x is the individual's average life-cycle earnings and $\theta_1 > \theta_2 > \theta_3$.

¹²The reason why we model the intergenerational links in the form of both accidental and voluntary bequests this way is to avoid the inclusion of an additional state variable regards to the parent's ability as is done in De Nardi (2004).

The law of motion for x is given by

$$x_{t+1} = \frac{[t-1]x_t + \tau_{ss} \min\{y_t; y_{\max}\}}{t}, \quad (8)$$

and $x_T = x$.

Retirement benefits under the Notional system Under a notional system each agent has an 'account' where his or her contributions are accumulated at a rate of return which is defined by the Government. From a private perspective it therefore functions similarly to a compulsory savings plan (with returns which are taxed or subsidized). Formally,

$$\tilde{a}_{t+1} = \tilde{a}_t(1 + \bar{r}_n) + \tau_{ss,t}y_t$$

where $\tilde{a}$ denotes the accumulated life-cycle contributions up to age t . $\bar{r}_n$ is the exogenous 'interest rate' determined by the government, which we allow to depend on age. $\tau_{ss,t}$ is the contribution rate which may also be age-dependent.

Upon retirement, agents are entitled to a benefit

$$b(\tilde{a}) = \frac{(1 + \bar{r})^{T_R}}{\sum_{t=1}^{T_R} (1 + \bar{r})^{t-1}} \tilde{a} \quad (9)$$

where $\tilde{a}$ denotes the accumulated life-cycle contributions at the time of retirement, T_R is the average number of retirement years and $\bar{r}$ is the exogenous interest rate determined by the government. We allow the system to run deficits or surpluses that must be financed by resources raised from the general tax system.

Although the system creates incentives that are identical to those produced by a private (compulsory) system, it is still a PAYG system. The private accounts are simply accounting devices. Because benefits are still funded by the contribution of current workers and/or current taxes, the disconnect between private and aggregate savings remains, with important consequences for capital accumulation.

With regards to labor supply, the system has the potential to reduce the incentive effects of social security financing, by reducing the intra-temporal labor wedge. This is accomplished by its tying more closely contribution to benefits. Note, however that the inter-temporal allocation of effort is still distorted even when all intra-temporal distor-

tion is eliminated. The downside of making benefits more closely tied to contributions is that the system is expected to be less distributive. Note however, that the overall progressivity of the system should be taken into account when one evaluates the tax prescriptions we derive.

The flow budget constraint that individuals in working age face in our model economy under the benchmark social security system is

$$a_{t+1} + (1 + \tau_c)c_t = [1 + r(1 - \tau_k)]a_t + \tilde{y}_t - T(\tilde{y}_t) + tra \quad \forall t, \quad (10)$$

for $t \leq T$, $\tilde{y}_t := y_t - \tau_{ss} \min\{y_t, \bar{y}\}$, $y_t := wn_t s_t e^{u+z_t}$.

After retirement, the budget constraint is

$$(1 + \tau_c)c + a' = [1 + r(1 - \tau_k)]a + b(x) + tra \quad (11)$$

for $b(x)$ defined in (7). If the agent is not employed, then $y_t = 0$

Under the NDC, the budget constraint for workers is still (10), but $\tilde{y}_t$ is now defined simply as $y_t - \tau_{ss}y_t$. Similarly, after retirement, we have

$$(1 + \tau_c)c + a' = [1 + r(1 - \tau_k)]a + b(\tilde{a}) + \epsilon \quad (12)$$

for $b(\tilde{a})$ defined by (9)

Recursive Formulation of Households' Problem

Let $V_{w,t}(\omega_t)$ and $V_{o,t}(\omega_t)$ denote the value function of an individual aged $t < T + 1$ for the employment and non-employment case, respectively, where $\omega_t = (a_t, u, z_t, x_t) \in \Omega$ is the individual state. Also, $\omega' = (a', u, z', x')$ denote the next period individual's state. Thus, the optimization problem of employed individuals aged t can be recursively represented as follows.

$$V_{w,t}(\omega) = \max_{n, a' \geq 0} U_t(c, 1 - n) + \beta \mathbb{E}_{z'} [\max \{V_{w,t+1}(\omega'), V_{o,t+1}(\omega')\}], \quad (13)$$

subject to (10).

We have only used individual state variables in ω . Prices do enter the value function as well and in solving the model we will need to find the equilibrium prices by explicitly

taking into account how they enter the policy functions associated with (14). Still for economy in notation we drop them from ω .

For non-employed agents, no labor supply decision is made. Thus, the bellman equation can be written as:

$$V_{o,t}(\omega) = \max_{a' \geq 0} U_t(c, 1) + \beta \mathbb{E}_{z'} [\max \{V_{w,t+1}(\omega'), V_{o,t+1}(\omega')\}], \quad (14)$$

subject to (10).

Retired agents also do not choose their labor supply and we assume that, after retirement, they start deriving utility from bequests as well. A retired agents' value function is, therefore,

$$V_t(\omega) = \max_{a' \geq 0} U_t(c, 1) + \beta [\psi_{t+1} \mathbb{E}_{z'} V_{t+1}(\omega') + (1 - \psi_{t+1}) \nu(a')]. \quad (15)$$

Finally, considering that agents die for sure at age T , we have $V_{T+1}(\omega_{T+1}) = \nu(a)$.

2.5 Recursive competitive equilibrium

In all that follows we describe the recursive equilibrium in a steady state. This greatly simplifies the presentation. As we have already mentioned also it dispenses with the need to maintain different notations for age and time.

Our equilibrium definition refers to the benchmark. An analogous definition applies to the economy under the NDC, with $\tilde{a}$ substituting for x as a state variable and with the suitable replacement of budget constraints.

At each point in time, agents differ from one another with respect to age t and to state $\omega = (a, u, z, x) \in \Omega$. Agents of age t identified by their individual states ω , are distributed according to a probability measure λ_t defined on Ω , as follows. Let $(\Omega, F(\Omega), \lambda_t)$ be a space of probability, where $F(\Omega)$ is the Borel σ -algebra on Ω : for each $\eta \subset F(\Omega)$, $\lambda_t(\eta)$ denotes the fraction of agents aged t that are in η .

Given the age distribution, λ_t , $Q_t(\omega, \eta)$ induces the age $t + 1$ distribution λ_{t+1} as follows. $Q_t(\omega, \eta)$ determines the probability of an agent at age t and state ω to transit to the set η at age $t + 1$. $Q_t(\omega, \eta)$, in turn, depends on the policy functions in (14), and on the exogenous stochastic process for z .

Definition Given the policy parameters, a **recursive competitive equilibrium** is a collection of value functions $\{V_t(\omega)\}$, policy functions for individual asset holdings $d_{a,t}(\omega)$, for consumption $d_{c,t}(\omega)$, for labor supply $d_{n_w,t}(\omega)$, prices $\{w, r\}$, age dependent but time-invariant measures of agents $\lambda_t(\omega)$, transfers ϵ such that:

- i) $\{d_{a,t}(\omega), d_{n_w,t}(\omega), d_{c,t}(\omega)\}$ solve the dynamic problems in (14);
- ii) individual and aggregate behaviors are consistent, that is:

$$\begin{aligned} K &= \sum_{t=1}^T \mu_t \int_{\Omega} d_{a,t}(\omega) d\lambda_t \\ N &= \sum_{t=1}^T \mu_t \int_{\Omega} d_{n_w,t}(\omega) s_t(\omega) \exp(u + z_t) d\lambda_t \\ C &= \sum_{t=1}^T \mu_t \int_{\Omega} \{d_{c,t}(\omega)\} d\lambda_t; \end{aligned}$$

- iii) $\{w, r\}$ are such that they satisfy the optimum conditions (2) and (1);
- iv) The final good market clears:

$$C + G + \delta K = K^\alpha N^{1-\alpha};$$

- v) given the decision rules, $\lambda_t(\omega)$ follows the law of motion:

$$\lambda_{t+1}(\eta) = \int_{\Omega} Q_t(\omega, \eta) d\lambda_t \quad \forall \eta \in F(\Omega);$$

- vi) the overall amount of wealth bequeathed by the deceased households correspond to the aggregate initial wealth endowment:

$$\mu_1 \int_{\Omega} a_1(\omega) d\lambda_1 = \sum_{t=T_r}^T \mu_t \int_{\Omega} (1 - \psi_{t+1}) d_{a,t}(\omega) d\lambda_t$$

where T_r denote the retirement age.

vii) taxes are such that the government's budget constraint,

$$\begin{aligned} \sum_{t=1}^T \mu_t \int_{\Omega} T_t \left(d_{n_w,t}(\omega) s_t(\omega) \exp(u + z_t) - \right. \\ \left. T_{ss} (d_{n_w,t}(\omega) s_t(\omega) \exp(u + z_t)) \right) d\lambda_t + \tau_c C + \tau_k r K + \\ \sum_{t=1}^T \mu_t \int_{\Omega} T_{ss} (d_{n_w,t}(\omega) s_t(\omega) \exp(u + z_t)) d\lambda_t = G + B \quad (16) \end{aligned}$$

are satisfied every period, where B are the total benefits that are due, given the social security system that we consider.

Item (vii) is redundant if conditions (i)–(vi) hold.

3 Heuristics

An NDC has two defining features: i) the disconnection between private and aggregate savings that characterizes all PAYG systems, and; ii) the 'actuarial fairness' that is generated by its reliance on individual accounts. By actuarial fairness one only means that the ratio between the present value of benefits to contributions is the same for all agents. It mean neither that the private valuation of benefits equal the private value of contributions nor even that the difference between the two coincide across agents.¹³ This has important consequences for the labor supply 'incidence' of mandatory contributions.

Consider an environment without uncertainty and with perfect capital markets. Under the current system, an agents' benefits are calculated by taking into account their earned income history according to (7) and (8), whereas under a NDC it is calculated according to (9).

