Ensaio Econômico

Escola de

Pós-Graduação

em Economia

da Fundação

Getúlio Vargas

Nº 786

ISSN 0104-8910

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Outubro de 2017

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S. Iachan, Felipe
Capital Budgeting and Risk Taking Under Credit
Constraints/ Felipe S. Iachan - Rio de Janeiro : FGV,EPGE,
2017

44p. - (Ensaio Econômico; 786)

Inclui bibliografia.

CDD-330

Capital Budgeting and Risk Taking Under Credit Constraints

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FGV/EPGE

April 29, 2016

Abstract

Credit constraints generate a hedging motive that extends beyond purely financial decisions by also distorting the selection and operation of real investment projects. We study these distortions through a dynamic model in which collateral constraints emerge endogenously. The hedging motive can be broken down into three components: expected future productivity, leverage capacity, and current net worth. While constrained firms behave as if averse to transitory fluctuations in net worth, additional exposure to factors related to persistent productivity innovations or credit capacity fluctuations increases their value. The most constrained firms abstain from financial hedging while still distorting real decisions to reflect the hedging motive. Firm-level volatility is influenced by capital budgeting distortions, which contribute as a potential explanation for the higher volatility of lower net-worth firms.

Keywords: capital budgeting, credit constraints, project selection, investment, risk exposure.

JEL codes: G31, G32, E22.

1 Introduction

Limited external financing creates a hedging motive in corporate decisions, as improvements in the alignment between internal resources and investment opportunities create value.¹ This need for hedging can be reflected on the costly use of financial instruments such as futures and derivatives, but even in the absence of their use, the need induces distortions on the selection and operation of real investments. Capital budgeting, attitudes towards risk exposure, and financial planning become

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¹See Froot *et al.* (1993) for a canonical reference.

intertwined. They respond to a marginal value of resources to the firm, which differs from market prices and is shaped by the interaction of its own investment decisions and exogenous factors, such as the evolution of profitability and credit conditions. To shed light on these interactions, we study a dynamic model of project selection, investment, and financing under credit constraints.

When contemplating alternative capital investments, firms face projects with different exposures to risk factors, correlations with their core business, and financing possibilities. To make matters less abstract, let us illustrate these features with a stylized airline industry example. An airline might decide to expand into a specific route. By doing so, it exposes its revenues to demand factors that shape the fares it can charge and the occupancy rates it can achieve. The choice of operating a route between Boston and New York exposes cash-flows in a specific way to underlying risks in the industry, such as fluctuations in business travel or regional economic downturns. Other decisions involve the type of capital goods used: for example, more fuel-efficient planes reduce an airline's exposure to fuel price shocks. However a plane that is more efficient for a specific route might be less redeployable and thus face a thinner secondary market. As consequence, it can be less useful as collateral and expose the firm to more risk in its ability to secure financing.²

To formally study the proposed interaction, we analyze a model in which neoclassical firms choose investment projects understanding that each mix of projects requires a specific financial plan. This plan describes investment levels, borrowing, and hedging policies. Projects are allowed to differ on the revenues they generate, including their exposure to particular shocks, and also on the type of capital goods they use. The key set of financial constraints comes from limited enforcement of repayment promises and can be rewritten as simple collateral constraints, as in Rampini and Viswanathan (2010).

A brief summary of our results follows. Firms that find themselves constrained distort both real investment and financial decisions to reflect the value of internal funds across time and states of the world. When contemplating alternative projects, such firms also go beyond the evaluation of cash flows from operations and place a premium on a project's ability to attract cheaper collateralized financing. The most constrained firms borrow as much as possible and abstain from financial hedging, but still distort their real decisions to reflect the hedging motive. Additionally, whenever shocks to their revenues are sufficiently persistent, even a transitory increase in exposure to these shocks raises the value of the firm, making smaller firms more willing to take on these risks.

To understand the distortions in capital budgeting, it is useful to characterize what drives the variation of the marginal value of internal funds. This value can be described as a forward looking product of marginal returns until a moment in which the firm is sufficiently well-capitalized as to pay out dividends. Importantly, it is not a standard return on investment that matters but a levered return on internal funds, for which credit capacity plays a central role. Overall, we can identify three determinants of the marginal value of internal funds: expected productivity, leverage capacity,

²See Benmelech and Bergman (2009, 2011) for empirical evidence of the impact of collateral quality on credit conditions and their volatility.

and current net worth. We illustrate the operation of each one of these separately.

We first study the effects of productivity shocks with different degrees of persistence. Productivity here should be understood to broadly encompass total factor productivity, input costs, demand fluctuations, and the combination of any additional factors that shape investment profitability. Persistent shocks cause firms to value increased exposure to their underlying sources in the present. The origin of this effect lies in the need to partially self-finance. A persistent productivity improvement has two consequences. First, it increases cash-flows, which can be used to fund investment and mitigate the effects of credit constraints. Second, it also increases the desired investment levels and, as a consequence, the marginal value of investment.

Whenever a shock is sufficiently persistent, the second effect dominates. The firm's investment needs increase by more than cash-flows can cover. As a consequence the firm finds itself relatively more constrained after positive productivity innovations. Conversely, when the innovation is negative, investment drops by more than cash-flows contract and downsizing actually frees up resources. When contemplating persistent shocks, firms face tighter constraints during a growth phase that follows a positive innovation than during a downsizing phase that follows a negative one. As a consequence, they are willing to pay a premium for funds that correlate positively with their own productivity process. An investment project that additionally loads on any persistent determinant of future productivity, even before any information is revealed,³ becomes effectively more valuable to constrained firms. This is because it generates cash-flows that are better aligned with future investment needs. Therefore, financially-constrained firms are more willing to take any risk that correlates sufficiently with their future productivity.

This willingness to take on additional risk can show up as capital budgeting distortions, as we illustrate with a two-period simplified set-up. In this framework, firms face a risk factor with a given persistence and have access to two investment projects. The behavior of the risk factor in the long-run shapes the investment needs for the firm, while in the short-run it shapes the cash-flows. The projects differ in their short-run exposure only: one is riskier, having cash-flows that covary more with the risk factor. A capital budgeting distortion emerges. For reasonable levels of persistence, firms invest more in the riskier project due to the endogenous hedging demand, lowering its expected return below the one for the safer project. The distortion is present both for firms that use financial instruments for hedging and for firms that are so constrained as to refrain from using these instruments altogether. This result can help rationalize the capital budgeting distortions identified in Krüger *et al.* (2015), although through a rationale that is different and complementary to the one offered in that paper.⁴

³The convexity of the profit function with respect to prices and productivity originates from the possibility of adjusting factor demands and output to information about these factors, and is not the source of the attitude that favors increased risk-exposure, as we discuss later.

⁴In Krüger *et al.* (2015), the starting point of the analysis is the potential use of a rule-of-thumb behavior in organizations, that consists of maintaining a single weighted-average cost of capital (WACC) estimate for discounting cash-flows with heterogeneous risk exposures. As a consequence of the WACC Fallacy, more investment would flow into riskier divisions. The result we present uses a different reasoning instead considering a constrained optimal

This effect is exactly reversed regarding transitory shocks. These have little to no effect on the value of future investment but change cash-flows. As such, they only create a mismatch between available funds and optimal investment. In the limit case, productivity shocks without any persistence behave as exogenous fluctuations in net worth. While constrained firms might still fail to use financial instruments to hedge against those risks, they are willing to distort their real decisions whenever that reduces their exposure to transitory factors.

We also evaluate attitudes towards a possible tightening of credit constraints. Constrained firms are concerned about levered returns on their own funds, which rise when more credit can be obtained. As a consequence, as more leverage becomes feasible in a state with more relaxed credit conditions, they place a higher premium on internal resources. Analogously, they do not have incentives to ensure resources for situations in which credit conditions deteriorate and thus do not ensure against a possible credit contraction. Indeed, projects that show more exposure to credit conditions increase the value of the firm: a project that is more exposed generates more funds exactly when these can be more leveraged through additional borrowing. This result illustrates how fluctuations in credit conditions might induce additional risk-taking and why firms might opt not to hedge against states where credit becomes scarce.

While the results are initially studied analytically and illustrated through simple two-period examples, an infinite-horizon version of the model is evaluated quantitatively in the last section of the paper. We start from parameter values that are reasonable in view of previous empirical studies and endow firms with projects that have different exposures to the factors that shape profitability while having the same expectation. Constrained firms are shown to favor riskier projects, distorting investment levels in their favor and, because of decreasing returns to scale, leading to lower returns at the margin.

In this setting, firms have multiple instruments for dealing with possible shocks, as they can defer dividend payments, accumulate net worth, tailor the availability of those resources with financial hedging, and also distort their investment decisions. We show that each of these instruments is used.⁵ Even in the presence of many alternative instruments, distortions in capital budgeting are present. They are especially noticeable for firms that are smaller than the median of the stationary distribution and decrease for larger firms, stabilizing at small values for the top percentiles. Quantitatively, these distortions in capital allocation across projects are typically significantly smaller than differences that would be induced by a one percent return differential across investment projects.

We also show that the emergence of these distortions is not a consequence of the rich set of

response to the hedging motive induced by credit frictions. Nonetheless, for reasonably high persistence levels for the risk factors that shape revenues, the distortions induced go in the same direction.

⁵An empirical equivalent can be found in the foreign-exchange risk literature. Allayannis *et al.* (2001) shows that operational and financial risk management coexist, with the latter dominating quantitatively. Bartram *et al.* (2010), using a different strategy, indicates that financial instruments contribute to a 40% decrease in exposure, while operational risk management and price pass-through contribute to a 10-15% decrease.

Bartram (2008) and Choi and Jiang (2009) raise concerns about the measurement of risk exposure in the broader literature, emphasizing the effect of endogenous operational strategies that transform high underlying risk exposures into small measured fluctuations in outcomes.

financial instruments to which firms have access in the baseline framework, but the contrary. If firms can only issue and accumulate riskless assets, with the only contingency being provided by the behavior of dividends and real investment, then capital budgeting distortions increase in magnitude. The logic behind this result is that financial hedging is a substitute for capital budgeting distortions and that any additional impediments for financial planning push manifestations of the hedging motive towards real investment decisions.

We additionally conduct an extensive sensitivity analysis, which illustrates that the closer to constant returns to scale the revenue function is and the more persistent the shocks are, the larger the distortions. Interestingly, transitory shocks can change the sign of the hedging demand, as expected from the theoretical analysis, but only lead to distortions which are quantitatively negligible. The intuition for this asymmetry between transitory and persistent shocks is that dividend postponement and financial hedging are sufficient instruments for dealing with transitory shocks, but less effective against persistent ones given their long-lasting effects. Also, the more attractive borrowing is relative to lending, the larger capital budgeting distortions. The intuition for this result is the same as before: the more expensive potential substitutes for dealing with the hedging demand become, the more the hedging motive is manifested in capital budgeting.⁶ Last, we allow collateral constraints and prices to respond to economic uncertainty, and study their consequences. The key conclusion is that an anti-cyclical behavior of downpayment requirements has important amplification effects over the hedging demand.

Relationship to the literature - In its approach towards financial contracts, this paper follows Rampini and Viswanathan (2010, 2013), which propose a model in which enforcement constraints can be reduced to collateral constraints. This is similar to Kiyotaki and Moore (1997), but allows for the trading of state-contingent assets. The focus of those papers is on analyzing financing and risk management. Capital budgeting, as in most of the literature studying financial frictions, is reduced to the choice of the scale of investment in a single technology available.⁷ These papers have had success in explaining some empirical regularities and previous puzzles, such as the absence of risk management for the firms typically understood to be the most constrained, and the cross-sectional profile of leasing decisions.

The contribution of the current paper relative to this literature is twofold. Most importantly, we analyze capital budgeting, focusing on the consequences of a hedging motive that is displayed in real decisions even for firms that abstain from financial hedging. As in previous contributions, the most constrained firms choose to borrow as much as possible and do not leave any slack for financial hedging. Once project selection is taken into account, these firms in particular find that distorting real decisions becomes a useful tool, acting as a substitute for financial hedging. Additionally, in empirically reasonable cases, these distortions actually make constrained firms favor riskier projects.

⁶Some suggestive empirical evidence in favor of the balancing of relative costs of alternative instruments can be found in Petersen and Thiagarajan (2000) and Kim *et al.* (2006).

⁷Consider for instance Albuquerque and Hopenhayn (2004); Bolton *et al.* (2011); Clementi and Hopenhayn (2006); DeMarzo *et al.* (2012); He and Krishnamurthy (2012); Holmström and Tirole (1998); or Krishnamurthy (2003).

As an intermediate step towards studying project selection, we review the determinants of the hedging motive. While the risk management literature has particularly emphasized the role of variations in net worth in shaping this motive, it has devoted less attention to state-contingent factors behind the marginal value of funds. We shed light on its dependence on both the expected productivity of a marginal investment, which is intrinsically related to the persistence of shocks, and on leverage possibilities, which are related to credit conditions exogenous to the firm.

The current paper is also related to a literature on capital budgeting in environments with frictions, which has two main strands. The first strand studies allocative distortions and efficiency losses that originate from conflicts of interest between owners and privately-informed self-interested managers.⁸ In this paper, we study how capital market distortions might translate into distortions in capital budgeting, even in the absence of any such internal agency conflicts.

The second literature strand features a macroeconomic perspective. Stylized examples of project selection have appeared in literature concerned with the aggregate consequences of financial frictions.⁹ This paper contributes to that strand by providing a more thorough analysis of the incentives in investment selection, risk taking, and risk management among financially constrained firms. The simple assumption of decreasing returns to scale also adds predictions for behavior along the cross section of firms that are not present in the previous literature.

This paper also speaks to recent literature on endogenous volatility, which has attempted to better understand how trade-offs faced by firms help account for the empirical pattern of volatility across countries (Koren and Tenreyro, 2013) and along the business cycle (D’Erasmus and Moscoso Boedo, 2013). This paper adds to that discussion by illustrating first that financially constrained firms have incentives to load on persistent risks to facilitate self-financing. This not only helps to account for some empirical regularities in the higher volatility of smaller firms,¹⁰ but also points out that increases in risk exposure can actually be an optimal response to the limited access to external funds.

Some particularly closely-related papers deserve a longer discussion. Vereshchagina and Hopenhayn (2009) study entrepreneurial risk-taking, in the presence of borrowing constraints. They show that given that entrepreneurs have a real option to stop their projects and become employees in other firms, they effectively become risk-loving for sufficiently low wealth. As a consequence, they are willing to choose riskier projects, even in the absence of a premium, which helps account for the surprisingly low returns found in empirical studies of entrepreneurship. This result originates from

⁸For instance, Harris and Raviv (1996, 1998); Rajan *et al.* (2000); Stein (2002) study difficulties in the allocation of resources to a manager or multiple divisions with conflicting interests. An excellent survey of work prior to the last decade is available in Stein (2003).

⁹For instance, Aghion *et al.* (2010), Greenwood and Jovanovic (1990), and Matsuyama (2007, 2008). Eisfeldt and Rampini (2007) discuss how credit constraints affect the composition of investment across used and new vintages of capital and provide empirical evidence that more constrained firms favor cheaper, used capital.

¹⁰Consider the evidence for cross-country comparisons in Koren and Tenreyro (2007) which show that firms in less developed countries concentrate on more volatile sectors and on D’Erasmus and Moscoso Boedo (2013) which draw a comparison of mean volatility across the larger-firm COMPUSTAT database versus smaller firms for the Kauffman Firm Survey. Similarly, Davis *et al.* (2007) point out correlations between measures of growth-volatility and typical proxies of financial constraints such as size, age, and publicly-traded status.

a non-convexity in the value function that is induced by the discreteness in occupational choice. As a consequence, entrepreneurs are willing to hold more of any risks, even risk that is uncorrelated with the productivity of their activity. The risk-taking studied in the present paper, however, does not rely on such non-convexity. The value function is concave in net worth for each state, but the marginal value of funds is state-contingent. Therefore, constrained firms exhibit a hedging motive in their decisions and evaluate risks differently depending on how correlated they are with that value.

For the same reason, despite the presence of an enforcement problem, distortions in risk taking do not originate from the same mechanisms as in the risk shifting and asset substitution literature.¹¹ Since contracts properly account for possible deviations and assets are observable to competitive lenders, all investment distortions originate from the dispersion in marginal value of funds to the firm and not from a conflict of interest between equity and debt holders.

Another paper, Almeida *et al.* (2011), studies capital budgeting distortions induced by costly access to external funds. It relies on a reduced-form approach, describing the choices across a small number of pre-specified projects which differ in liquidity and riskiness. It makes a key assumption that projects are uncorrelated. As a consequence, it finds that more constrained firms should increase both financial and operational hedging and thus end up with lower volatility. This result is however at odds with the empirical evidence both across countries with different degrees of financial development and across firm types.¹² The current paper generalizes and qualifies their conclusions by illustrating formally how project changes can be evaluated and how firms react in different ways to shocks which are more or less informative about future opportunities.

Organization - The remainder of the paper is organized as follows. Section 2 proposes the baseline model of capital budgeting, financing, and risk management, and provides an analysis of the key endogenous variable behind firm's decisions: the marginal value of internal funds. Distortions in the allocation of capital budgets are the focus of Section 2.2, while Section 2.3 extends the general set-up to incorporate mutually-exclusive projects and irreversibilities. These general results are then specialized in Section 3 by imposing more structure on specific elements of the model, with examples that illustrate how firms evaluate exposure to productivity shocks with different degrees of persistence and credit capacity shocks. Section 4 provides quantitative evaluations in a steady state and a thorough sensitivity analysis. The final section concludes.

2 Model

We start by introducing a model of a firm's financial decisions, taking a set of available investment projects as given. This initial set-up extends the results from Rampini and Viswanathan (2010), a risk management model in which state-contingent borrowing is limited by endogenous collateral

¹¹Jensen and Meckling (1976) is the seminal reference of this literature. Landier *et al.* (2011) study investment selection in the presence of risk-shifting incentives and provide some empirical evidence based on mortgage origination.

¹²The cross-sectional evidence is reviewed in footnote 10. Additionally, in the empirical financial development literature, better creditor protection is linked to lower firm level volatility in Claessens *et al.* (2001), which studies cross-country firm level evidence.

constraints, to incorporate capital budgeting decisions. We first use this baseline model to analyze how limited credit, productivity innovations, and leverage possibilities shape the value the firm places on funds across states of the world. This marginal value of internal funds is the key variable driving distortions in corporate assessment of risky projects.

Guided by that discussion, we then further discuss project selection. Section 2.2 provides some general results on how potentially constrained firms assess investment in risky projects while Section 2.3 deals with irreversibility in decisions regarding project characteristics and mutually-exclusive projects, respectively. These results are later specialized through examples in section 3.

The benchmark set-up is the following. Time is discrete and indexed by $t = 0, 1, \dots, T$, with $T \leq +\infty$. Uncertainty is described by an exogenous event tree. The initial state s^0 is a singleton, and $s^t \in S^t$ denotes the history known at time t . We define the transition probabilities between node s^t and its successors s^{t+1} , $\pi(s^{t+1}|s^t)$, in the usual way and let $\pi(s^t)$ denote the unconditional probability of state $s^t \in S^t$ being reached.

The economy is populated by two types of risk-neutral agents. One has access to production technologies: we call them *firms*. The other group is composed of *lenders* who, without direct access to a production technology, provide external funding to firms.

A firm maximizes the expected discounted dividend stream according to

$$E \left[\sum_{t=0}^T \beta^t d_t \right],$$

where $\beta \leq 1$.

Different investment projects entail different exposures of cash-flows to the most relevant risk factors, such as input and output prices as well as both idiosyncratic and aggregate productivity shocks. Projects might also differ in other relevant ways such as by involving capital goods that can be more or less easily redeployed for alternative uses, serve as better collateral, have different exposures to price fluctuations, or have different depreciation rates. In our airline example from the introduction, these were embedded in the decisions of which routes to explore and which aircraft to choose.

We represent a project type by $j \in \mathcal{J} = \{0, 1, \dots, J\}$. We assume that project selection is observable to lenders and can be contracted on. For concreteness, we allow projects to differ along these three dimensions: how much output is generated in each contingency given previous capital investment, the price of the capital goods used by the project, and the project's recovery rate.¹³ We can think of the first as the exposure of cash-flows to risks, of the second as the fluctuations in the relevant cost of investment/divestment, and of the third as sensitivity to variations in credit conditions. Thus the project type determines the evolution of the production function, capital

¹³Input and output price changes can be thought of as comprising part of the fluctuations in the productivity of capital. Changes in depreciation rates and depreciation shocks represent only a small departure from the consequences of capital price changes and will not be discussed.

prices, and credit constraints as functions of the exogenous uncertainty embedded in s^t .

Formally a firm running a given project type $j \in \mathcal{J}$ uses a type-specific capital good, which is traded at a price $q^j(s^t)$. Capital k_{t+1}^j of this type, purchased and installed in state s^t , generates revenues $F^j(k_{t+1}, s^{t+1})$ and $(1 - \delta)$ units of depreciated s^{t+1} capital of the same type. Here $F^j(\cdot, s^{t+1})$ is a differentiable and concave production function.

Lenders have a discount factor of $R^{-1} \geq \beta$, have deep pockets, and are not subject to commitment problems, so they are willing to buy and sell contingent claims at an expected rate of return of R .¹⁴ Markets are complete in the sense that assets based on all contingencies can be traded, i.e., a full spanning notion. However, the firm's ability to issue claims on its output is limited by commitment problems.

At date t , after production takes place, a firm can renege on any of its outstanding debt. If that happens, lenders can only recoup a fraction $\theta^j(s^t)$ of the capital goods of type j . We will refer to $\theta^j(s^t)$ as a *recovery rate*. Therefore, given a level of capital goods $\{k_t^j\}_{j \in \mathcal{J}}$ used across the different projects, lenders can at most expect obtain to recover a total of ¹⁵

$$\sum_{j \in \mathcal{J}} \theta^j(s^t) q^j(s^t) (1 - \delta) k_t^j.$$

After reneging on its debt, the firm can go back to capital markets with net worth equal to all of the cash-flows it absconded with plus the fraction $(1 - \theta^j(s^t))$ of the depreciated capital stock of each capital good j .

An extension of Rampini and Viswanathan (2010) shows that, in this setting, the enforcement constraints to be imposed on the firm's problem greatly simplify: the outstanding level of debt in any state cannot exceed how much a lender would recover if the firm chose to default. Additionally, without actually imposing any restrictions on the maturity structure of repayments, the financial contract can be implemented with state-contingent short-term debt.

We use these results and write the firm's recursive problem as

$$V(w, s^t) \equiv \max_{d \geq 0, \{k^j \geq 0\}_{j \in \mathcal{J}}, \{b(s^{t+1})\}} d + \beta E_t [V(w(s^{t+1}), s^{t+1})] \quad (1)$$

subject to resource flow constraints,

$$w + R^{-1} E [b(s^{t+1})] \geq d_t + \sum_{j \in \mathcal{J}} q^j(s^t) k^j, \quad (2)$$

¹⁴Rampini and Viswanathan (2013) and Li *et al.* (2015) justify the reasonableness of a strict inequality in $R < \beta^{-1}$ with the tax-base reduction benefit of most debt instruments. A strict inequality additionally ensures that an unconstrained firm, subject to a sufficiently long sequence of negative shocks, can eventually find itself constrained again in the future, as in Cao *et al.* (2013) and Rampini and Viswanathan (2013).