Let us simplify the discussion and assume that there in no uncertainty and capital markets are perfect. Then, the value of one dollar paid as contribution at age t is simply the present value of the increase in benefits to which the agent is entitled, given this increase in contribution. Although the benefits in the current system are not directly related to the contributions, for the purposes of calculating the labor wedge, the con-

¹³Each persons' valuation depends on one's own discount rates, which differ across agents in an incomplete and imperfect market environment.

nection between one's labor income history and benefits implies that the labor wedge is smaller than what is implied by the contribution rate, τ_{ss} .

The non-linearity of the benefit schedule and the cap on benefits implies that wedges are history dependent. Indeed, consider an agent in his last working year whose earnings exceed $\bar{y}$ but whose history of earnings implies that he or she is still below the benefit cap. Any extra dollar this agent earns increases his or her benefit without any extra contribution. It is a work subsidy. Had the same agent had a history of earnings leading him or her to exceed the cap, then there would be no extra gain. This is but one example of history dependence induced by the current social security system.

Now, for the same agent the labor wedge induced by the NDC is age- but not history-dependent. Indeed, let $\theta(\bar{r})$ denote the ratio between the value accumulated in the notional account and the present value of benefits. It is easy to see that $\theta(r) = 1$, and, $\theta'(\bar{r}) < 0$. Then, the wedge induced by the notional system is $\tilde{\tau}_{ss}^t = \tau_{ss}(1 - \theta(\bar{r})((1 - \bar{r})/(1 - r))^{T-t})$. For any $\bar{r} < r$ it is increasing in the distance from retirement $T - t$.

Uncertainty and Market Imperfections To understand the intra-temporal distortions induced by the social security system when uncertainty and capital markets imperfections are taken into account, disregarding consumption taxes for simplicity, we calculate the labor wedges as follows.

$$\frac{\partial_l U_t(c_t, l_t)}{\partial_c U_t(c_t, l_t)} = [1 - T'(\tilde{y}_t)] [1 - \tau_{ss}] + \beta \mathbb{E} \left[\frac{\alpha_{t+1}}{\partial_c U_t(c_t, l_t)} \right] \frac{dx_{t+1}}{dn_t}, \quad (17)$$

where α_{t+1} is a Lagrange multiplier associated with the law of motion for x .¹⁴

The first term in the right hand side of (17) shows the fiscal wedge that would be calculated if contributions were unrelated to benefits as in a pure Beveridge social security system. The fact that contributions are capped at a maximum level of earnings, $\bar{y}$, this term alone shows social security contributions reducing the progressivity of labor wedges introduced by the labor income tax schedule.¹⁵

¹⁴Using the recursive formulation (14) from Section 2.4, it is not hard to see that the multiplier α_{t+1} in (17) is equal to the partial derivative of $V_{t+1}(\omega')$ with respect to x' , $\partial_{x'} V_{t+1}(\omega')$. For the NDC, $\tilde{a}$ substitutes for x as a state, and (18) substitutes for (10) as the budget constraint. $\tilde{\alpha}_{t+1}$ in (19) is equal to the partial derivative of $V_{t+1}(\omega')$ with respect to $\tilde{a}'$, $\partial_{\tilde{a}'} V_{t+1}(\omega')$.

¹⁵For $y_t > \bar{y}$, we have $\tilde{y}_t = wn_t s_t e^{u+z_t} - \tau_{ss} \bar{y}$ which implies, $d\tilde{y}_t = ws_t e^{u+z} dn_t$, whereas for $y_t < \bar{y}$, $\tilde{y}_t = wn_t s_t e^{u+z} (1 - \tau_{ss})$, leading to $d\tilde{y}_t = ws_t e^{u+z} (1 - \tau_{ss}) dn_t$. Similarly, $dx_{t+1} = \frac{\tau_{ss}}{t} dy_t$ for $y_t < \bar{y}$, and 0 for $y_t > y_{\max}$.

This is not the whole story, of course. In the U.S. system, as in most countries, contributions are not pure taxes since benefits *do* depend on contributions. In particular, benefits depend on the history of contributions, hence income. To understand this history-dependence it is important to note that benefits are a non-linear function of x , a state variable that depends on the history of contributions according to (8).¹⁶

In the economy with the notional defined contribution system the budget constraint for a working agent is given by

$$a_{t+1} + (1 + \tau_c)c_t = [1 + r(1 - \tau_k)]a_t + \tilde{y}_t - T(\tilde{y}_t) + \epsilon \forall t, \quad (18)$$

for $t \leq T$, $\tilde{y}_t := y_t - \tau_{ss}y_t$, $y_t := wn_t s_t e^{u+z_t}$, whereas the notional account accumulates according to

$$\tilde{a}_{t+1} = \tilde{a}_t r_n + \tau_{ss}y_t.$$

Ignoring consumption taxes for simplicity, labor wedges are of the form

$$\frac{\partial_l U_t(c_t, l_t)}{\partial_c U_t(c_t, l_t)} = \left\{ [1 - T'(\tilde{y}_t)] [1 - \tau_{ss}] + \beta \mathbb{E} \left[\frac{\tilde{a}_{t+1}}{\partial_c U_t(c_t, l_t)} \right] \tau_{ss} \right\} w s_t e^{u+z}, \quad (19)$$

where $\tilde{a}_{t+1}$ is a Lagrange multiplier associated with the law of motion for $\tilde{a}$.

There are important differences between (17) and (19) which are worth highlighting. First, the non-linearities in $\tilde{y}_t$ as a function of y_t are absent in $\tilde{y}_t$, hence, ignoring benefits, social security contributions do not change progressivity.¹⁷ Second, $\tilde{a}_{t+1}$ is always positive which, as the example in footnote 16 shows, need not be the case for α_{t+1} defined in (17). Third, unless an agents' borrowing constraint is binding, history independent contribution rates τ_{ss} lead to history-independent, though possibly age dependent, labor wedges.

Another crucial feature of NDC's which we emphasize in our policy experiments is that there is no cross-subsidy built into the system. No redistribution via social security takes place *if the system's budget is balanced*.

¹⁶The multiplier α reflects the marginal value of increasing x , thus encoding this history dependence. As an example consider α_T for an agent such that, $x_{T-1} > y_{\max}T/(T-1)$, then $\alpha_T = 0$, since earnings, y_{T-1} for this agent will have no consequences for the benefits he will receive.

¹⁷For the particular specification of taxes we use, $y(1 - \tau_{ss}) - T(y(1 - \tau_{ss})) = \xi[y(1 - \tau_{ss})^{1-\varrho} - (1 - \tau_{ss})^{1-\varrho}\xi y^{1-\varrho}]$.

4 Calibration

To carry out our quantitative analysis, we need first to find values for all the parameters of the model. We accomplish this by calibrating the model to the U.S. economy.

The population age profile $\{\mu_t\}_{t=1}^T$ depends on the population growth rate, g_n , the survival probabilities, ψ_t , and the maximum age, T , that an agent can live. Agents enter the economy at age 16 and live for 75 years, $T = 75$, so that the real maximum age is 90 years.

Data on survival probability by age were extracted from [Bell and Miller \(2005\)](#). Given the survival probabilities, the population growth rate is chosen so that the age distribution in the model replicates the dependency ratio observed in the data. By setting $g_n = 0.0105$, the model generates a dependency ratio of 17.27%, which is close to the dependency ratio observed in the data for the year 2000.

To calibrate the preference parameters we proceed as follows. First, we choose the discount factor β in such a way that the equilibrium of our benchmark economy implies a capital-output ratio around of 3.2, which is the value observed in the data. Then we fix the parameter γ to 4.0, from micro evidence, and choose the share of leisure in the utility function, ρ_t , to match average ours for different age groups. In particular, we assume that $\rho_t = \rho_0 + \rho_1 t$. To calibrate ρ_0 , we use the average working hours for ages 19 – 40 and for ρ_1 the average between 41 – 60. The first group works on average 37.86 while the second 40.37 of their time endowment. For the last 5 years we specify a new profile $\rho_t = \rho_{60} + \rho_2 t$. We calibrate ρ_2 to match the average hours during those last five years equal to 35.16.¹⁸

Individuals face an age-dependent fixed cost of working. We use a parametric function $\chi_t = \nu_0 + \nu_1 t + \nu_2 t^2$. The larger the fixed cost, the smaller the incentive to be employed. We calibrate (ν_0, ν_1, ν_2) using the average employment rate at three stages of the life cycle as targets: early working years (ages 21-35), middle ages (35-50), and the rest of working life cycle (ages 51-65).

The parameter ϕ_1 represents the weight on the utility from bequeathing. Since it measures the strength of bequest motives, we calibrate ϕ_1 so that the ratio of the median wealth held by the individuals aged 75 and above to that of all individuals is 1.8, as reported in the Statistical Abstract of the United States (US Census Bureau 2009). The

¹⁸The data for hours worked are from IPUMS (U.S. Department of Commerce, Bureau of the Census 2005).

term ϕ_2 is the shifter of bequests in the utility function. It reflects the extent to which bequests are luxury goods, affecting the bequest distribution, especially the high end of it. Thus, we follow [Nardi and Yang \(2014\)](#) and set ϕ_2 to match the 90th percentile of bequest distribution normalized by income as reported by [Hurd and Smith \(2002\)](#).

The parameters that characterized the stochastic component of individuals productivity are $(\sigma_u^2, \varphi_z, \sigma_\epsilon^2)$. Several authors have estimated similar stochastic process for labor productivity. Controlling for the presence of measurement errors and/or effects of some observable characteristics such as education and age, the literature provides a range of $[0.88, 0.96]$ for φ_z and of $[0.10, 0.25]$ for σ_ϵ^2 . In this article, we rely on the estimates of [Kaplan \(2012\)](#), setting $\varphi_z = 0.96$ and $\sigma_\epsilon^2 = 0.02$. Then, σ_u^2 was chosen for the Gini index for labor income in the model to match its counterpart in the data, which is nearly 0.45. The value obtained for σ_u^2 is in line with the estimates in [Kaplan \(2012\)](#) who provides a point estimate of 0.06 for this parameter. We discretize the two shocks in order to solve the model, using seven grid points to represent the permanent shock and eleven grid points for the persistent shock. For expositional convenience, we refer to the two extremes of the grid for the permanent shock as low and high ability.

The values of the technological parameters (α, δ) are also in Table 1. We chose a value for α based on the U.S. time series data from the National Income and Product Accounts (NIPA). The depreciation rate, in turn, is obtained by $\delta = \frac{I/Y}{K/Y} - g$. We set the investment-product ratio I/Y equal to 0.25 and the capital-product ratio K/Y equal to 3.2. The economic growth rate, g , is constant and consistent with the average growth rate of GDP over the second half of the last century. Based on data from Penn-World Table, we set g equal to 2.3%, which yields a depreciation rate of 5.8%.

The age-efficiency profile for the exogenous model is set to be consistent with the values estimated in [Kaplan \(2012\)](#), which are based on the average hourly earnings by age in the PSID. We use a second order polynomial to smooth this profile and extend it to cover ages from 16 to 65.

Finally, we specify the others parameters related to government activity. First, we set government consumption, G , to 17% of the output of the economy under the baseline calibration. Following the literature, we assume a consumption tax of 6% and a capital income tax rate of 30%.¹⁹ The parameter ϱ , which governs the progressivity of the labor income taxes, is calibrated to match the actual average tax rates. Marginal tax rates are chosen to raise enough revenue to balance the government budget constraint. The value

¹⁹See, for example, [Fuster et al. \(2007\)](#)

Table 1: Parameter Values - Baseline Calibration

Parameter	Value	Source/Target
β	0.976	$K/Y = 3.2$
γ	4.00	Micro evidence
ρ_t	-	age-profile of mean hours
χ_t	-	age-profile of employment
σ_u^2	0.06	Gini index of 0.45
φ_z	0.96	Kaplan (2012)
σ_ϵ^2	0.02	Kaplan (2012)
δ	0.06	Investment to GDP ratio
α	0.36	NIPA
B	0.89	$w = 1$
ϱ	0.09	Average tax rates
η_1, η_2	-24.5, 50.6	Wealth distribution
τ_c	0.06	Fuster et al. (2007)
τ_k	0.30	Fuster et al. (2007)
ξ	0.83	Balanced budget
τ_{ss}	0.062	S.S. legislation
$\theta_1, \theta_2, \theta_3$	0.90, 0.32, 0.15	S.S. legislation
y_1, y_2, y_{max}	$0.21y_m, 1.29y_m, 2.42y_m$	S.S. legislation

we find for ξ_0 is 0.83.²⁰

As for Social Security, we set the normal retirement age to 65, which corresponds to 45 in the model. Contributions are of the form $T_{ss}(y) = \tau_{ss} \min \{y, y_{\max}\}$, where $y_{\max}$ is the contribution ceiling. We set the payroll tax rate at τ_{ss} at 6.2%. The marginal replacement rates in the progressive Social Security payment schedule $(\theta_1, \theta_2, \theta_3)$ are set at their actual respective values of 0.9, 0.32 and 0.15. The bend points where the marginal replacement rates change (y_1, y_2) and the maximum earnings $y_{\max}$ are set equal to the actual multiples of mean earnings used in the U.S. Social Security system so that y_1 , y_2 and $y_{\max}$ occur at 0.21, 1.29 and 2.42 times average earnings in the economy.

5 The Baseline Reform

Our first set of results is displayed in Table 2 and Figures 1 and 2, which we explain next.

Impact Measures

Before we start describing our results some explanation regarding welfare and insurance measurements is due.

Inequality We measure inequality using the Gini coefficient. We consider the before and after tax incomes and welfare. We do a separate measurement for retirees.

Income is labor plus capital income for workers and social security benefits plus capital income for retirees.

Insurance We define the insurance coefficient ϕ through

$$\phi = 1 - \frac{\text{Cov}(d\tilde{c}_{i,t}, dz_{i,t})}{\text{Var}(dz_{i,t})} \quad (20)$$

where $\tilde{c} = \log(c)$ to measure the the amount of consumption insurance for each possible policy. ϕ is 0 if changes in z are translated directly into a variation in consumption with a unit coefficient. When idiosyncratic exogenous changes in productivity do not translate into chances in consumption, ϕ is equal to 1.

²⁰Recall that $\xi_0(1 - \varrho)y^{-\varrho}$ is the marginal retention rate, $1 - T'(y)$.

It is important to bear in mind that it is only when preferences are separable that a fully insured agent would not change his or her consumption in response to a change in the temporary idiosyncratic productivity term, $z_{i,t}$. Here, we allow for non-separability, which leads consumption to optimally respond to changes in z .²¹ Still, we follow the literature – e.g., — in using this measure, which should, however be taken with some caution.

Welfare The welfare variation (CEV) is calculated as follows: Let $V_1(\omega)$ denote the expected utility of an agent who starts life at state ω under the policy we aim at evaluating. Then, define

$$V_1^0(\omega) = \mathbb{E} \left[\sum_{t=1}^T \prod_{s=1}^t \psi_s (1 + \Delta)^{(1-\rho_t)(1-\gamma)} U_{t,0}(c_t, 1 - n_t) \right]$$

where $U_{1,0}(c_t, 1 - n_t)$ is the flow utility attained by the agent under the benchmark at age t . Our relevant measure of welfare variation is

$$CEV = \min_{\Delta} [\mathbb{E}_{\omega} V_1^0(\omega) - \mathbb{E}_{\omega} V_1^1(\omega)] . \quad (21)$$

Note how we take full advantage of our iso-elastic specification for preferences.

CEV is computed separately for low and high types (with regards to u) agents.

CEV Decomposition We decompose the CEV in variations that are due to improved insurance – Δ_{unc} —, and those that are due to a more efficient use of aggregate resources — Δ_{lev} . Define $C_{0,t}$ and $L_{0,t}$ as average consumption and average hours worked by t years old agents at the benchmark, respectively, and $C_{1,t}$ and $L_{1,t}$ as the same statistics

²¹The marginal utility of income changes according to $d \log U_c(t) = -d \log c_t + (1 - \gamma) d \log U(t)$. Thus, as long as the Bernoulli utility $U(t)$ varies due to changes in labor supply, consumption will optimally respond.

at the alternative tax system. This allows us to implicitly define Δ_{lev} through

$$\sum_t \beta^{t-1} \prod_{j=1}^t \psi_t \left(((1 + \Delta_{lev}) C_{0,t})^{1-\rho_t} (1 - L_{0,t})^{\rho_t} \right)^{1-\gamma} = \sum_t \beta^{t-1} \prod_{j=1}^t \psi_t \left(C_{1,t}^{1-\rho_t} (1 - L_{1,t})^{\rho_t} \right)^{1-\gamma}. \quad (22)$$

As for Δ_{unc} use $(c_{t,0}, l_{t,0})_t$, to define the benchmark equilibrium allocations, and $(c_{t,1}, l_{t,1})_t$, the equilibrium allocations under the alternative policy. Next, implicitly define p_0 and p_1 , through²²

$$\sum_t \beta^{t-1} \prod_{j=1}^t \psi_t \left(((1 - p_0) C_{0,t})^{1-\rho_t} (1 - L_{0,t})^{\rho_t} \right)^{1-\gamma} = \mathbb{E} \left[\sum_t \beta^{t-1} \prod_{j=1}^t \psi_t \left(c_{t,0}^{1-\rho_t} (1 - l_{t,0})^{\rho_t} \right)^{1-\gamma} \right],$$

and

$$\sum_t \beta^{t-1} \prod_{j=1}^t \psi_t \left(((1 - p_1) C_{1,t})^{1-\rho_t} (1 - L_{1,t})^{\rho_t} \right)^{1-\gamma} = \mathbb{E} \left[\sum_t \beta^{t-1} \prod_{j=1}^t \psi_t \left(c_{t,1}^{1-\rho_t} (1 - l_{t,1})^{\rho_t} \right)^{1-\gamma} \right].$$

Then,

$$\Delta_{unc} \equiv \frac{1 - p_1}{1 - p_0} - 1. \quad (23)$$

Warm Glow and Welfare With warm glow preferences it is the act of bequeathing that is valued by agents. Hence, the final allocation of resources need no longer suffice to define welfare. This raises some subtle issues for normative analysis. In particular, should the utility gain from bequeathing be included in welfare evaluation?

At stake is the fact that one could artificially increase welfare by inducing higher

²²In both expressions below, $\mathbb{E}$ denotes the unconditional expectation operator. That is, we take expectations with respect to all possible permanent types and histories.

bequests and transferring resources back to the donors. To avoid these possibilities the welfare assessment we use – equation (21) – ignores the intrinsic (warm glow) utility derived from the act of bequeathing.

Main Findings

We now describe the main findings for our baseline reform.

Policy Variables We start our analysis with age-independent contributions (second column) and then consider the optimal age-dependent policies (third column). All relevant statistics for each policy are compared with the benchmark equivalent (first column).

In the first four rows in Table 2 we compare the relevant policy parameters for the benchmark and the alternative policies we study. $\tau_{ss,0}$ is the contribution rate for the agent at the time he or she joins the labor market. The contribution rate, $\tau_{ss,t}$, increases with age by $\tau_{ss,1}/10$ per year but and is capped below by 0, i.e., $\tau_{ss,t} = \max \{ \tau_{ss,0} + \tau_{ss,1} \times t/10; 0 \}$. At the benchmark, the age-independent social security contribution rate is held at the statutory value of 6.2%. Because at the benchmark contributions are not age-dependent, $\tau_{ss,0} = 0.062$ and $\tau_{ss,1} = 0$.