¹⁵Allowing for recovery of a fraction of output would not lead to any major departure from the results presented later.

$$w(s^{t+1}) = \sum_{j \in \mathcal{J}} \{F^j(k^j, s^t) + q^j(s^{t+1})(1 - \delta)k^j\} - b(s^{t+1}), \quad (3)$$

for each $s^{t+1}|s^t$, as well as state-contingent collateral constraints

$$b(s^{t+1}) \leq \sum_{j \in \mathcal{J}} \theta^j(s^{t+1}) q^j(s^{t+1})(1 - \delta)k^j. \quad (4)$$

Here, w is the firm's net worth at s^t , k^j is the quantity it uses of the capital good specific to project j it chooses to deploy, and $b(s^{t+1})$ is the outstanding amount of debt that it leaves to be due at state s^{t+1} . We allow the problem to be non-stationary; the dependence on time is implicit in its dependence on the node s^t . A stationary structure with Markovian transitions is an important particular case, which we explore in section 4. This flexibility does not introduce any significant additional burden and is useful for allowing both the simple finite-horizon examples that serve as illustrations in Section 3 as well as a stationary infinite-horizon problem.¹⁶

Equation 2 describes the origins of resources, with net worth and debt on the left-hand side, and their uses on dividend payments and capital purchases on the right-hand side. Equation 3 describes the evolution of net worth, taking into account cash-flow generated, the value of the capital stock, and promised debt repayments. The last set of constraints, in the form of equation 4, represents the endogenous borrowing constraints.

There is an even simpler formulation of the firm's problem. We can define the downpayment required per unit of capital good of type j in node s^t as

$$\wp^j(s^t) \equiv q^j(s^t) - E[R^{-1}\theta^j(s^{t+1})(1 - \delta)q^j(s^{t+1})|s^t] \quad (5)$$

and financial slack, or unused borrowing capacity, as

$$h(s^{t+1}) \equiv \sum_{j \in \mathcal{J}} \theta^j(s^{t+1})(1 - \delta)q^j(s^{t+1})k^j - b(s^{t+1}). \quad (6)$$

In expression 5, the downpayment requirement is defined as the minimum a firm needs to pay in order to deploy a unit of capital, i.e., how much it spends when it finances a purchase at a unit price $q^j(s^t)$ by borrowing all that lenders are willing to marginally lend against that collateral. In expression 6, *financial slack* is the difference between how much the collateral value of the firm's capital stock is in state s^t , i.e., the borrowing capacity of the firm against that state, and how much the firm is actually pledging to pay from that state onward. That is, a firm that borrows less than the maximum it could is said to be saving financial slack.

¹⁶For the infinite-horizon problem with Markovian transitions, one can apply Proposition 1 in Rampini and Viswanathan (2013) in order to ensure existence, uniqueness, and concavity of the value function, as convexity and monotonicity of the constraint set can be directly verified for 1. For finite-horizon set-ups, existence and uniqueness of the value function is established by moving recursively from terminal nodes towards their predecessors.

The firm's recursive problem can then be rewritten as

$$V(w, s^t) = \max_{\{d, \{k^j\}_j, \{h(s^{t+1})\}_{s^{t+1}|s^t}\} \geq 0} d + \beta E_t [V(w(s^{t+1}), s^{t+1})] \quad (7)$$

s.t.

$$w \geq d + E[R^{-1}h(s^{t+1})] + \sum_j \wp^j(s^t) k^j \quad (8)$$

and

$$w(s^{t+1}) = \sum_{j \in \mathcal{J}} \{F^j(k^j, s^{t+1}) + (1 - \theta^j(s^{t+1})) q^j(s^{t+1}) (1 - \delta) k^j\} + h^j(s^{t+1}). \quad (9)$$

The Envelope Theorem ensures that the multiplier on the first constraint equals the *shadow value of net worth* to the firm, $\frac{\partial V(w, s^t)}{\partial w}$.¹⁷ We will also call it the *marginal value of internal funds*, interchangeably.

The solution to the recursive maximization problem is then characterized by the following set of first-order conditions:

$$k^j : \beta E_t \left[\frac{\partial V(w(s^{t+1}), s^{t+1})}{\partial w(s^{t+1})} \left(\frac{\partial F^j(k^j, s^{t+1})}{\partial k^j} + (1 - \theta^j(s^{t+1})) q^j(s^{t+1}) (1 - \delta) \right) \right] \leq \wp^j(s^t) \frac{\partial V(w, s^t)}{\partial w}, \quad (10)$$

$$d : 1 \leq \frac{\partial V(w, s^t)}{\partial w}, \quad (11)$$

and

$$h(s^{t+1}) : \beta R \frac{\partial V(w(s^{t+1}), s^{t+1})}{\partial w(s^{t+1})} \leq \frac{\partial V(w, s^t)}{\partial w}, \quad (12)$$

each of which holds as an equality if the relevant choice variable is strictly positive.

Equation 10 represents the firm's capital investment Euler equation for project $j \in \mathcal{J}$. Guided by it, we go on to define the leveraged marginal return on investment in project j as

$$R_{lev}^j(k^j, s^{t+1}) \equiv \frac{\frac{\partial F^j(k^j, s^{t+1})}{\partial k^j} + (1 - \theta^j(s^{t+1})) (1 - \delta) q^j(s^{t+1})}{\wp^j(s^t)}.$$

This represents the variation in net worth induced by a marginal investment in capital good j associated to the maximum borrowing possible against its collateral value. We can therefore rewrite that investment Euler equation as

$$\frac{\partial V(w, s^t)}{\partial w} \geq \beta E_t \left[\frac{\partial V(w(s^{t+1}), s^{t+1})}{\partial w(s^{t+1})} R_{lev}^j(k^j, s^{t+1}) \right], \quad (13)$$

¹⁷As financial slack enters linearly the constraints, an interior solution is typically not guaranteed and Benveniste and Scheinkman (1979) does not apply without adaptations. One possibility for proving differentiability and the envelope result lies using Corollary 5.2 from Milgrom and Segal (2002) together with uniqueness in the Lagrangian multipliers from 7, which can be established by using the first-order conditions associated with problem 7.

which holds with equality whenever operation of that project takes place.

The capital accumulation equation indicates the importance of two endogenous variables: the value of internal funds and the marginal levered return. It also indicates that they are intrinsically related. Their behavior is key for understanding how credit constraints influence the decisions of constrained firms, not only in terms of financial planning, but also in terms of their real investment decisions. Therefore, we will take a deeper look at these variables first.

2.1 The value of net worth

Standard dynamic programming arguments establish that the value function, $V(w, s^t)$, is concave in w , so that the marginal value of net worth is decreasing.¹⁸ Additionally, when the production function is strictly concave, this marginal value reaches one for a sufficiently high net worth.

The concavity of the value function has been pointed out as a reason for risk management, along the lines of the argument first put forward by Froot *et al.* (1993): financially constrained firms become averse to fluctuations in net worth, since these fluctuations prevent firms from deploying adequate levels of capital across states of the world and create dispersion in the value of internal funds across these states. However, there is also state-dependence in the firm's problem. Besides net-worth, leverage possibilities and expected productivity play central roles, as we now illustrate concisely.

Let us first simply impose Inada conditions for project $j = 0$ to ensure its operation in all states. Let τ denote the random variable that describes the first dividend payment after time t , which is an endogenous decision. Then, at s^τ in which a dividend payment occurs,

$$\frac{\partial V(w, s^\tau)}{\partial w} = 1,$$

and at s^t

$$\frac{\partial V(w, s^t)}{\partial w} = E_t \left[\beta^{\tau-t} \prod_{h=t+1}^{\tau} R_{lev}^0(k^0(s^h), s^h) \right], \quad (14)$$

where $k^0(s^h)$ stands for the level of capital chosen at s^{h-1} .¹⁹

Therefore, the marginal value of resources within the firm in state s^t depends directly on the composition of the forward levered returns on investment up to the first moment the firm finds itself sufficiently capitalized and pays out a dividend. All else being equal, the worse-capitalized a firm is, the higher these returns are; the same is true for more-levered and more-productive firms.

As a sample case to build intuition, let the set of projects $\mathcal{J}$ be a singleton and let us temporarily drop the dependence of the notation on projects. Let time be finite and indexed by three dates,

¹⁸Proposition 1 in Rampini and Viswanathan (2013) extends to the current set-up.

¹⁹Equation 14 provides a measure of financial constraints and a moment condition that can be taken to data. The marginal value of internal funds can be computed as an expected product of levered returns until the next dividend payment. This path can actually be computed according to any investment in which the firm engages, so that many moments are generated.

$t \in \{0, 1, 2\}$, the production function be separable as $F(k_{t+1}, s^{t+1}) = A(s^{t+1}) k_{t+1}^\alpha$, with $\alpha \in (0, 1)$, and capital be fully pledgeable as in Kiyotaki and Moore (1997), so that $\theta(s^t) = 1$, for all t, s^t . We focus on $t = 1$, one period before liquidation dividends are paid out for sure.

There, whenever $\frac{\partial V(w, s^1)}{\partial w} > 1$, the firm is effectively constrained in its capital deployment decisions, and uses maximal leverage, investing all its net worth in capital by purchasing $k_2(s^1) = \frac{w(s^1)}{\wp(s^1)}$. In that case, the marginal value of internal funds is

$$\frac{\partial V(w, s^1)}{\partial w} = \beta \alpha \frac{E[A(s^2) | s^1]}{\wp(s^1)} k(s^1)^{\alpha-1} = \beta \alpha \frac{E[A(s^2) | s^1]}{\wp(s^1)^\alpha} w(s^1)^{\alpha-1}. \quad (15)$$

We can point out three effects in place. The expected productivity effect, embedded in the $E[A(s^2) | s^1]$ term, pushes resources towards being more valuable in higher productivity states. The leverage effect, embedded in the reciprocal of the downpayment requirement, increases the value of resources when the credit conditions are looser and the downpayment is lower. Notice that decreasing returns to scale dampen this effect, but do not change its sign. Finally, we have the induced risk aversion effect, which is the effect most emphasized in the risk management literature. This effect originates from the concavity of the production function and makes sure that, ceteris paribus, firms with lower net worth face more severe distortions, deploy less capital, and have higher marginal returns to investment.

There is an interesting consumption-based asset pricing analogy that can be drawn with the three effects behind equation 15. Merton (1975) solved a portfolio problem in continuous-time in the presence of underlying states that follow a diffusion. That solution is always explicit up to the determination of the value function, which can still be obtained in some particular cases, and it decomposes the demand for risky assets into two parts.

One is a *myopic demand*, in which the individuals choose their exposure to risky assets as a function of return differentials. The propensity to increase exposure to assets with higher returns is disciplined by the variance in their returns weighted by the endogenous concavity of the value function. In our set-up, the concavity that disciplines the demand for risky investment in projects is manifested in $\frac{\partial^2 V}{\partial w_t^2}(w(s^1), s^1) \leq 0$, which follows from differentiation of equation 15.

Additionally, the second component in the demand for risky assets is what Merton called the *hedging demand*. It emerges in the consumer's problem from the interaction of income and substitution effects that make $\frac{\partial V(w, s^1)}{\partial w}$ state-dependent.²⁰ In Equation 15, an analogous state-dependence manifests itself for the firm through the effects of future expected productivity and leverage possibilities. If both are increasing (decreasing) in the underlying state, so is the marginal value of funds, and a hedging demand will give firms incentives to increase (decrease) their exposure to that state.