The optimal age-independent contribution is slightly lower than the current one, at $\tau_{ss,0} = 0.05$. Another variable of interest is the rate of return on the NDC accounts, r_n . In all exercises, we adjust r_n in order for the total mass of benefits to remain fixed at the same percentage of GDP found in the data, $G_{ss}/Y = 3.98\%$. Very low values for r_n mean that the present value of benefits are very low when compared to the contributions. This increases the labor wedge, when compared to that induced by the labor income tax alone. When, instead, r_n is high compared to the rate of discount then the analysis is more involved: the labor wedge is reduced if the agent is not constrained — as equation (19) shows — but increased at the margin if the agent is borrowing constrained. Indeed, from the first term in (19), taxable income is reduced by τ_{ss} , hence the period retention rate, i.e., the fraction of an agents earned income which is held by the agent in that period is $1 - \tau_{ss} - T'(\tilde{y})[1 - \tau_{ss}]$. When we compare with (19) it is apparent that whereas the marginal retention for the period is unaffected by r_n , it does affect the labor wedge through the last term in curly brackets. By increasing r_n the Government reduces the labor wedge *for the same level of taxable income*.

The other important policy consequences of changing the return on the NDC's the Government is that wedges are altered in an age-dependent way. That is, if r_n is greater than the discount rate, a subsidy which *increases* in distance from retirement is being given to workers. One might desire to calculate the implicit rate of return for the benchmark system, but the non-linearity in the benefit system implies that there are multiple history-dependent rates of return. For the system based on age-independent contributions, r_n is found to be $r_n = 3.94\% > 3.54\% = r(1 - \tau_k)$. That is, for all unconstrained agents, social security contributions act as an age-dependent work subsidy. Of course, the younger the worker the more likely he or she is to be constrained, meaning that these subsidies mostly apply to middle-aged workers.

Another important direct measure of policy is presented in the last three lines of Table 2's top panel: the replacement ratio for low and high types. While the overall ratio is kept fixed, the replacement ratio drops significantly for the low type agents and increases for the high type agents. This is a direct reflection of the type of actuarial fairness that we alluded to in the introduction.

The parameter ξ_0 from the labor income tax schedule $T(y) = y - \xi y^{1-\varrho}$ adjusts to keep the government's budget balanced. The progressivity parameter ϱ is held fixed at the benchmark value, $\varrho = 0.09$, in all policies considered. The NDC deficit is financed with lower labor income taxes – higher ξ_0 – thus suggesting overall efficiency gains.

As for the optimal age-dependent contribution, it is increasing in age with no contributions for the first 32 working years; it is only at age 57 that contributions are first made. In all the exercises displayed in Table 2, $\tau_{ss,0}$ is adjusted in such a way that the social security revenue as a share of GDP is kept at its benchmark value, $R_{ss}/Y = 3.23\%$. For the system with age-dependent contributions, $r_n = 7.97\% > 2.98\% = r(1 - \tau_k)$: the rate of return on the notional account is higher than the market rate of return. For all those who are net savers, the contribution acts like a work subsidy. Given that productivity and earnings increase with age, the NDC system is reducing the overall progressivity of government policy. Also, in this case ξ_0 is increased – marginal and average tax rates reduced at all levels of y – relative to the benchmark.

Effect on Aggregate Variables The lines following those displaying the policy variables in Table 2 describe the aggregate variables of the economy. Moving to an NDC leads to increases in GDP, mild in the case of age-independent contributions, 1.45%, and large, 9.10%, in the case of age-dependent contributions. In both cases the capital stock

Current vs. NDC Systems: Baseline Reform

Variable	Benchmark	Age-Indep	Age-Depend
$\tau_{ss,0}$	0.062	0.050	-5.68
$\tau_{ss,1}$	—	—	0.99
r_n	—	3.94%	7.97%
ξ_0	0.817	0.820	0.833
Y	0.484	0.491	0.528
K/Y	3.208	3.253	3.508
Average Hours	0.418	0.422	0.427
Labor/hours	0.734	0.731	0.745
Labor force part.	0.901	0.900	0.912
w	1.009	1.018	1.062
r	5.22%	5.06%	4.26%
G_{ss}/Y	3.98%	3.98%	3.98%
R_{ss}/Y	3.23%	3.23%	3.23%
θ_{low}	0.65	0.32	0.42
θ_{mean}	0.34	0.34	0.33
θ_{high}	0.21	0.34	0.28
Gini			
Income Before Taxes	0.591	0.597	0.597
Income After Taxes	0.489	0.508	0.508
Assets	0.725	0.679	0.694
retirees	0.373	0.482	0.481
Insurance coefficient			
Low	0.375	0.384	0.388
all	0.322	0.330	0.315
High	0.345	0.360	0.341
Welfare			
CEV	—	-0.82%	4.81%
CEV low type	—	-1.03%	5.26%
CEV high type	—	-1.35%	2.43%
CEV uncertainty	—	-1.15%	-1.41%
CEV level	—	0.32%	6.24%

Table 2: Current and Optimal NDC System: The table displays the values for the policy parameters $\tau_{ss,0}$ and $\tau_{ss,1}$ which jointly determine the social security contribution according to $\tau_{ss,t} = \min\{\tau_{ss,0} + t\tau_{ss,1}/10, 0\}$, ξ_0 determining the level of taxes according to $T(y) = y - \xi y^{1-\rho}$; the equilibrium value for the variables GDP (Y), capital-output ratio (K/Y), average hours (Avg hours) conditional on working, Labor force participation, and Labor/hours, the ratio between hours weighted by productivity and average hours; wages (w), real interest rates (r); Social security expenditures (G_{ss}/Y) and revenues (R_{ss}/Y) as a share of GDP replacement ratio for each type (θ_i). Inequality is calculated using the GINI coefficient whereas the insurance coefficient is given by (20). Welfare (CEV) for the benchmark and the optimum is calculated for all agents and for the highest and lowest u agents using (21). CEV uncertainty is calculated from (23) and CEV level from (22). From left to right, the three columns display the results for the benchmark, the system with age-independent and the system with age-dependent contributions.

is larger, 2.87% and 19.30%, respectively, which explains most of the GDP growth we found; a simple accounting exercise indicates a direct impact of 1.03% and 6.95%, respectively. Hours and participation also increase, partially in response to higher wages, explaining the rest.

Note, however, that in the case of age-dependent contributions labor supply responses only takes place when contributions become positive, i.e., when the social security 'work subsidy' kicks in, despite higher wages and lower taxes. It feels a little odd that the optimal system would induce this type of non-smoothness, but recall that we are forcing both the size of the system, as measured G_{ss}/Y , and the deficit $G_{ss} - R_{ss}$ to remain at the benchmark values. This 'odd' policy is optimal within those that respect these constraints.

Many studies find welfare gains by reducing the size of social security and allowing the capital stock to replace transfers in a PAYG system as an important source of asset accumulation.²³ In a sense, these gains obtain by ignoring the 'legacy debt'. To avoid these issues, in all our experiments the SS system measured by the total amount of benefits as a fraction of GDP is held fixed. Our goal is to avoid imposing on a single generation the cost of financing the pensions of the currently old while still requiring them to save for retirement. Still, the changes in work incentives both across ages and productivity types encourages aggregate capital accumulation despite our keeping transfer promises as a share of GDP fixed.

Figures 1 and 2, for example, highlight the subtle ways in which the reform affects the pattern of savings through the non-separability in preferences. As incentives are created for an important increase in labor supply close to retirement, the complementarity between work and consumption, drives a pattern of accumulation that entails the accumulation of more assets up to the point where work incentives arise followed by a stronger decumulation at this point. We return to the role of capital accumulation in Section 6.5 where we force the capital stock to remain fixed by increasing the public debt.

Distribution The last three lines in the top panel of Table 2 display the reform impact on the replacement ratio on average and for high and low productivity agents. Note that the average replacement ratio does not change much. This is almost by construction, since we are holding both contributions and benefits fixed as a share of GDP. Things

²³The effect on capital accumulation is equivalent to reducing the public debt.

are very different when we consider the different types of agents. While in the benchmark there is a significant difference between the replacement ratio for low productivity agents, $\theta_{low} = 0.65$, and that for high productivity agents $\theta_{high} = 0.21$, under the NDC the values of $\theta_{low} = 0.42$ and $\theta_{high} = 0.28$ are clear sign that the system induce substantially less redistribution. Indeed, if we consider the case of age-independent, there is basically no difference in the replacement ratio for the two different groups.

The distribution of labor income is not significantly altered by any of the policies considered.²⁴ Before tax inequality does not vary in any meaningful way with the reforms. The same is not true for after tax inequality. In fact, the system is less progressive as one may find evidence in the difference between after and before tax inequality. It is not a large difference, though.

Both reforms have the important consequence of substantially increasing the Gini for retirees, which is higher than at the benchmark while substantially lower than that for whole population under any of the NDC reforms. Both reforms therefore approximate the inequality among retirees to that of the overall population. The very low Gini for retirees that we observe at the benchmark is probably associated with an inefficient policy. Indeed, if one thinks about the second best policy - from mechanism design programs, e.g., [Farhi and Werning \(2013\)](#); [Golosov et al. \(2016\)](#) – it entails an ever increasing dispersion of consumption along agents' working lives, which is the opposite of what the low Gini for retirees entails.²⁵

Of course, in the second best the planner must be able to control each person's savings, which is something we are not allowing here. Still, the underlying principle that incentives are more cheaply provided by spreading consumption in time is more general. This, which is the main force behind the ever increasing pattern of inequality along the life-cycle implies that inequality ought not to be lower for retirees.

While income inequality increases, wealth inequality declines. It is expected that, as the social security system becomes less distributive, the pattern of savings among the different groups becomes more similar. However, as one can see in the case of age-dependent contributions, it is the rapid decumulation of assets by high types that take place right before retirement that explains why wealth for this group is lower than at the benchmark for a relevant fraction of their lives. For low types, wealth is larger than at

²⁴The slight increase in inequality that we observe is most probably due to the impact of wage w increases.

²⁵A word of caution here. These results are true when preferences are separable in consumption and effort, which we are not assuming here.

the benchmark at all ages. It remains true that low types but not high types spend all their wealth as they approach the end of their lives.²⁶

Insurance and Welfare NDC systems are commonly said not to sufficiently provide insurance and redistribution. Indeed, as we have explained in the introduction, the use of NDC's make the social security system actuarially fair, thus eliminating all intragenerational distributive features of a budget balanced system.

Beyond the insurance against longevity, social security is not usually thought of as an insurance program. Still, the fact that in almost all countries agents are guaranteed some income in old age frees them to run assets down to zero if needed to smooth consumption. The NDC reform we study all but eliminates a guaranteed minimum income for the elderly poor.²⁷ An agent who has not accumulated any income in his or her account cannot afford to run down all his or her assets without risking starvation in retirement. Once agents start contributing, then the resources in their account play a similar role to that of any benefit promises. Indeed, since NDC accounts are illiquid, the balance in these accounts play, in principle, the same role of any social security promise. It is not clear in this case that the amount of insurance provided by an NDC system should differ substantially from that provided by the current system.

To assess whether this is the case, we must be able to measure insurance and redistribution both in the benchmark and under the NDC system. The first thing to note is that self-insurance through borrowing and savings may still be possible even when no proper insurance is available. In a world with homogeneous agents, no government consumption and perfect capital markets, we can measure the amount of insurance received by an agent by comparing the net present value of his consumption with the net present value of his earnings. In our model economy, only saving is allowed. This means that there is less consumption smoothing than it would be possible with perfect markets self-insurance. There is some room for self-insurance, but, because capital markets are not perfect, agents face the risk of being illiquid even when shocks are temporary.

Somewhat surprisingly, we observe an increase in the insurance coefficient if we move to an age-independent NDC system, with the insurance coefficient increasing for both low and high types. If, instead, one allows contributions to be age-dependent, then the overall insurance coefficient displays a slight decrease, but the effect is heteroge-

²⁶This is due to the non-homothetic warm glow motive – see eq. (5).

²⁷We retain a minimum consumption guarantee of 10% of mean income.

neous across types. Insurance is higher for low types and lower for high types.

Of course, at the end of the day, it is welfare we are interested in. The bottom panel of Table 2 shows that welfare decreases if we replace the current U.S. SS system by an NDC system with age-independent contributions. The small increase in efficiency does not compensate for the worse distributive/insurance properties of the new system. Welfare declines for both high and low productivity agents.

Things are different when we allow contributions to depend on age. In this case, welfare increases by almost 5% driven by a very large increase in welfare driven by the behavior of aggregate variables — $\Delta_{lev} = 6.24\%$ — which greatly compensates the worsening of distribution/insurance — $\Delta_{unc} = -1.41\%$. A substantial fraction of the welfare gain is, therefore, driven by a larger capital stock. And, what explains this larger stock? As one can see from Figure 1, asset accumulation is larger from the beginning of the life-cycle, which might seem counter-intuitive since it is only when work subsidies kick in that we see a large increase in labor force participation, hence earnings. Note, however, that the marginal utility of consumption increases with labor supply. By tilting work more towards later in life, the NDC system induces asset accumulation to start early on and ultimately leads to more accumulation.

6 Extensions and Robustness

6.1 Holding the tax system fixed

Thus far, we have held the social security deficit at the benchmark level. To accomplish this, the social security contribution was adjusted in such a way as to preserve R_{ss}/Y at the benchmark level, 3.23%. Changes in allocation lead to changes in tax revenues if tax schedules are held fixed. To avoid fluctuations in G and hold the overall Government budget at 16% of GDP, the benchmark value, ξ_0 , was adjusted in each exercise. The question we address now is what happens if we leave the tax system fixed and allow the social security deficit to vary. Were we simply to relax the assumption that the social security deficit was fixed then there would necessarily be a welfare gain. The nature of the question we try to answer here is, instead, to restrict the tax instruments at their benchmark values. In this case, contributions must be adjusted to hold the unified government budget at their benchmark level. Our findings are displayed in Table 3.

When compared to our baseline reform, a very slight decline in the contribution

Current vs. NDC Systems: Same labor income tax

Variable	Benchmark	Age-Indep	Age-Depend
$\tau_{ss,0}$	0.062	0.048	-2.52
$\tau_{ss,1}$	—	—	0.45
r_n	—	4.09%	11.88%
ξ_0	0.817	0.817	0.817
Y	0.484	0.491	0.523
K/Y	3.208	3.254	3.456
Average Hours	0.418	0.422	0.425
Labor/hours	0.734	0.731	0.749
Labor force part.	0.901	0.901	0.914
w	1.009	1.018	1.053
r	5.22%	5.06%	4.41%
G_{ss}/Y	3.98%	3.98%	3.98%
R_{ss}/Y	3.23%	3.05%	2.10%
θ_{low}	0.65	0.34	0.41
θ_{mean}	0.34	0.34	0.33
θ_{high}	0.21	0.32	0.29
Gini			
Income Before Taxes	0.591	0.597	0.597
Income After Taxes	0.489	0.508	0.509
Assets	0.725	0.679	0.691
retirees	0.373	0.482	0.491
Insurance coefficient			
Low	0.375	0.387	0.378
all	0.322	0.330	0.311
High	0.345	0.360	0.338
Welfare			
CEV	—	-0.77%	4.01%
CEV low type	—	-0.99%	4.61%
CEV high type	—	-1.36%	1.96%
CEV uncertainty	—	-1.14%	-1.19%
CEV level	—	0.36%	5.21%

Table 3: Current and Optimal NDC System: The table displays the values for the policy parameters $\tau_{ss,0}$ and $\tau_{ss,1}$ which jointly determine the social security contribution according to $\tau_{ss,t} = \min\{\tau_{ss,0} + t\tau_{ss,1}/10, 0\}$, ξ_0 determining the level of taxes according to $T(y) = y - \xi y^{1-\rho}$; the equilibrium value for the variables GDP (Y), capital-output ratio (K/Y), average hours (Avg hours) conditional on working, Labor force participation, and Labor/hours, the ratio between hours weighted by productivity and average hours; wages (w), real interest rates (r); Social security expenditures (G_{ss}/Y) and revenues (R_{ss}/Y) as a share of GDP replacement ratio for each type (θ_i). Inequality is calculated using the Gini coefficient whereas the insurance coefficient is given by (20). Welfare (CEV) for the benchmark and the optimum is calculated for all agents and for the highest and lowest u agents using (21). CEV uncertainty is calculated from (23) and CEV level from (22). From left to right, the three columns display the results for the benchmark, the system with age-independent and the system with age-dependent contributions.

rate is accompanied by a small increase in r_n , for the case of age-independent contributions. The social security deficit increases by 0.18% of GDP. Aggregate variables barely change when compared with the baseline, constant deficit, reform. The welfare loss is slightly lower.

If we allow contributions to be age-dependent, they are chosen in such a way as to start at 56 years. This is just one year younger than what was optimal when the social security deficit was held fixed. However, the contribution rate now grows at less than half the speed that was optimal when the social security deficit was held fixed. The social security deficit grows from 0.75% to 1.88% of GDP.

The impact on all aggregate variables is slightly attenuated when compared to our baseline reform. There is a slight decrease in hours but a slight increase in labor force participation.

When compared with the benchmark allocation, both hours and labor force participation still increase as a consequence of increased work incentives, both due to the direct impact of social security contributions and to changes in wages. Earnings and the associated tax revenues increase more than GDP, since wages increase, thus guaranteeing budget balance.

Almost all statistics are similar under either assumption about the social security deficit. As for welfare, we find that the system with fixed deficit represents a larger improvement from the benchmark than the system with a fixed labor income tax schedule. The difference is more than explained by Δ_{lev} , since Δ_{unc} is larger (smaller losses) in the latter case.

6.2 No Fiscal Exemptions

In the U.S. and in our baseline reform taxable income is calculated as total earnings minus social security contributions. Because the system is progressive, high levels of contribution imply reduced taxable income which lower income taxes for high earners combining for an overall reduction in progressivity. Although social security contributions typically add to income taxes in increasing wedges, they actually reduce them when $r_n > r(1 - \tau_k)$, at least for unconstrained workers. In all cases, they reduce progressivity when taxable income is defined net of social security contributions. Do these deductions increase welfare?

To answer this question we assume that labor income taxes are based on total earn-

ings, instead of earnings net of social security contributions. Because there is no deduction, marginal and average rates can be significantly reduced — $\xi_0 = 0.847$ against $\xi_0 = 0.817$ at the benchmark — while still preserving the overall government budget balance. In addition, contributions also decrease as $\tau_{ss,0}$ adjusts to keep the social security deficit unchanged. Welfare gains are much higher in this case — $CEV = 6.68\%$ versus $CEV = 4.01\%$ — despite very similar capital stocks, and are more than explained by CEV level. It is apparent that whatever the reason to define taxable income net of contributions, it does not lead to welfare gains.

6.3 Restriction $r_n = r(1 - \tau_k)$

In all our exercises, we have allowed the rate of return on the notional accounts to adjust to keep the size of the social security system, G_{ss}/Y , at its benchmark value. Whenever age-dependent contributions were used, they were accompanied by very high rates of notional return. Such policy choices converted social security contributions into large work subsidies at the later stages of one's working life. The purpose of the next exercise is to evaluate how restricting the return r_n not to differ from the net return that agents can obtain in capital markets changes optimal policy. Since r_n is equal to $r(1 - \tau_k)$ in this exercise, we allow $\tau_{ss,0}$ to adjust to keep G_{ss}/Y unchanged.

The most surprising feature of this policy is that it leads to a large social security surplus — 2.02% of GDP. Since r_n is much lower than in the baseline reform — 2.62% as opposed to 7.97% — agents have to contribute more in order to keep the size of social security constant. In fact, $\tau_{ss,0}$ goes from -5.68 at the baseline reform to -5.33, increasing the social security revenue as a share of output, R_{ss}/Y , by 2.77% percentage points. This increase in turn allows labor income taxes to be reduced across the board with ξ_0 going from 0.817 to 0.879, inducing average hours worked to increase by almost twice as much compared as under the baseline reform — 4.24% as opposed to 2.15%. In contrast with the baseline reform, labor to hours ratio falls to 0.728, indicating that the larger increase in hours is concentrated among low skilled workers. Of all policies considered here this is the one that induces the largest increase in capital stock, 28%. Indeed, when such large increase in capital is considered little of the 11.77% increase in GDP remains to be explained.