We consider next how these three effects interact when firms evaluate alternative investment

²⁰In Merton's problem, with a diffusion for s_t , the cross partial $\frac{\partial^2 V(w, s^t)}{\partial w_t \partial s_t}$ is well-defined. Here, one needs to evaluate discrete changes of $\frac{\partial V(w, s^t)}{\partial w}$ across states.

projects, as well as how they shape the determination of the optimal financial policies. Unfortunately, unlike in Merton's problem, an explicit solution cannot be obtained due to the presence of non-negativity constraints and decreasing returns to scale. A few additional lessons about the risk attitudes of constrained firms and optimal distortions in their capital budgeting processes emerge nonetheless. For example, the importance of persistence in productivity processes in shaping the hedging demand is discussed in sections 3.1 and 4.2 while leverage and credit conditions are the subject of sections 3.2 and 4.3. The following subsection takes the marginal value of funds as given and discusses the induced distortions in resource allocation.

2.2 Evaluating projects

Suppose the firm always invests in some project $j = 0$ in its constrained optimal solution, which can be ensured by imposing Inada conditions for that project. Let that firm evaluate the relative gains from allocating additional resources to some project $j = 1$ relative to the baseline alternative, $j = 0$. From the capital investment Euler equations (eq. 13) for these two projects, it follows that

$$E_t \left[\frac{\partial V(w(s^{t+1}), s^{t+1})}{\partial w(s^{t+1})} (R_{lev}^1(k^1, s^{t+1}) - R_{lev}^0(k^0, s^{t+1})) \right] \leq 0, \quad (16)$$

with equality if investment occurs in both projects at the optimum. Whenever the inequality is strict when evaluated at $k^1 = 0$, the firm should refrain from investing anything in project $j = 1$.

As is standard in the asset pricing literature, we proceed towards rewriting that condition in a covariance form. To simplify the notation, we first define a return differential and a normalized marginal value of funds.

Definition. Let $\Delta R(k^1, s^{t+1}) \equiv R_{lev}^1(k^1, s^{t+1}) - R_{lev}^0(k^0, s^{t+1})$ denote the excess levered return between these two projects as a function of investment in project $j = 1$, and let

$$m(s^{t+1}) \equiv \frac{\partial V(w, s^{t+1}) / \partial w}{E_t[\partial V(w, s^{t+1}) / \partial w]}$$

denote the normalized marginal value of funds at $t + 1$ at the optimum.

Manipulation of expression (16) leads to a decision rule that implies a decision to not operate project $j = 1$ whenever

$$E_t[R_{lev}^1(0, s^{t+1})] + Cov_t(m(s^{t+1}), \Delta R(0, s^{t+1})) < E_t[R_{lev}^0(k^0, s^{t+1})]. \quad (17)$$

A few features call our attention. First, given that firms cannot borrow arbitrary amounts, project selection is always comparative: at the margin any two projects compete for internal funds

and become mutually exclusive. Under decreasing returns to scale, firms that are more constrained have higher leveraged marginal returns, and consequently, face naturally higher hurdle rates.

Second, the relevant return that is taken into account is a leveraged return, not a simple return on investment. A project that is capable of raising more collateralized financing allows for a lower downpayment, and as a consequence, requires fewer resources to be displaced from other profitable opportunities the firm might have.

Third, firms that are constrained take into account a covariance term: projects that pay out more in the states in which the value of internal resources is higher are preferred. A lower-return project might be preferred over a higher-return project if it pays out more in the states in which the firm is more constrained. Results in Section 3.1 illustrate that when productivity is persistent, firms are actually more constrained after positive, rather than negative, productivity innovations. As a consequence, equation (17) would indicate a positive covariance between $\frac{\partial V(w(s^{t+1}), s^{t+1})}{\partial w(s^{t+1})}$ and $R_{lev}^0(k^0, s^{t+1})$. It follows that diversification away from the baseline project lowers the value of the firm, even if the alternative project offers moderately higher expected returns.

Notice also that even in the absence of any technological interactions, such as economies of scope, frictions in access to external funding are capable of generating both substitution and complementarity across projects. Substitution is present when two contemporaneous projects that cannot be fully externally financed compete for the use of the firm's resources. A complementarity arises across time, since projects that offer payouts that covary positively with the marginal value of net worth help finance the firm's most productive investment opportunities. Therefore, although the firm is always maximizing the total net present value (NPV) of dividends by construction, it is not maximizing NPV project-by-project. A project is evaluated in light of its capital requirements, its ability to attract external funding, and its ability to generate additional funding for the most valuable investment opportunities.

Additionally, the standard net-present value criterion can be recovered as a particular case. If we make the discount factor of lenders and firms the same, by setting $\beta = R^{-1}$, and look at firms that are effectively unconstrained and pay out dividends at s^t , these face $\frac{\partial V(w, s^t)}{\partial w} = 1$. It follows that $\frac{\partial V(w(s^{t+j}), s^{t+j})}{\partial w(s^{t+j})} = 1$ holds for any node s^{t+j} which is a successor of s^t . Then equation 17 collapses back into the first-best rule of optimal investment: a firm should undertake a project if, and only if, it has a positive net-present value.

2.3 Myers' adjusted present value criterion, irreversible, and mutually exclusive decisions

In a seminal contribution, Myers (1974) was the first to study interactions of financing and investment decisions in the presence of frictions. The paper analyses a once-and-for-all investment decision in the presence of two exogenous functions that introduce the consequences of frictions in a reduced-form manner: a debt capacity function, which describes how much a firm could borrow as a function of investment in the different projects, and a value of cash-inflows, which allows the

value of the firm to increase more than one-to-one with each dollar of cash-flow generated.

We now show that our first-order conditions for capital investment in the different projects can be interpreted in light of Myers's seminal adjusted present value (APV) formulas. The main contribution of the current paper in this dimension lies in incorporating a recursive stochastic structure, and in describing the fundamentals behind objects that show up in Myers (1974) as shadow values and reduced-form functions that describe benefits of additional cash-flows or credit capacity.

First, we rewrite the capital investment Euler equation in a given project j (equation 10) as

$$\begin{aligned} & \beta E_t \left[\frac{\partial V(w(s^{t+1}), s^{t+1})}{\partial w(s^{t+1})} \left(\frac{\partial F^j(k^j, s^{t+1})}{\partial k^j} + q^j(s^{t+1})(1 - \delta) \right) \right] \\ & + E \left[\left(R^{-1} \frac{\partial V(w, s^t)}{\partial w} - \beta \frac{\partial V(w(s^{t+1}), s^{t+1})}{\partial w(s^{t+1})} \right) \theta^j(s^{t+1})(1 - \delta) q^j(s^{t+1}) | s^t \right] \leq q^j(s^t) \frac{\partial V(w, s^t)}{\partial w}, \end{aligned} \quad (18)$$

with equality whenever $k^j > 0$ and some capital is allocated to the project.

Equation 18 indicates that we can think of an adjusted present value criterion for capital investment as involving two benefits on the left-hand side: an adjusted discounted value generated by the marginal investment and a borrowing capacity change. On the right-hand side, we have the current capital cost of a marginal investment in project j properly weighted by the marginal value of internal funds, $\frac{\partial V_t(w(s^t), s^t)}{\partial w(s^t)}$.

The discounted value term includes both the marginal output and the liquidation value of the capital stock, and is analogous to the discounting of free cash-flows from projects. Importantly, however, it discounts these flows according to the marginal value to the constrained firm and not according to market prices. An additional term incorporates the consequences of capital purchases in enhancing borrowing capacity.

Marginal capital purchases increase a firm's ability to raise external financing to the extent that they increase the available collateral. Since borrowing is limited by commitment problems, these funds are possibly cheaper than internal funds. A premium on borrowing capacity emerges for firms that find themselves against their borrowing constraints.²¹ As a consequence of limited borrowing capacity, projects are not only evaluated by the net cash-flow increases they induce, but also according to their ability to attract cheaper collateralized funding. While the APV criterion became notorious in the practice and teaching of Applied Corporate Finance for taking into account the tax advantage of a relaxation of borrowing capacity, its original formulation allowed for more general benefits, as with the premium on collateralized borrowing that emerges in equation 18.

In the original APV derivation, decisions to undertake a project are made once and for all. In our setting, previously made decisions which cannot be revised at each period require keeping an

²¹From equation (12), a binding collateral constraint at s^{t+1} , which limits borrowing at s^t implies $R^{-1} \frac{\partial V_t(s^t)}{\partial w_t} - \beta \frac{\partial V_{t+1}(s^{t+1})}{\partial w_{t+1}} > 0$.

additional state variable in the description of value function. Indeed, this approach has additional benefits, as it allows some further insights to be derived.

For instance, under many circumstances a firm faces decisions that are mutually exclusive, since they involve different ways of conducting a single operation. For example, an airline might expand the the business class in a given route at the cost of having less seats available for the economy class. This is a decision that changes the behavior of its revenues, as the demand for business travel and economy-class travel have different exposures to shocks and price elasticities. Other decisions can change the type of capital goods used, hence the costs of investment and its usefulness as collateral. Although not entirely irreversible, many of these decisions might only be infrequently adjustable.

We model these partially-irreversible and mutually-exclusive decisions in a general way, by allowing the firm to choose an additional state variable that is carried over to the future. We denote this variable by a parameter γ from a menu Γ of mutually exclusive alternatives, where Γ is a closed interval in $\mathbb{R}$ for simplicity. In the case of an irreversible investment scale decision in a given project, $\gamma \in \Gamma$ denotes a previously taken decision regarding that level. Other mutually-exclusive decisions, for example the risk-exposure implicit in the allocation of a single aircraft and crew to one of multiple different markets, can be represented with the same notation.

Now, the firm's problem can be solved in two steps: for each fixed decision $\gamma \in \Gamma$ the optimal financial policy describing borrowing, investment, and hedging can be obtained. Let $V(w, \gamma, s^t)$ denote the value achieved when decision γ is implemented and net worth is w_0 . Thus

$$V(w, \gamma, s^t) \equiv \sup_{d \geq 0, \{k^j\}_j \geq 0, \{b(s^{t+1})\}_{s^{t+1}|s^t}} d + \beta E[V(w(s^{t+1}), \gamma, s^{t+1})] \quad (19)$$

subject to resource flow constraints,

$$w + R^{-1}E[b(s^{t+1})] \geq d + \sum_{j \in J} q^j(\gamma, s^t) k^j, \quad (20)$$

$$w(s^{t+1}) = \sum_{j \in \mathcal{J}} \left\{ F^j(k_{t+1}^j, \gamma, s^t) + q^j(\gamma, s^{t+1}) (1 - \delta) k^j \right\} - b(s^{t+1}), \quad (21)$$

for each $s^{t+1}|s^t$, as well as state-contingent collateral constraints

$$b(s^{t+1}) \leq \sum_{j \in \mathcal{J}} \theta^j(\gamma, s^{t+1}) q^j(\gamma, s^{t+1}) (1 - \delta) k^j. \quad (22)$$

Here $F^j(k^j, \gamma, s^t)$, $q^j(\gamma, s^{t+1})$, and $\theta^j(\gamma, s^{t+1})$ allow for dependence of revenue functions, capital-goods price processes, and recovery rates on the partially irreversible decision γ . For the case in which capital levels are not readjustable dynamically, as in the original Myers set-up, one can simply add a $k^j = k(\gamma)$ constraint without any additional modification.

If decision $\gamma \in \Gamma$ can be revised at node s^t , then the optimal decision is the solution to

$$\gamma^* \in \arg \max_{\gamma \in \Gamma} V(w, \gamma, s^t).$$

This framework can encompass both the case of a partially-irreversible investment decision made at s^t or the selection of a project γ in a menu of mutually-exclusive alternatives Γ .

We will proceed by characterizing the consequences of local changes in γ , as these are more tractable, and properties of large changes can always be obtained as an integral of marginal changes. We therefore evaluate locally the combined impact of changes in the cash-flow process, the expenditures on capital goods, and of different recovery rates which are induced by an potential change in a partially irreversible decision. The final impact of a different decision on the value of the firm is a composition of the effects through these three channels.