Welfare gains are larger in this case — $CEV = 5.88\%$ —, with the gains in level more than compensating the decline in redistribution and insurance — $\Delta_{lev} = 8.20\%$

Current vs. NDC Systems: Robustness

Variable	Benchmark	No fiscal benefit	$r_n = (1 - \tau_k)r$	Same $T(\cdot)$
$\tau_{ss,0}$	0.062	-5.73	-5.32	-5.68
$\tau_{ss,1}$	—	0.99	0.99	0.99
r_n	—	8.11%	2.64%	7.97%
ξ_0	0.817	0.847	0.879	0.917
Y	0.484	0.530	0.541	0.523
K/Y	3.208	3.507	3.680	3.469
Average Hours	0.418	0.426	0.436	0.425
Labor/hours	0.734	0.749	0.728	0.746
Labor force part.	0.901	0.916	0.895	0.912
w	1.009	1.061	1.091	1.055
r	5.22%	4.26%	3.78%	4.37%
G_{ss}/Y	3.98%	3.98%	3.98%	3.98%
R_{ss}/Y	3.23%	3.23%	6.03%	3.23%
θ_{low}	0.65	0.41	0.42	0.42
θ_{mean}	0.34	0.33	0.33	0.33
θ_{high}	0.21	0.30	0.26	0.28
Gini				
Income Before Taxes	0.591	0.598	0.592	0.597
Income After Taxes	0.489	0.518	0.515	0.507
Assets	0.725	0.691	0.700	0.694
retirees	0.373	0.505	0.455	0.482
Insurance coefficient				
Low	0.375	0.349	0.383	0.394
all	0.322	0.296	0.314	0.318
High	0.345	0.333	0.348	0.342
Welfare				
CEV	—	8.15%	5.88%	3.12%
CEV low type	—	7.64%	5.99%	4.00%
CEV high type	—	7.87%	2.85%	0.57%
CEV uncertainty	—	-2.33%	-2.30%	-1.17%
CEV level	—	10.46%	8.20%	4.29%

Table 4: **Current and Optimal NDC System:** The table displays the values for the policy parameters $\tau_{ss,0}$ and $\tau_{ss,1}$ which jointly determine the social security contribution according to $\tau_{ss,t} = \min\{\tau_{ss,0} + t\tau_{ss,1}/10, 0\}$, ξ_0 determining the level of taxes according to $T(y) = y - \xi y^{1-\epsilon}$; the equilibrium value for the variables GDP (Y), capital-output ratio (K/Y), average hours (Avg hours) conditional on working, Labor force participation, and Labor/hours, the ratio between hours weighted by productivity and average hours; wages (w), real interest rates (r); Social security expenditures (G_{ss}/Y) and revenues (R_{ss}/Y) as a share of GDP replacement ratio for each type (θ_i). Inequality is calculated using the GINI coefficient whereas the insurance coefficient is given by (20). Welfare (CEV) for the benchmark and the optimum is calculated for all agents and for the highest and lowest u agents using (21). CEV uncertainty is calculated from (23) and CEV level from (22). The column in the middle assumes that income taxes are based on total earnings instead of income net of SS contribution while the rightmost holds the rate of return on the Notional Account.

against $\Delta_{unc} = -2.30\%$.

6.4 Same labor income tax and S.S. deficit

In the last column of Table 4, we also consider the possibility of a reform where both the SS deficit and the labor income tax function are kept at their benchmark values. In this case, we allow $\tau_{ss,0}$ to adjust to keep contributions as a fraction of GDP constant. Since ξ_0 is fixed, the wasteful spending, G , adjusts to balance the government budget constraint. As before, we use the return on notional accounts, r_n , to keep the size of social security benefits, G_{ss}/Y , unchanged.

When compared to the baseline reform, this exercise provides a sort of decomposition of the welfare gains associated with changes in ξ_0 . The reform still leads to relevant welfare gains — $CEV = 3.12\%$ — with gains concentrated in lower types, while the CEV for high types agents falls 76%. As in all reforms considered here, gains are driven by the level component — $CEV_{lev} = 4.29\%$ — that exceeds the losses due to worsened insurance and redistribution — $CEV_{unc} = -1.17\%$. Thus, allowing income taxes to adjust leads to a quantitatively relevant welfare improvement, explaining most of the gains for high productivity individuals. The fact that the capital stock does not increase as much — 17% against 19.4% in the baseline — does not appear to suffice for explaining the difference.

6.5 Fixed Capital Stock

As we have already mentioned, the NDC structure induces an increase in the demand for safe assets that induces a large increase in capital stock. This explains about 2/3 of the output growth, hence, a large fraction of welfare gains. What we do in this section is to ask what happens if the government responds to the increase in asset demand by issuing more public debt. With this policy response the capital stock remains fixed at its benchmark value. Toward this end, the asset market clearing condition is re-written as follows:

$$D = \sum_{t=1}^T \mu_t \int_{\Omega} d_{a,t}(\omega) d\lambda_t - K$$

where D denotes public debt.

Since changes in debt have sizable effects on labor income tax — making it harder

Current vs. NDC Systems: Alternative Adjustments			
Variable	Benchmark	Same $T(\cdot)$	K fixed
$\tau_{ss,0}$	0.062	-2.52	-2.52
$\tau_{ss,1}$	—	0.45	0.45
r_n	—	11.88%	12.31%
ξ_0	0.817	0.817	0.817
Y	0.484	0.523	0.489
K/Y	3.208	3.456	3.175
Average Hours	0.418	0.425	0.420
Labor/hours	0.734	0.749	0.743
Labor force part.	0.901	0.914	0.917
w	1.009	1.053	1.004
r	5.22%	4.41%	5.34%
G_{ss}/Y	3.98%	3.98%	3.98%
R_{ss}/Y	3.23%	2.10%	2.01%
θ_{low}	0.65	0.41	0.42
θ_{mean}	0.34	0.33	0.33
θ_{high}	0.21	0.29	0.29
Gini			
Income Before Taxes	0.591	0.597	0.587
Income After Taxes	0.489	0.509	0.507
Assets	0.725	0.691	0.676
retirees	0.373	0.491	0.506
Insurance coefficient			
Low	0.375	0.378	0.391
all	0.322	0.311	0.324
High	0.345	0.338	0.353
Welfare			
CEV	—	4.01%	0.90%
CEV low type	—	4.61%	-0.14%
CEV high type	—	1.96%	3.03%
CEV uncertainty	—	-1.19%	-1.50%
CEV level	—	5.21%	2.40%

Table 5: **Current and Optimal NDC System:** The table displays the values for the policy parameters $\tau_{ss,0}$ and $\tau_{ss,1}$ which jointly determine the social security contribution according to $\tau_{ss,t} = \min\{\tau_{ss,0} + t\tau_{ss,1}/10, 0\}$, ξ_0 determining the level of taxes according to $T(y) = y - \xi y^{1-\varrho}$; the equilibrium value for the variables GDP (Y), capital-output ratio (K/Y), average hours (Avg hours) conditional on working, Labor force participation, and Labor/hours, the ratio between hours weighted by productivity and average hours; wages (w), real interest rates (r); Social security expenditures (G_{ss}/Y) and revenues (R_{ss}/Y) as a share of GDP replacement ratio for each type (θ_i). Inequality is calculated using the GINI coefficient whereas the insurance coefficient is given by (20). Welfare (CEV) for the benchmark and the optimum is calculated for all agents and for the highest and lowest u agents using (21). CEV uncertainty is calculated from (23) and CEV level from (22). The column in the middle holds SS and the labor income tax function. The rightmost column holds K fixed by allowing G to adjust.

to interpret the effects of holding capital fixed — we keep ξ_0 at its benchmark value and again adjust the government expenditure, G , to satisfy the government budget constraint. Thus, the results of this section — presented in Table 5 — are directly comparable to the results displayed in Table 3, which are also shown in Table 5 for convenience.

We find that, to absorb all the extra savings, the debt to GDP ratio increases by 76.88% of GDP! The additional debt services requires the government expenditures as a fraction of GDP, G/Y , to drop from 16% to 12.27%. While still positive, welfare gains are substantially lower than if the planner simply allowed the capital stock to increase — $CEV = 0.90\%$ as opposed to $CEV = 4.01\%$. A substantially smaller increase in CEV level — $\Delta_{lev} = 2.40\%$ with fixed capital versus $\Delta_{lev} = 5.21\%$ with capital adjustment — explains the difference. It is, however, worth noting that Δ_{unc} , too, is lowered with this policy.

Interestingly, in contrast with all other reforms considered here, which allow capital to adjust, it is now the high types who benefit — $CEV = 3.03\%$ — from the reform; low types experience welfare losses — $CEV = -0.14\%$. The introduction of age-dependent taxes induces an increase in asset demand that, in the other exercises, leads to a higher capital stock which benefits mostly the low productivity individuals. When the government responds to this higher demand by issuing more debt, it is exactly the low productivity agents that miss out on the potential gains.

Also important, if the reduction in G/Y were accounted for in the welfare analysis, then, for any reasonable assumption about the welfare impact of government expenditures it is unlikely that the overall welfare variation would remain positive. Of course, in contrast with the cases in which the capital stock is larger in the steady state, the fiscal slack that higher savings create allows the planner to reduce the dead weight burden in the transition. Hence, whether the government accommodates increased savings with more debt or simply allow for capital to accumulate determines whether welfare gains accrue along the transition or in the long run.

6.6 No Extensive Margin

Age-dependent taxes are typically rationalized through its tagging properties: age is an exogenous variable that is strongly correlated with productivity, a variable on which the planner would want to condition taxes. The appeal of such instruments may be reduced if workers are able to altogether leave the market at some moments in their careers. This

is exactly what happens in our economy, where workers respond to changes in taxes and contributions by both adjusting hours and by temporarily withdrawing from the labor market. How relevant is this margin for the importance age dependent contributions? This is the question we go after in this Section.

Despite the theoretical differences, we find small changes in welfare gains compared to the baseline reform, 5.03% against 4.81%. Interestingly, the capital increase is essentially the same found in our baseline exercise. When we hold the income tax function fixed, our findings are once again very similar to those reported in Table 3. Contrary to our presumption, the extensive margin does not seem to reduce the power of age-dependent contribution.

7 Voluntary Contributions

While pensions systems around the world are typically based on mandatory contributions, in most cases they are also supplemented by incentives for savings toward retirement.²⁸ The underlying view is that using a voluntary system one avoids the tax-like aspect of social security financing. In this section we take this aspect to the limit by ridding the system from all forms of mandatory contribution. We consider an NDC system in which all social security contributions are voluntary. Arguably, if contributions were not compulsory there would be no (additional) distortions in labor supply choices induced by a social security scheme.

It is important to emphasize that the reform we propose is not equivalent to eliminating the social security system. First, we allow for contributions to social security to have favorable tax treatment, $r_n \neq r(1 - \tau_k)$. Second, we choose this rate to induce the system to preserve its size, thus avoiding the aforementioned spurious gains due to ignoring the legacy debt. Finally, NDC's are very special assets as we shall explain next.

The system provide important incentives for participation. First, if the social security rate is larger than the rate that agents can earn by investing in bonds this provides a reason. Second, social security 'investments' are converted in annuities upon retirement. Because the only risk faced by agents after retirement is the longevity risk, this is a very desirable asset to hold. Finally, there is another institutional detail that interacts may play a role in determining a motive for social security voluntary contributions. When

²⁸Favorable tax treatment of retirement savings, e.g., 401 (k) plans, are a very common example of such policies.

Current vs. NDC Systems: Intensive margin only

Variable	Benchmark	Fixed S.S. Deficit	Fixed Tax rate
$\tau_{ss,0}$	0.062	-4.848	-1.920
$\tau_{ss,1}$	—	0.850	0.35
r_n	—	7.89%	10.77%
ξ_0	0.821	0.836	0.821
Y	0.494	0.537	0.531
K/Y	3.174	3.484	3.435
Average Hours	0.401	0.413	0.411
Labor/hours	0.796	0.786	0.787
w	1.003	1.058	1.049
r	5.34%	4.33%	4.48%
G_{ss}/Y	4.01%	4.01%	4.01%
R_{ss}/Y	3.26%	3.26%	2.27%
θ_{low}	0.66	0.374	0.336
θ_{mean}	0.34	0.333	0.33
θ_{high}	0.21	0.311	0.32
Gini			
Income Before Taxes	0.601	0.604	0.603
Income After Taxes	0.488	0.509	0.510
Assets	0.718	0.681	0.678
retirees	0.387	0.503	0.509
Insurance coefficient			
Low	0.399	0.405	0.403
all	0.349	0.341	0.339
High	0.385	0.366	0.363
Welfare			
CEV	—	5.03%	4.40%
CEV low type	—	4.70%	4.60%
CEV high type	—	1.55%	1.11%
CEV uncertainty	—	-0.85%	-0.28%
CEV level	—	5.89%	4.69%

Table 6: Current and Optimal NDC System: The table displays the values for the policy parameters $\tau_{ss,0}$ and $\tau_{ss,1}$ which jointly determine the social security contribution according to $\tau_{ss,t} = \min\{\tau_{ss,0} + t\tau_{ss,1}/10, 0\}$, ξ_0 determining the level of taxes according to $T(y) = y - \xi y^{1-\varrho}$; the equilibrium value for the variables GDP (Y), capital-output ratio (K/Y), average hours (Avg hours) conditional on working, Labor force participation, and Labor/hours, the ratio between hours weighted by productivity and average hours; wages (w), real interest rates (r); Social security expenditures (G_{ss}/Y) and revenues (R_{ss}/Y) as a share of GDP replacement ratio for each type (θ_i). Inequality is calculated using the GINI coefficient whereas the insurance coefficient is given by (20). Welfare (CEV) for the benchmark and the optimum is calculated for all agents and for the highest and lowest u agents using (21). CEV uncertainty is calculated from (23) and CEV level from (22). In this table only adjustments in the intensive margin are allowed. The exercise displayed in the middle column is identical to our baseline counterfactual while the exercise in the rightmost column holds labor income tax fixed.

$r_n > r(1 - \tau_k)$, social security contributions reduce labor wedges as one can see from (19).²⁹ The downside of saving for retirement through a notional account is that an NDC is a very illiquid investment. It does not substitute for other forms of investments for precautionary savings purposes.

As we have mentioned for the policy experiment we run here the planner chooses an implicit rate of return for social security which is high enough to entice agents to voluntarily contribute to the system but not too high to increase the size of the system. This means that, while many social security systems provide incentives for agents to save for retirement we make no such assumption here: we cannot a priori determine whether the rate of return r_n required to keep the system at its benchmark size is or lower than $r(1 - \tau_k)$.³⁰

We consider three different policies according that differ from one another by the contribution ceiling, i.e., the largest fraction of one's income that is allowed to be saved through the NDC. In all cases we were able to find a value for r_n which induced a Social Security System with a size measured as a percentage of GDP equal to the benchmark.

Main Findings Our main findings for this session are displayed in Table 7, and Figure 3. At the top of each column of Table 7 we define a ceiling for the voluntary contribution. This represent the maximum share of a workers' labor income that may be invested in the social security system. When this ceiling, $t_{\max}$, is set at 10% of a worker's income, average contributions decline from 6.2% to 3.85%. To induce agents to reach this value the planner must set a return on the notional accounts $r_n = 3.92\%$, which is larger than net return on the risk free asset $r(1 - \tau_k) \approx 3.54\%$. Although agents contribute less than in the benchmark, the higher return, r_n , keeps the system's size, as measured by total benefits paid, at 3.98% of GDP. Interestingly, the system's deficit actually declines. This is explained by the fact that it is mostly high earners that choose to contribute.

The pattern of contributions, displayed in Figure 3 is markedly different for the two values for the ceiling, showing that the constraint binds for a substantial fraction of savers. In both cases contributions are biased toward older ages, as one expects. It also

²⁹See also our policy exercise in Section 6.2.

³⁰Our search for such a rate relies on the idea of a positively (in r_n) sloped and continuous aggregate demand for social security. We have already offered some reasons why we should expect this to be the relevant case. In particular, the fact that social security is an illiquid asset that pays a *constant* flow *contingent* on one being alive, means that as one takes a larger position on it, its marginal value compared to an alternative investment which may be used for self-insurance.

Current vs. NDC Systems: Voluntary Contribution

Variable	Benchmark	Contribution Ceiling ($t_{\max}$)		
		10%	20%	30%
avg τ_{ss}	6.20%	3.85%	4.18%	4.26%
r_n	—	3.92%	3.82%	3.78%
ξ_0	0.817	0.823	0.828	0.830
Y	0.484	0.493	0.495	0.497
K/Y	3.208	3.254	3.281	3.290
Average Hours	0.418	0.423	0.424	0.424
Labor/hours	0.734	0.733	0.733	0.733
Labor force part.	0.901	0.903	0.902	0.903
w	1.009	1.018	1.022	1.023
r	5.22%	5.06%	4.97%	4.95%
G_{ss}/Y	3.98%	3.98%	3.98%	3.98%
R_{ss}/Y	3.23%	3.44%	3.73%	3.83%
θ_{low}	0.65	0.11	0.10	0.10
θ_{mean}	0.34	0.34	0.33	0.33
θ_{high}	0.21	0.31	0.28	0.27
Gini				
Income Before Taxes	0.591	0.594	0.594	0.594
Income After Taxes	0.489	0.501	0.497	0.496
Assets	0.725	0.675	0.680	0.682
retirees	0.373	0.480	0.462	0.457
Insurance coefficient				
Low	0.375	0.389	0.390	0.389
all	0.322	0.335	0.332	0.332
High	0.345	0.361	0.358	0.359
Welfare				
CEV	—	0.11%	0.73%	0.88%
CEV low type	—	0.13%	0.74%	0.86%
CEV high type	—	-0.30%	0.40%	0.67%
CEV uncertainty	—	-1.19%	-1.15%	-1.18%
CEV level	—	1.21%	1.89%	2.07%

Table 7: **Current and Optimal NDC System:** The first line displays the average contribution rate chosen by agents, and the second the rate of return on the notional accounts. ξ_0 determines the level of taxes according to $T(y) = y - \xi y^{1-\varrho}$. Following are the equilibrium value for the variables GDP (Y), capital-output ratio (K/Y), average hours (Avg hours) conditional on working, Labor force participation, and Labor/hours, the ratio between hours weighted by productivity and average hours; wages (w), real interest rates (r); Social security expenditures (G_{ss}/Y) and revenues (R_{ss}/Y) as a share of GDP replacement ratio for each type (θ_i). Inequality is calculated using the GINI coefficient whereas the insurance coefficient is given by (20). Welfare (CEV) for the benchmark and the optimum is calculated for all agents and for the highest and lowest u agents using (21). CEV uncertainty is calculated from (23) and CEV level from (22). The column labels $t_{\max} = x$ indicate the contribution ceilings. The column labels $t_{\max} = x$ indicate the contribution ceilings.

resembles the pattern induced by the mandatory age-dependent contribution when the contribution ceiling is 30%.

Prices, w and r , and most aggregate statistics for this experiment are not significantly affected by the reform: wages slightly increase, rates slightly decrease leading to similar hours and GDP.

As one can see from Table 7, what is really made different by the policy is the distribution of replacement ratios across groups of agents, which becomes very regressive. This is in contrast with the strong progressivity that characterizes the benchmark. Indeed, while the average replacement ratio is preserved with the reform, low type agents have their replacement ratio significantly reduced. High productivity agents, on the contrary, increase their replacement ratios. Because we are keeping the size of the system fixed, as r_n induces high productivity agents to increase their savings on this vehicle, it crowds out low productivity agents. What this behavior highlights is the fact that this system is more attractive at the margin for high productivity agents, which has probably to do with the fact that they are on average more wealthy at birth as well.