Towards that end, let us define the net investment at s^t in the standard way as $i^j(s^t) \equiv k^j(s^t) - (1 - \delta) k_t^j(s^{t-1})$, where $k^j(s^t)$ is used in production at $t + 1$. Additionally, we write the endogenous borrowing capacity from s^{t+1} towards its predecessor s^t as

$$BC(\{k^j\}_j, \gamma, s^{t+1}) \equiv \sum_j \theta^j(\gamma, s^{t+1}) (1 - \delta) q^j(\gamma, s^{t+1}) k^j(s^t), \quad (23)$$

and the firm's net cash-flow as

$$NCF(\{k^j(s^t), i^j(s^t)\}_j, \gamma, s^t) \equiv \sum_j [F^j(k^j, \gamma, s^t) - q^j(\gamma, s^t) i^j(s^t)]. \quad (24)$$

Given those two definitions, we can study the consequences of a project selection decision which changes both cash-flows and the firm's financing conditions. Expression 25 below can be seen as a generalization of equation 18 for the case of partially irreversible decisions and mutually-exclusive projects.

Proposition 1. *The shadow value of a marginal change in the partially irreversible decision $\gamma \in \Gamma$ is given by*

$$\frac{\partial V(w, \gamma, s^t)}{\partial \gamma} = E_t \left[\sum_{h=t}^{\tilde{T}} \beta^{h-t} \frac{\partial V(w^*(s^h), \gamma, s^h)}{\partial w(s^h)} \left(\frac{\partial NCF(k_h^*, i_h^*, \gamma, s^h)}{\partial \gamma} + \mu(s^h) \frac{\partial BC(k_h^*, \gamma, s^h)}{\partial \gamma} \right) \right], \quad (25)$$

where $\mu(s^h) \equiv \frac{\partial V(w(s^{h-1}), s^{h-1}) / \partial w(s^{h-1})}{\beta R(\partial V(w(s^h), s^h) / \partial w(s^h))} - 1 \geq 0$ is a normalization of the multiplier on the collateral constraint that limits borrowing between s^{h-1} and s^h and $\tilde{T} \leq \infty$ is the (potentially random) variable describing the next decision revision time.

Going back to the firm's choice among all possible alternatives, we can also use equation 25 to describe the firm's project selection in the following way. Any interior solution needs to satisfy

$\frac{\partial V(w, \gamma, s^t)}{\partial \gamma} = 0$. This also becomes a sufficient condition for an interior optimum whenever (23)-(24) define net-cash flows and borrowing capacity that are concave in γ . In this case, we can think in terms of a firm that takes as given the value of internal funds and the premium on borrowing capacity obtained from the operation of the optimal project, and acts as if maximizing the sum of discounted cash-flows plus the premium-adjusted borrowing capacity.

Even in more general cases, a root in equation (25) is a necessary condition for an interior optimum, allowing us to rule out projects that can never be optimal. It also serves to illustrate how different decisions, such as favoring a project with marginally riskier or safer cash flows, can influence the value of the firm. We use it for characterizing one of the examples in the next section, which involves a permanent choice of a type of capital goods from a set of mutually-exclusive alternatives.

3 Project selection and risk taking

After the general descriptions of the environment and criteria for evaluating project selection, we now study which qualitative consequences are induced by the hedging motive. For this purpose, we analyze two simple examples, which illustrate how limited access to external finance changes risk-taking incentives of firms. In both examples, there are three dates, $t \in \{0, 1, 2\}$. Discount factors are the same for firms and lenders, $\beta = R^{-1} = 1$. Additionally, in both environments, technologies are described by separable single-factor neoclassical production functions of the form $F^j(k_t^j, s^t) = A^j(s^t) f(k_t^j)$, with smooth and strictly concave $f(k_t) = k_t^\alpha$ for $\alpha \in (0, 1)$.

3.1 Persistent productivity

Uncertainty is described by a random variable ϵ which can take one of two values, $\epsilon \in \{\epsilon_l, \epsilon_h\}$, with $-1 < \epsilon_l < 0 < \epsilon_h$. The shock ϵ has zero mean and is fully learned at $t = 1$.²² As a consequence, states at $t = 1$ can be related one-to-one with the realization of ϵ . Each state $s^1 \in \{s_l^1, s_h^1\}$ of these has a unique successor, which we label $s^2 \in \{s_l^2, s_h^2\}$, and $\epsilon(s_t^t) = \epsilon$ for $t = 1, 2$ and $\iota = l, h$. We study the allocation of investment across two projects that have different exposures to the underlying state.

Project $j = 0$ has a revenue total factor productivity (TFP) which is described by

$$A^0(s^1) = \bar{A}(1 + \epsilon(s^1))$$

and

$$A^0(s^2) = \bar{A}(1 + \epsilon(s^2))^\rho,$$

for $s^1 \in \{s_l^1, s_h^1\}$ and $s^2 \in \{s_l^2, s_h^2\}$.

²²Any residual uncertainty would be inconsequential, as the firms pay out all of their resources as liquidation dividends at $t = 2$, so we abstract from it.

This particular stochastic process is chosen to incorporate, in a concise way, the consequences of innovations to the natural logarithm of TFP which decay at a rate ρ , as in the autoregressive process of order 1 commonly used in the empirical literature. This autocorrelation parameter ρ will be key for understanding endogenous risk-taking, since we will show that a firm's attitudes towards exposure to the shock ϵ intrinsically depend on its persistence.

Firms have access to another project, $j = 1$, which is safer in the short-run but which displays the same long-run behavior as project $j = 0$. Formally, its productivity at date $t = 1$ is described by

$$A^1(s^1) = \bar{A}(1 + \sigma\epsilon(s^1)),$$

for $\sigma < 1$, while its $t = 2$ productivity is again

$$A^1(s^2) = \bar{A}(1 + \epsilon(s^2))^\rho,$$

for $s^1 \in \{s_l^1, s_h^1\}$ and $s^2 \in \{s_l^2, s_h^2\}$.

As a consequence, the output from each project $j \in \{0, 1\}$ in each state s^t is given by

$$F^j(k_t^j, s^t) = A^j(s^t) (k_t^j)^\alpha,$$

for some parameter $\alpha \in (0, 1)$ describing the scale-elasticity. For simplicity, we assume that the capital goods used in both projects are identical, have unit prices, and are fully collateralizable, so that $\theta^0(s^t) = \theta^1(s^t) = 1$ always. We let depreciation be positive ($\delta > 0$), so the downpayment requirement is also constant across time and states of the world. We simply call this common downpayment requirement $\wp$.

Notice that by allocating funds to investment in the safer project ($j = 1$) as opposed to the riskier project ($j = 0$), the firm can reduce its short-run exposure to the shock and, as a consequence, lower the volatility of its cash-flows. The first-best capital allocation serves as a useful benchmark, as it requires equalization of expected marginal productivities. In this set-up, expected return equalization simply means equal capital investment across both projects. Therefore, by comparing quantities invested in each project, we can understand the underlying endogenous attitudes towards risk.

Our main theoretical result for this section, which will be further explored in the quantitative analysis later, is offered below.

Proposition 2. *Consider the model described above, in which credit-constrained firms allocate a $t = 0$ capital budget across a riskier ($j = 0$) and a safer ($j = 1$) project that share a common expected return. Then,*

1. *In the first-best, capital allocations are the same across both projects, i.e., $k_1^0(s^0) = k_1^1(s^0)$.*
2. *There exists a threshold for the persistence parameter, $\bar{\rho}(\alpha)$, such that*

- (a) whenever $\rho > \bar{\rho}(\alpha)$, firms invest more in the riskier ($j = 0$) than in the safer ($j = 1$) project. That is, $k_1^0(s^0) \geq k_1^1(s^0)$, with strict inequality for all firms that are constrained to invest below first-best levels at $t = 0$. Consequently, the expected marginal productivity in the safer project is higher than in the riskier project.
- (b) whenever $\rho < \bar{\rho}(\alpha)$, firms invest less in the riskier ($j = 0$) than in the safer ($j = 1$) project. That is, $k_1^0(s^0) \leq k_1^1(s^0)$, with strict inequality for all firms that are constrained to invest below first-best levels at $t = 0$. Consequently, the expected marginal productivity in the safer project is lower than in the riskier project.
- (c) the threshold function $\bar{\rho}(\alpha)$ is decreasing in α and satisfies $\bar{\rho}(\alpha) < 1 - \alpha$. Additionally, as $\sigma \rightarrow 1$, $\bar{\rho}(\alpha) \rightarrow 1 - \alpha$.

The intuition behind Proposition 2 is the following. There are two forces at play when a shock hits the firm: expected productivity and the availability of resources. Together they shape the marginal value of funds and determine whether resources are more valuable after a positive shock ($\epsilon_h > 0$) or a negative one ($\epsilon_l < 0$).

On one hand, a positive shock raises marginal returns from $t = 1$ onward for any fixed level of capital used. As a consequence, first-best levels of capital investments are higher after positive shocks than after negative ones. This effect is larger the more persistent is the shock, since persistence makes returns from additional investment more closely related to the current conditions. This effect is also stronger the closer the production function is to constant returns to scale ($\alpha = 1$), since the returns to scale parameter α shapes how strong the optimal response of investment is to TFP.

On the other hand, cash-flows also increase upon a positive productivity shock. As a consequence, firms have more resources to employ after a positive shock than after a negative one. In the presence of decreasing returns to scale, this force makes resources scarcer and more valuable after negative shocks rather than after positive shocks.

Therefore, which of these two forces dominates depends centrally on whether the optimal investment response to a productivity shock is higher or lower than the current cash-flow response. Whenever persistence is sufficiently high given the returns to scale, firms place a higher value on resources that are used for funding expansions after a positive shock than on resources used for covering cash-flow shortfalls after negative shocks. Since the riskier project offers a profile of cash-flows that is more aligned with the funding needs of a constrained firm, more resources are allocated to it. As a consequence, its expected marginal returns are lower than the ones offered by the safer alternative, which offers a worse matching of flows with the firm's funding needs.

The last item of the proposition also highlights that in this simplified framework, as σ approaches one and the projects become indistinguishable, an even simpler condition emerges for evaluating whether constrained firms seek increased or decreased risk exposure. Whenever the sum of the elasticity to scale (α) and the persistence (ρ) parameter exceeds one, investment needs are stronger than the cash-flow response and the marginal value of funds becomes pro-cyclical with respect to the

TFP shock. It is possible to extend this part of the proposition to show that, if firms could control exposure at that symmetric situation, they would value increased exposure, i.e., $\partial_\sigma V(w_0, \sigma, s^0) \geq 0$ whenever $\alpha + \rho > 1$ (respectively, firms would value decreased exposure, i.e., $\partial_\sigma V(w_0, \sigma, s^0) \leq 0$, whenever $\alpha + \rho < 1$).²³ Levels of both the returns to scale and persistence parameters that are close to one each are pervasive in the literature, as discussed in Section 4.

This situation is illustrated by Figure 1. In the left-hand panel, date $t = 0$ capital budgeting and financial decisions are depicted. Capital investments in the riskier ($j = 0$) and safer ($j = 1$) projects are plotted along with financial slack left for future states. Firms can be split according to three categories, as delimited by the vertical dotted lines.

In the leftmost region of the panel, net worth is small. Firms are so constrained that they refrain from using any of their debt capacity for conserving financial slack. Consequently, only distortions in capital budgeting can be used for hedging purposes. These distortions manifest themselves as larger investments in the riskier project, which has cash-flows that are better aligned with the firms' investment needs.

Figure 1: The persistent shock case, with $\rho = 0.8$. The left-hand side panel plots capital budgeting and financial policies at date $t = 0$. The right-hand panel plots the marginal value of internal funds ($\frac{\partial V_t}{\partial w_t}$) at $t = 0$ and the two possible states at $t = 1$.

In the intermediate region, firms still face binding borrowing constraints. They use both some financial instruments as well as capital budgeting distortions to deal with their hedging demand. Last, in the rightmost region of the panel, firms have enough net worth to fund the first-best scale in all possible situations. Investment is never constrained for these firms and the hedging demand vanishes. As a consequence, there are also no distortions in capital budgeting.

²³For formal statement and proof, following the approach of Proposition 1, please refer to previous working paper versions of this article.

The panel on the right-hand side plots the marginal value of funds at date $t = 0$ and across both $t = 1$ states. Two features are worth noting. First, the ordering of these marginal funds. Firms are always relatively more constrained at date $t = 0$ than in the growth state $s^1 = \epsilon_h$ at date 1. They are always unconstrained at ϵ_l , where they simply downsize. This rationalizes the direction of the hedging demand, which favors exposure to the underlying shock. Second, as one moves to the right, towards firms that have higher net worth, all marginal returns approach the market rate which was normalized to one while their dispersion is reduced. This illustrates the weakening of the hedging demand for less constrained firms and its ultimate disappearance for unconstrained ones.

Figure 2: The transitory shock case, with $\rho = 0$. The left-hand side panel plots capital budgeting and financial policies at date $t = 0$. The right-hand panel plots the marginal value of internal funds ($\frac{\partial V_t}{\partial w_t}$) at $t = 0$ and the two possible states at $t = 1$.

Figure 2 plots the same two panels for a fully transitory shock, with $\rho = 0$. In this case, resources are always more valuable after negative shocks to cash-flows, and first-best investment levels do not respond to the shock. The hedging demand points towards reducing exposure and we can see that capital budgeting favors the safer ($t = 1$) alternative.

In Section 4, we revisit the discussion of persistence and endogenous risk-taking for empirically reasonable parameter values in an infinite-horizon quantitative exploration of the model.

3.2 Credit capacity shocks

The environment described in the previous section served to illustrate the connections between the persistence of shocks to productivity and the evaluation of cash-flow risk. In the presence of sufficiently persistent shocks, firms do not seek to save resources for the lowest productivity states, since growth concerns outweigh cash-flow insurance.

Shocks to their credit capacity work much in the same way: they reduce a firm's leverage ability and, as a consequence, the return it can make on internal funds. In this section, we explicitly introduce a credit cycle: more or less external funding can be obtained against the same collateral depending on credit conditions which are exogenous to the firm. Firms can choose among capital goods with different exposure to this shock, as well as seek insurance through standard financial instruments.

We illustrate how firms can sacrifice net worth and investment levels in low credit capacity states in order to invest more when credit conditions are more favorable. Again, they opt not to insure against negative shocks, and additional exposure to such shocks can be shown to increase the value of the firm.

The consequences from exposure to credit shocks are illustrated through an application of the framework that involves mutually exclusive decisions, which was developed in Section 2.3. In particular, we endow the firm with a single date $t = 0$ decision that determines which type of (mutually exclusive) capital goods it uses to operate a given production function. In the airline example, this would concern a possible aircraft choice for a given route. Since the firm will operate a single project, $j = 0$, we drop dependence to simplify notation.

In the model, these capital goods differ in their exposure to fluctuations in credit conditions. We model this as a collateralization parameter $\theta(\gamma, s^t)$ that depends both on the decision $\gamma \in [\underline{\gamma}, \bar{\gamma}]$ and in the underlying state s^t . This state is exogenous to the firm and describes credit market conditions. For presentation clarity, and to fully isolate the effects from credit constraints and variable leverage, we assume that no matter what decision is undertaken, capital prices and productivity are the same. In the same way, to illustrate in isolation the role of exposure to shocks and not the overall level of collateralization (which are likely to go hand in hand in any application) we assume that the expectation of $\theta(\gamma, s^t)$ does not vary across alternatives.

To formalize this set-up, let the realization of one of two possible states be learned at $t = 1$. Therefore, we let $s^1 \in \{s_h^1, s_l^1\}$. From $t = 1$ into $t = 2$ the event tree evolves trivially: there is a singleton as a successor of either s^1 . We refer to them as $s^2 \in \{s_h^2, s_l^2\}$. The states s_h^1, s_l^1 imply a one-to-one mapping with $\theta(\gamma, s^2)$, which denotes how much lenders expect to recover if the firm decides to walk away from its debt right after production at $t = 2$. We let

$$0 < \theta(\gamma, s_l^2) < \theta(\gamma, s_h^2) < 1$$

and order states by their credit conditions saying that $s_l^t < s_h^t$ for $t = 0, 1$.

Variation in these recovery rates changes how much credit can be obtained against the same collateral in a way that is orthogonal to any movements that could be happening in collateral prices. It is the simplest way to introduce a credit cycle which is unrelated to the productivity of investment.²⁴

²⁴Modeling a credit fluctuation through a change in the recovery rate instead of the prices of capital goods has the advantage of generating an effect which is entirely orthogonal to productivity, since investment in capital produces

All other variables in the environment are constant across decision, time, and states. Productivity is constant, $A(\gamma, s^t) = A$ for all t , $s^t \in S^t$, and $\gamma \in [\underline{\gamma}, \bar{\gamma}]$. For simplicity, we also let $\theta_1(\gamma, s^1) = E[\theta_2(\gamma, s^2)] = \theta$ for all $s^1 \in S^1$ and $\gamma \in [\underline{\gamma}, \bar{\gamma}]$. We assume that $\theta_2(\gamma, s^2)$ is differentiable in γ and that higher γ is associated with higher risk exposure. Additionally, the depreciation rate is set to zero, $\delta = 0$, implying that all firms find themselves constrained in all states.²⁵ This assumption is made for simplicity, to directly illustrate how sufficiently constrained firms would evaluate exposure to credit fluctuations while avoiding the need to describe several cases.

As a consequence of the variation in the recovery rate, the firm's borrowing capacity given any investment level depends on the underlying state. A firm that invests k_2 at $t = 1$ can borrow up to

$$BC(k_2, \gamma, s^1) = \theta(\gamma, s^2) k_2,$$

implying a downpayment requirement that is reduced when credit conditions improve, since

$$\wp(\gamma, s^1) = 1 - \theta(\gamma, s^2).$$

The variation in the downpayment requirement is directly responsible for making the return on internal funds increase as credit conditions improve. As a consequence, firms both fail to insure their investment at s_l^1 and prefer capital goods whose degree of collateralization is more sensitive to credit conditions²⁶. This is formalized in the next two results.

Lemma 1. *Consider the environment described above. Then, for each $\gamma \in [\underline{\gamma}, \bar{\gamma}]$,*

1. *every firm faces shadow values of net worth which are pro-cyclical with respect to the credit fluctuations, i.e., $\frac{\partial V(w(s_h^1), \gamma, s_h^1)}{\partial w(s_h^1)} > \frac{\partial V(w(s_l^1), \gamma, s_l^1)}{\partial w(s_l^1)}$. As a consequence, firms never save financial slack towards the low collateralization state.*
2. *capital investment is increasing in the underlying state, $s_h^1 > s_l^1$, so that $k_2(s_h^1) > k_2(s_l^1)$.*

Proposition 3. *Consider the environment described above, where firms face an irreversible decision that determines their exposure to credit fluctuations. Then, it is optimal to choose the decision that leads to the highest possible exposure, i.e., $\gamma = \bar{\gamma}$, since for any $\gamma \in [\underline{\gamma}, \bar{\gamma}]$, we have*

$$\frac{\partial V_0}{\partial \gamma} = c(\gamma) \left[\left(\frac{\partial V(w(s_h^1), s_h^1)}{\partial w(s_h^1)} - 1 \right) k_2(s_h) - \left(\frac{\partial V(w(s_l^1), s_l^1)}{\partial w(s_l^1)} - 1 \right) k_2(s_l) \right] > 0, \quad (26)$$

both output and some capital after depreciation at $t+1$. Therefore, a reduction in $q(\gamma, s^{t+1})$ directly makes investment less productive.

Although it is hard to motivate a literal change in a recovery rate along the business cycle, we can interpret shocks to this variable as any shocks that affect how much a lender is willing to offer against a given amount of collateral. For instance, a deterioration of adverse selection in credit markets would have similar effects.

²⁵With zero depreciation and no discounting, additional capital investment always dominates dividend payments at $t = 0, 1$. Effectively, each firm always finds itself constrained, as it is always below the first-best level of capital investment.

²⁶In the sense of having a higher variance of the recovery rate for a given mean.

where $c(\gamma)$ is a constant of proportionality.

There is a simple interpretation of the effects identified in equation (26) above.

When credit constraints are relaxed at s_h^1 , the firm can borrow more for every purchased unit of capital. This borrowing generates funds valued at $\frac{\partial V(w(s_h^1), s_h^1)}{\partial w(s_h^1)}$, a value that exceeds the cost of their repayment at $t = 2$, where the marginal value of funds is unitary. The difference between these two values is the premium on borrowing capacity.

There are two reasons why the relaxation of borrowing constraints in the high-leverage state (s_h^1) more than offsets an equivalent tightening in the low-leverage state (s_l^1). The first reason is that the value of being able to borrow more for each unit of capital is higher in the former than in the latter. The second reason is that the increase in borrowing capacity interacts with more units of capital, since leverage is also higher at s_h^1 .

We have thus shown that leverage capacity plays a key role in endogenous attitudes and can also distort project selection towards investments that favor additional exposure. The next section concentrates again on endogenous risk taking, but now in an infinite horizon setting where firms have multiple instruments for dealing with their hedging demands.

4 A numerical assessment

In this section, we quantitatively evaluate how significant the capital budgeting distortions induced by the presence of credit frictions can be. We center our numerical analysis on the production and collateralization parameters obtained in Li *et al.* (2015) for the airline industry, which are broadly consistent with the ones obtained for similar industries. The key productivity process persistence and return to scale parameters are also in line with the values obtained in a literature from a Macroeconomic tradition. This literature uses mostly revenue and investment data, which is sufficient for this subset of parameters.²⁷ We additionally conduct an extensive sensitivity analysis and discuss the economic forces behind each of the comparative statics studied.

We use a 3-state Tauchen-Hussey Markov chain approximation to a log-total-factor-productivity that follows an auto-regressive process of order 1, with persistence $\rho = 0.829$ and variance $\sigma = 0.098$. The single-factor production function has an scale elasticity parameter $\alpha = 0.918$. The rate of depreciation for capital goods is set to $\delta = 0.043$, their prices to unity, and the degree of collateralization to $\theta = 0.493$. We set the firm's discount factor over delayed dividends to $\beta = 1.05^{-1}$

²⁷A few papers attempt to estimate both TFP processes and returns to scale parameters from firm level revenue data. Cooper and Haltiwanger (2006) find an elasticity to scale of 0.89 and a persistence of 0.59. Khan and Thomas (2003) identify an elasticity of 0.9 and persistence of 0.92. Midrigan and Xu (2013) document autocorrelations of output of 0.9 over a one year horizon and proceed towards a calibration that combines an elasticity to scale of 0.85 and a combination of persistent and transitory shocks. That combination leads to 85% of the cross-sectional variance being accounted by a fully permanent component, which we take as evidence of the high degree of persistence of TFP in the data. Constant returns to scale are assumed in Collard-Wexler *et al.* (2011), which studies firms across multiple countries and identify a mean persistence of 0.85.

and the perceived cost of borrowed funds to $R = \beta^{-1}(1 - \tau(1 - \beta))$, where the wedge $\tau = 20$ is motivated by the tax benefit of debt.²⁸

All of the following analysis is conducted around the stationary distribution of firms and the intermediate state in the Markov chain, from which each firm has equal probabilities of suffering a positive or a negative shock. To study distortions in capital budgeting we endow the firm with two projects that differ whenever business conditions lie in this intermediate state. Both projects have the same expected TFP but different exposures to the underlying risk. The riskier $j = 0$ project is a mean preserving spread of the original TFP process, with twice the dispersion in output. Project $j = 1$ is riskless.²⁹ In this way, if the firm maintains a balanced allocation of resources across both projects, the dispersion in its cash-flows is unaltered relative to the original single-project parametrization. By distorting capital allocation across these investments, away from symmetry, the firm can manipulate its risk exposure, at the cost of an expected output sacrifice. Notice that this example maximizes the risk-exposure distinction between the two projects, subject to a common mean and non-negative exposures. As such, the magnitudes we will characterize should be seen as an upper-bound for this parametrization.