Despite this change in replacement ratios, when compared to our baseline reform, the inequality among retirees does not change when the ceiling is kept at 10% and it actually reduces when the ceiling is raised first to 20%, then to 30%. This change in inequality among retirees is however compensated by a better distribution of income across workers: the Gini coefficient for before and after taxes income barely changes, while the Gini for retirees display a substantial increase. We had previously suggested that there is too much redistribution for retirees. All reforms considered in this section which seems to be moving in the right direction from this perspective. However, while there are important gains they are not overwhelming: in the case of a low ceiling, a worsening of distribution and insurance — $CEV_{unc} = -1.19\%$ — all but erases the efficiency gains — $CEV_{lev} = 1.21\%$ — attained by the reform.

The reform also induces a change in hours worked from later to earlier periods of one's career, a move away from what we would expect from the wage-efficiency profile. This may be caused by the impact of social security contribution on (effective) marginal tax rates.

When the ceiling is raised to 20%, then a notional return $r_n = 3.82\%$ leads to the same level of benefits as percentage of GDP as in the benchmark. This is higher than the net return on the risk free asset, $r(1 - \tau_k) \approx 3.48\%$, which shows that social security investments are more highly valued than the risk free asset. The top middle

panel in Figure 3 makes clear that agents wait until their income is highest to contribute, thus profiting most from the tax deduction. Somewhat surprisingly, we do not see a large difference in the life-cycle behavior of consumption, hours, participation and asset accumulation. The welfare gain we find, $CEV = 0.73\%$, while far from trivial are much lower than what we attained with mandatory, age-dependent contribution.

When we raise the contribution ceiling to $t_{\max} = 30\%$ welfare gains are slightly larger. Almost all aggregate statistics are very similar to those with $t_{\max} = 20\%$. While this is not conclusive, it suggests that the gains from further increasing the ceiling should not be very large.

Before we end this session it is important to insist on the fact that our results do not rely on removing what is the very nature of PAYG systems: the separation between aggregate and private savings. While one can read our exercises as implying the elimination of a proper social security system, we have made sure that the welfare costs attached to the legacy debt were not eliminated.

8 Conclusion

We ask the counterfactual question of how much more welfare would be attained if instead of the current social security system the U.S. government relied on an NDC of equal size. By preserving the size of the system we isolate the issues of intra-generational redistribution and equity from those of inter-generational equity.

What we find is that it is not the move to an NDC that may per se increase welfare. It is the possibility of designing an age-dependent system that better aligns incentives that provides these gains. In all the exercises for which relevant gains obtain, these accrue mostly to the least productive individuals through general equilibrium effects: the incentives for asset accumulation induced by NDC's financed with age-dependent contributions lead to a higher capital stock that mostly benefit these agents whose wealth is essentially their labor. Our findings also suggest that there is too much redistribution/insurance at old age in the current system, but not in the system as a whole. That is, in all the reforms that we consider, only those for which the Gini among retirees increase substantially welfare gains obtain. They are not accompanied by as much increase in the income Gini for the population as a whole and an actual decrease in the wealth Gini.

We also show that eliminating all forms of mandatory contribution is no panacea. Welfare increases when contributions are voluntary, with incentives designed to keep

the size of the system fixed, but substantially less than under the optimal reform which preserves the mandatory nature of the system.

References

- S Rao Aiyagari. Uninsured idiosyncratic risk and aggregate saving. *Quarterly Journal of Economics*, 109(3):659–684, 1994. [2.3](#)
- James Andreoni. Giving with impure altruism: Applications to charity and ricardian equivalence,. *Journal of Political Economy*, 97(6):1447–1458, 1989. [2.3](#)
- Alan J. Auerbach and Ronald Lee. Notional defined contribution pension systems in a stochastic context: Design and stability. In Jeffrey Brown, Jeffrey Liebman, and David A. Wise, editors, *Social Security Policy in a Changing Environment*, chapter 2, pages 43 – 68. University of Chicago Press: Chicago IL, 2009. [8](#)
- Felicite Bell and Michael Miller. Life tables for the united states social security area 1900-2100. Actuarial Actuarial Study 120, Social Security Administration, 2005. [4](#)
- Roland Bénabou. Unequal societies: Income distribution and the social contract. *American Economic Review*, 90(1):96–129, 2000. [2.4](#)
- Roland Bénabou. Tax and education policy in a heterogeneous-agent economy: What levels of redistribution maximize growth and efficiency? *Econometrica*, 70(2):481 – 517, 2002. [2.4](#)
- Sören Blomquist and Luca Micheletto. Age-related optimal income taxation. *Scandinavian Journal of Economics*, 110(1):45 – 71, 2008. [10](#)
- Juan Carlos Conesa, Sigiri Kitao, and Dirk Krueger. Taxing capital? not a bad idea after all. *American Economic Review*, 99(1):25–48, 2009. [2.3](#)
- Carlos E. da Costa and Marcelo Santos. Age-dependent taxes with endogenous human capital formation. *International Economic Review*, forthcoming, 2017. [10](#)
- Mariacristina De Nardi. Wealth inequality and intergenerational links. *Review of Economic Studies*, 71(3):743 – 768, 2004. [2.3](#), [12](#)
- Peter A. Diamond and Peter A. Orszag. *Saving Social Security: A Balanced Approach*. Brookings Institution Press, 2004. [1](#)
- Andrés Erosa and Martin Gervais. Optimal taxation in life-cycle economies. *Journal of Economic Theory*, 105:338–369, 2002. [10](#)
- Emmanuel Farhi and Iván Werning. Insurance and taxation over the life cycle. *Review of Economic Studies*, 2013. forthcoming. [9](#), [5](#)

- Martin Feldstein. The effects of taxation on risk taking. *Journal of Political Economy*, 77(5):755 – 764, 1969. [2.4](#)
- L. Fuster, A. Imrohoroglu, and S. Imrohoroglu. Elimination of social security in a dynastic framework. *Review of Economic Studies*, 74:134–45, 2007. [1](#), [19](#)
- John Geanakoplos, Olivia S. Mitchell, and Stephen P. Zeldes. Social security money’s worth. In Olivia S. Mitchell, Rober J. Myers, and Howard Young, editors, *Prospects for Social Security Reform*. University of Philadelphia Press, 1999. [1](#)
- M. Golosov, M. Troshkin, and A. Tsyvinski. Redistribution and social insurance. *American Economic Review*, 106(2):359–386, 2016. [9](#), [5](#)
- Jonathan Heathcote, Kjetil Storesletten, and Ginaluca Violante. Optimal tax progressivity: An analytical framework. *Quarterly Journal of Economics*, 132(4):1693 – 1754, 2017. mimeo. [2.4](#)
- Robert Holzmann and Edward Palmer, editors. *Pension Reform: Issues and Prospects for Non-financial Defined Contribution (NDC) Schemes*. The World Bank, 2006. [1](#)
- Michael D. Hurd and James P. Smith. Expected bequests and their distribution. *NBER WORKING PAPER SERIES*, 2002. [4](#)
- Marek Kapička. Quantifying the welfare gains from history dependent income taxation. Mimeo. Cerge, January 2020. [11](#)
- Greg Kaplan. Inequality and the lifecycle. *Quantitative Economics*, 3(11-014):471–525, 2012. [4](#), [1](#), [4](#)
- Marios Karabarbounis. A road map for efficiently taxing heterogeneous agents a road map for efficiently taxing heterogeneous agents a road map for efficiently taxing heterogeneous agents. *American Economic Journal: Macroeconomics*, (forthcoming), 2016. [10](#)
- Kevin M. Murphy and Finis Welch. Perspectives on the social security crisis and proposed solutions. *American Economic Review*, 88(2):142 – 150, 1998. [1](#)
- Richard A Musgrave. *The Theory of Public Finance*. New York, 1959. [2.4](#)
- Mariacristina De Nardi and Fang Yang. Bequests and heterogeneity in retirement wealth. *European Economic Review*, 2014. [4](#)
- Shinichi Nishiyama and Kent Smetters. Does social security privatization produce efficiency gains. *Quarterly Journal of Economics*, 122(4):1677 – 1719, 2007. [6](#)
- William B. Peterman. The effect of endogenous human capital accumulation on optimal taxation. *Review of Economic Dynamic*, forthcoming, 2016. [10](#)
- Stephanie Stantcheva. Optimal taxation and human capital policies over the life cycle.

[Lead article] *Journal of Political Economy*, 125(6):1931 – 1990, 2017. [9](#)

William S. Vickrey. *Agenda for Progressive Taxation*. The Ronald Press Company, 1947. [11](#)

Matthew Weinzierl. The surprising power of age-dependent taxes. *Review of Economic Studies*, 78(4):1490–1518, 2011. [9](#)

A Figures

Figure 1: **Benchmark and Optimal Allocations - Mandatory Contributions:** The three panels above display the evolution of labor market participation, consumption and assets along the life-cycle for the baseline reform. For participation, only the ages before retirement are displayed.

Figure 2: **Benchmark and Optimal Allocations - Mandatory Contributions:** The two panels above display the consumption and assets along the life-cycle for low and high type agents for the baseline reform.

Figure 3: **Benchmark and Optimal Allocations - Voluntary Contribution:** The four panels above display the evolution of the average contribution rate, labor market participation, consumption and assets along the life-cycle. For participation, only the ages before retirement are displayed. We consider two alternative contribution ceilings: 10% and 30%.

Os artigos publicados são de inteira responsabilidade de seus autores. As opiniões neles emitidas não exprimem, necessariamente, o ponto de vista da Fundação Getulio Vargas.

EPGE Escola Brasileira de Economia e Finanças

Diretor Geral: Rubens Penha Cysne

Vice-Diretor: Aloisio Araujo

Coordenador de Graduação: José Gustavo Feres

Coordenadores de Pós-graduação Acadêmica: Humberto Moreira & Lucas Jóver Maestri

Coordenadores do Mestrado Profissional em Economia e Finanças: Ricardo de Oliveira Cavalcanti & Joísa Campanher Dutra & Maria Teresa Duclos