We first study optimal capital budgeting and financial policies. Firms choose possible dividend payouts, how much capital to allocate to each of the two projects, as well as how much financial slack to leave to each of the possible states. They have access to full set of contingent claims, although their borrowing is limited by commitment problems. We soon study how incompleteness in risk management instruments would further distort decisions.

Figure 3 plots capital budgeting and other financial decisions as a function of net worth, where one unit of net worth is normalized to represent the median level under the stationary distribution. A few properties of the optimal policy are worth emphasizing. First, the hedging motive indicates that resources are more valuable after positive state innovations than after negative ones. This manifests itself both in a higher investment in the riskier project and in the conservation of financial slack only to fund expansion in the highest state. Financial risk management is not used at all for a range of firms with extremely low values of net worth, and changes quickly in magnitude along the cross section of possible net worth levels. Similarly, only sufficiently large firms pay out dividends.

From Figure 3 the distortion in favor of the riskier project (while still noticeable) is dwarfed by the scale of the financial decisions. Figure 4 further explores the magnitude of this distortion in the cross section of firms. The vertical lines represent key percentiles of the stationary firm distribution. The median firm invests approximately 2.2% more in the riskier technology, bringing

²⁸In Li *et al.* (2015), the discount rate for the firm's own funds is set, prior to the estimation stage, to the riskless rate from U.S. Treasuries. It has been argued that T-Bills and T-Bonds might include relevant liquidity and safety premia and represent too low a benchmark for a riskless rate (see Krishnamurthy and Vissing-Jorgensen (2012), for instance). We set this discount rate to a higher value in our benchmark. For a sensitivity analysis, see Section 4.1.

²⁹In an asset pricing language, the projects differ in their β with respect to the risk factor underlying the Markov Chain.

Figure 3: Optimal policy as a function of current net worth.

Parameter values are $\rho = 0.829$, $\sigma = 0.098$, $\alpha = 0.918$, $\delta = 0.043$, $q_t^j = 1$, $\theta = 0.493$, $\beta = 1.05^{-1}$ and the perceived cost of borrowed funds to $R = \beta^{-1}(1 - \tau(1 - \beta))$, where the wedge $\tau = 20$ is motivated by the tax benefit of debt. Net worth is normalized by the median under the stationary distribution, while financial variables are normalized by the first-best level of capital.

its marginal return below the one for the riskless project. For a firm at the 5th percentile of the stationary distribution this distortion is 3.86%. By distorting investment away from expected marginal product equalization, towards the riskier project, firms increase their exposure to the risk factor that shapes revenue, thus creating endogenous volatility. Figure 3 can also be understood as describing the shape of the incentives to take on endogenous volatility, which would peak at a very low level of net worth (about 10% of the median firm) and gradually recede as firms grow larger either in the cross-section or throughout time.

Larger values for the distortion, above 6%, are reached by firms with a net worth which is only a small fraction of the median level, a situation which might apply to firms in an entry phase. Also, for these extremely low net worth values, which are not frequent under the stationary distribution, a non-monotonicity in the hedging-induced distortion is displayed. The economic intuition is the following. For extremely low net worth values, returns are very high and firms would grow even in the lowest of productivity states. In the same way that smaller firms refrain from using financial instruments for hedging, as they would lower investment levels and impede growth, the low-net-worth are less willing to sacrifice expected returns to obtain hedging benefits. Interestingly, this non-monotonicity is only a feature of the fully-dynamic environment, in which firms can postpone hedging for future dates; this feature is not present in the short-horizon set-up of Section 3.1.

The capital allocation distortion is monotonic around net worth values that are frequent under stationary distribution: it decreases once one focuses on larger firms, but never entirely disappears, reaching a plateau at around 0.66%. Since there is always a premium on borrowed funds and binding

Figure 4: Percentage distortion in capital allocation across the two projects of different risk exposure along the stationary distribution.

Parameter values are $\rho = 0.829$, $\alpha = 0.918$, $\delta = 0.043$, $q_t^j = 1$, $\theta = 0.493$, $\beta = 1.05^{-1}$ and the perceived cost of borrowed funds to $R = \beta^{-1}(1 - \tau(1 - \beta))$, where the wedge $\tau = 20\%$ is motivated by the tax benefit of debt. Net worth is normalized by the median under the stationary distribution.

collateral constraints, firms never become totally unconstrained and the hedging motive does not entirely disappear, despite the fact that a maximum scale is reached and dividends are eventually paid out after a sufficiently long sequence of good shocks.

Although we identify a meaningful discrepancy in capital allocation, its magnitude is significantly smaller than the difference that would be induced by heterogeneity in expected rates of returns for projects. For instance, in the first-best, a difference of 1% in expected TFP across the two projects would lead to a 12% difference in capital allocation. Therefore, a takeaway from this numerical experiment is that while hedging concerns are present, they are still expected to represent considerably smaller drivers of investment than rates of return. One reason is that firms have several instruments available for dealing with their hedging demands, such as dividend postponement, distortions in capital budgeting, and financial risk management. To what degree each instrument is used depends on its underlying costs and the parameters of the environment, as we discuss next.

4.1 Constraints to risk management: incomplete instruments

Although firms have access to many possible contractual contingencies, renegotiation possibilities, and financial instruments, one could reasonably argue that a benchmark that allows full-contingency in traded securities or designed contracts overstates actual risk management flexibility. Indeed, this is the main motivation behind parallel literatures both in Corporate Finance and Macroeconomics that impose some additional form of market incompleteness, such as restricting the set of possible assets issued or held by agents, with a special role allowed for riskless debt.

The presence of risk management distortions in capital budgeting is not a consequence of financial flexibility, but a response to imperfections that is actually amplified by additional obstacles to financial planning. In order to study the magnitude of this possible impact, we slightly modify the set-up studied so far. The modification lies in forcing collateralized debt to be constant across states of the world. In the absence of fluctuations in the prices of capital or in its degree of collateralization, as in the example in this section, this change is equivalent to imposing a constraint that financial slack is only allowed to vary over time, not over states. In this modified version of the model where financial instruments are incomplete, capital investment, the accumulation and liquidation of liquidity buffers, and dividend policies offer some important flexibility.

We plot the optimal policies (Figure 5) and the distortions in capital budgeting along the stationary distribution (Figure 6) using the same parameter values from the previous example. The median net worth is essentially the same as in the incomplete instruments benchmark, being just about 2% smaller.

Figure 5: Optimal policy as a function of current net worth, in the presence of incomplete set of financial instruments for risk management.

Parameter values are $\rho = 0.829$, $\sigma = 0.098$, $\alpha = 0.918$, $\delta = 0.043$, $q_t^j = 1$, $\theta = 0.493$, $\beta = 1.05^{-1}$, and the perceived cost of borrowed funds to $R = \beta^{-1}(1 - \tau(1 - \beta))$, where the wedge $\tau = 20$ is motivated by the tax benefit of debt. Net worth is normalized by the median under the stationary distribution, while financial variables are normalized by the first-best level of capital.

Interestingly, all firms in the intermediate productivity state find themselves against their borrowing constraints. It is only high-net worth, less-productive firms (in the lowest productivity

Figure 6: Percentage distortion in capital allocation across the two projects of different risk exposure along the stationary distribution, in the presence of an incomplete set of financial instruments for risk management. Parameter values are $\rho = 0.829$, $\alpha = 0.918$, $\delta = 0.043$, $q_t^j = 1$, $\theta = 0.493$, $\beta = 1.05^{-1}$, and the perceived cost of borrowed funds to $R = \beta^{-1}(1 - \tau(1 - \beta))$, where the wedge $\tau = 20\%$ is motivated by the tax benefit of debt. Net worth is normalized by the median under the stationary distribution.

state) that keep any liquidity buffers or unused debt capacity. This is essentially a feature of the 3-state approximation to the productivity process, which vanishes for finer approximations or some alternative parameter values.

Despite the hedging motive not being strong enough to make firms in this intermediate state leave financial slack, it induces distortions in capital budgeting that are from 50% to 200% higher than the size of the distortions induced when financial risk management is complete. This distortion still drops as net worth becomes larger, but stabilizes at 2.68% instead of 0.66% for sufficiently large firms.

This example serves as an illustration that as alternative financial instruments become less accessible, distortions in capital budgeting become significantly larger and also potentially more noticeable across the whole cross-section of firms, as opposed to a phenomenon mostly displayed by smaller-than-average firms.

4.2 The role of returns to scale, persistence, and other features of the environment

In this section, we describe how the magnitude of the capital allocation distortion responds to key parameters in the environment, such as persistence in productivity innovations, returns to scale, and discount factors.

As we anticipated in Section 3.1, returns to scale play a key role in shaping how strongly optimal investment responds to shocks in the business conditions. In this set-up, where firms allocate

resources across two technologies with different risk exposures, they play yet another important role. Returns to scale here determine how strongly returns from real investment respond to quantity distortions. The more strongly-decreasing these returns to scale are, the more costly it becomes in terms of expected output to move away from first-best capital shares (which in this simulation were designed to be equal). For this reason, returns to scale describe how firms trade-off the use of financial instruments and real capital allocation distortions.

As Figure 7a illustrates, there are noticeable distortions in capital allocation whenever the scale elasticity parameter, α , exceeds 0.75. Around this region, effects are mostly noticeable for very low quantiles of the stationary distribution of firms, as better capitalized firms use more financial risk management. They increase in magnitude along all quantiles, and eventually diverge to infinity, as the technology gets closer to constant returns to scale. Sensitivity to parameters is moderate around $\alpha = 0.9$ and any effects essentially disappear for $\alpha < 0.7$, since both investment sensitivity to productivity shocks drops considerably as well as real distortions become costly when compared to the use of financial instruments.

Another key parameter in our previous analysis was the persistence in business conditions. The more persistent shocks are, the stronger the hedging motive is. From the initial finite horizon theory, we also expected a sign change in the direction of this hedging distortion. Highly persistent shocks make sure that current conditions predict future profitability and investment needs. In this case, exposure to a factor that shapes current conditions helps fund future investment. Purely transitory shocks, on the other hand, only generate fluctuations in net worth, which are costly in the presence of imperfect financing, and should be mitigated.

Figure 7b plots optimal distortions along the stationary distribution for different values of persistence in the productivity process. Although the sign reversal which is expected in theory does occur around a persistence level of $\rho = 0.05$ or lower, it is barely visually or economically perceptible. The main lesson from this sensitivity analysis is the following: for the benchmark case, firms have adequate alternative instruments for dealing with transitory shocks, such as preemptive net worth accumulation and contingent financial contracts. For high levels of persistence two other features call our attention. First, as current factors become better predictors of future investment needs, effects increase. Second, the distribution of distortions fans out, indicating that the cross-sectional heterogeneity in the magnitude of distortions becomes more pronounced as persistence becomes larger.

Figures 7c and 7d plot the consequences of changes in the borrower's discount rate, as well on the wedge between that rate and the perceived cost of collateralized funding. Regarding the wedge, which we parametrize as in Li *et al.* (2015), we observe a large sensitivity to small values of τ but decreasing effects that roughly stabilize for values above $\tau = 0.2$. The discount factor for delayed dividends shows some sensitivity to values of $\beta > 0.95$. For instance, distortions roughly decrease to half their values if one moves from the benchmark case to $\beta = 0.98$. A large fraction of this sensitivity originates from the tax benefit interpretation of the wedge in the cost of funds and the

(a) Returns to scale

(c) Discount factor for delayed dividends

(e) Dispersion in productivity

(b) Productivity persistence

(d) Wedge between own and lender's discount factor

(f) Collateralization

implied parametrization. In the model we study, borrowing against collateral seems relatively cheap to firms and, as a mirror image, saving (or leaving financial slack for risk management purposes) relatively expensive. How so depends on the level difference between the lender’s discount factor (R) and the borrower’s (β). Under the Li *et al.* (2015) parametrization, this distance decreases fast as β approaches one. As a consequence, risk management through financial instruments becomes relatively cheap in that limit, firms accumulate assets in the stationary distribution, and any capital budgeting distortions are greatly reduced. If one studies a set-up with a constant additive wedge, which can be motivated by a setting in which stores of value used for backing liquidity carry a premium (as in Holmström and Tirole, 1998, 2011), most of this sensitivity disappears.

We also study the consequences of different levels of underlying volatility and the recovery value of collateral. Figure 7e plots the sensitivity analysis regarding the baseline dispersion in productivity. The most noticeable feature is a lack of monotonicity, with a peak level of distortions being identified for each quantile. Also, lower quantiles, representing more constrained firms, are more sensitive to changes in underlying volatility. From additional inspection of the underlying policies and stationary distribution, we conclude that once firms are subject to higher underlying volatility, they both increase the distortions in capital budgeting and delay dividends, opting to conserve more financial slack. This occurs at any fixed level of net worth. As a consequence, higher volatility shifts the stationary distribution of firms towards the right. This increase in scale of the firms at all key quantiles dominates the direct effect of more capital budgeting distortions for sufficiently high levels of volatility, implying that lower distortions are perceived in the stationary cross-section.

Regarding the recovery value of collateral, there is little change in the level of distortions in the cross section. As the recovery rate increases, the stationary net worth distribution is mostly stable and the capital levels approach the first-best. The stationary distribution only starts to shift to the left for extremely high levels, such as $\theta > 0.9$, but under that situation capital is almost perfectly pledgeable and firms rarely suffer a string of bad results that is long enough as to make them constrained again. As a consequence, we observe a drop in distortions in capital budgeting that is induced by a hedging demand.

4.3 Fluctuations in credit conditions, capital prices, and the downpayment requirement

The direction and intensity of the hedging motive is driven by the endogenous behavior of the marginal value of internal funds. We have shown in Section 2.1 that credit conditions play a key role in shaping this value, as a relaxation of credit constraints allows more leverage to be undertaken, and increases the return on the firm’s own funds. To the best of our knowledge this potential effect is new to the literature and has not been previously evaluated in a dynamic model.

The simplest way to induce pro-cyclical credit conditions in the studied framework is by allowing $\theta^j(s_t)$ to vary with the underlying state, which in the dynamic simulations up to now has only

affected cash-flows. We make this collateralization variable pro-cyclical, so that feasible leverage increases with productivity and revenues. To make matters simple, we keep the exposure to credit conditions constant across both projects, so distortions in investment away from symmetry are purely driven by the projects' heterogeneous exposure to cash-flow risk.

Figure 8 plots the distortions in capital budgeting across both projects for two levels of volatility in credit conditions, and can be directly compared to Figure 4, where there is no volatility at all. The main conclusion is that the more strongly pro-cyclical these conditions are, the more pronounced the hedging demand becomes. Additional consequences from an increase in volatility of credit conditions include a small reduction of the median net worth (about 2%) and an increase in the firm-size dispersion. Both effects are consistent with firms endogenously taking on more risk.

Figure 8: Pro-cyclical credit conditions.

Capital price fluctuations have an alternative consequence. Figure 9 plots the distortions in capital budgeting caused by making the price of the capital goods used in both projects pro-cyclical, while reverting the pledgeability parameter to be acyclical, as in our benchmark. The consequences are a mirrored image of the pro-cyclical relaxation of credit constraints. The hedging motive is dampened and essentially disappears in panel 9b, while firm size distributions are compressed. The reason is that because of the defining mean-reversion of the stochastic process we study, an increase in prices unequivocally increases downpayment requirements. As prices can only go down after a positive shock, the future value of the capital stock becomes significantly lower than the current price, making it harder to finance expansions. As a consequence, returns on investment become less exposed to current conditions. In panel 9b, we see evidence of an almost acyclical value of return on internal funds.

For sufficiently volatile capital good prices, above the levels plotted in the second panel, one can even achieve a sign reversal of that return. Once that happens, because of strongly pro-cyclical downpayment requirements, a change in the direction of the hedging demand would occur. This extreme scenario featuring anti-cyclical returns seems unlikely both in a theoretical general equilibrium framework as well as empirically.

The main lesson from this comparison is that the cyclical behavior of downpayment require-

Figure 9: Pro-cyclical prices of capital goods

ments, which are affected by both prices and credit conditions, can influence the hedging demand and incentives for endogenous risk taking. To make our benchmark simulations directly comparable to the current literature, we assumed acyclical downpayment requirements. If, however, one has evidence that these are anti-cyclical, then we should see incentives for additional exposure to persistent factors that shape the cycle. Further study of the empirical behavior of credit conditions and downpayment requirements can help shed light on optimal corporate hedging policies.

5 Conclusion

Without perfect access to credit, firms cannot fully rely on markets in order to invest efficiently. This gives rise to a hedging motive, which is reflected on both the financial and investment sides of corporate decisions.

We have paid especial attention to the induced investment distortions. For this, we have extended a standard model of corporate investment and financing with endogenous collateral constraints to incorporate explicit capital budgeting. The model offers a unified framework for studying the factors that shape the internal value of corporate funds and the interaction of alternative hedging instruments. It abstracts away from other potentially relevant elements, such as internal agency problems, adjustment costs, and indivisibilities. The analytical insights we have derived, however, are likely to be present (with some modifications) in all related frameworks.

We have shown that while the hedging motive creates a desire to smooth out transitory cash-flow fluctuations, empirically reasonable levels of persistence in productivity shocks cause constrained firms to be more willing to bear risks that correlate with their productivity processes. Therefore, since this form of risk-taking facilitates self-financing, we have illustrated a channel through which more constrained firms become endogenously more volatile.

We have also emphasized the importance of leverage for optimal hedging. Leverage makes internal funds complementary to external credit and can make resources more valuable to the firm when credit conditions are slacker. As a consequence, constrained firms might show a risk-taking

attitude towards their exposure to aggregate credit conditions.

Distortions in capital budgeting and the use of standard financial instruments for hedging are imperfect substitutes. We have shown that the costlier financial hedging becomes as an alternative, the more distortions one observes in capital budgeting. For instance, while financial markets might fail to offer products contingent on the factors that shape a particular firm's investment opportunities, real investment projects, by their own nature, are likely to offer exposure to these contingencies.

Last, future empirical work might help test and quantify the importance of capital budgeting distortions as well as further describe their interactions with cross-sectional heterogeneity and the evolution of financial market instruments.

Appendix

Proof of Proposition 1. Marginal changes in γ lead to changes in cash-flows, prices, and recovery rates; each of these contributes to part of the marginal effects into the value function at s^t . We use the envelope theorem recursively, until a node in which γ can be reoptimized is reached. After some algebra, the terms involved are represented by $E \left[\sum_{h=t+1}^{\tilde{T}} \beta^{h-t} \frac{\partial V(w(s^h), \gamma, s^h)}{\partial w(s^h)} \sum_j \frac{\partial F^j(k_h^*, \gamma, s^h)}{\partial \gamma} \right]$ for the discounted value of changes in cash-flows, $\beta^{h-t} \pi(s^h | s^t) \frac{\partial V(w, \gamma, s^h)}{\partial w(s^h)} \left\{ -i^{j,*}(s^h) + \mu(s^h) (1 - \delta) \theta^j(\gamma, s^h) k_h^{j,*}(s^{h-1}) \right\}$ for a change in each capital good price, and $\beta^{h-t} \pi(s^h | s^t) \frac{\partial V(w(s^h), \gamma, s^h)}{\partial w(s^h)} \mu(s^h) q^j(\gamma, s^h) (1 - \delta) k_h^{j,*}(s^{h-1})$ for a change in the recovery rate. Combining these terms and using definitions 23 and 24 for borrowing capacity and net cash-flows, we obtain expression 25. $\square$

Proof of Proposition 2.1. In the first-best, with $\beta = R = 1$, the timing of dividends is irrelevant for optimality. Optimal capital investments are separable across time and projects, so we can solve the simple $\max_{k_{t+1}^j} E \left[f(k_{t+1}^j) + (1 - \delta) k_{t+1}^j | s^t \right] - k_{t+1}^j$ for each $j \in \{0, 1\}$ and s^t , for $t = 0, 1$. The first-order conditions at s^0 imply $\alpha E_0 [A^j(s^1)] k_1^j(s^0)^{\alpha-1} = \delta$. As $E_0 [A^0(s^1)] = E_0 [A^1(s^1)]$, we have $k_1^0(s^0) = k_1^1(s^0)$. $\square$

Proof of Proposition 2.2. First, as an intermediate step, we characterize the value function at $t = 1$, which can be simply written as

$$V(w, s^1) = \max_{\{k^j\}_j, h \geq 0} \sum_j A^j(s^2 | s^1) f^j(k^j) + h(s^2 | s^1),$$

s.t. $\sum_j \phi k^j + h(s^2 | s^1) \leq w$. Here, there is no loss in value in setting $d(s^1) = 0$ given that s^2 is terminal, leading to a liquidation dividend, and $R = \beta = 1$. Given $A^0(s^2) = A^1(s^2) =$

$\bar{A}(1 + \epsilon(s^2))^\rho$, optimal capital deployments are the same across both projects, and we get

$$V(w, s^1) = \begin{cases} 2^{1-\alpha} A^j(s^2) \left(\frac{w}{\wp}\right)^\alpha, & \text{if } w \leq w^*(s^1) \\ V(w^*(s^1), s^1) + w - w^*(s^1), & \text{if } w > w^*(s^1), \end{cases}$$

where $w^*(s^1)$ solves $\alpha 2^{1-\alpha} A^j(s^2|s^1) \left(\frac{w}{\wp}\right)^{\alpha-1} = 1$, making the firm effectively unconstrained. Again, since $\beta = R^{-1}$, there is no loss in value of requiring $d(s^0) = 0$, which eliminates some cases of indeterminacy of the optimal policy. Then, the value function at s^0 can simply be written as

$$V(w_0, s^0) = \max_{\{k^j\}_j, \{h(s^1)\}_{s^1 \geq 0}} E[V(w(s^1), s^1)] \quad (27)$$

s.t. $w(s^1) = \sum_j A^j(s^1) (k^j)^\alpha + h(s^1)$ and $w_0 = \wp \sum k^j + E[h(s^1)]$. $\square$

Concavity can be easily verified. Taking first-order conditions and using the envelope theorem, we get

$$k^j : E \left[A^j(s^1) \frac{\partial V(w(s^1), s^1)}{\partial w} \right] \alpha (k^j)^{\alpha-1} - \wp \frac{\partial V(w_0, s^0)}{\partial w} = 0 \quad (28)$$

and

$$h(s^1) : \frac{\partial V(w(s^1), s^1)}{\partial w} - \frac{\partial V(w_0, s^0)}{\partial w} \leq 0, \text{ with equality if } h(s^1) > 0. \quad (29)$$

We first define an auxiliary function

$$\varphi(k^0, k^1, s^1) := \alpha 2^{1-\alpha} \left(\frac{\bar{A}}{\wp}\right)^\alpha (1 + \epsilon(s^1))^\rho ((1 + \epsilon(s^1)) k^0 + (1 + \sigma \epsilon(s^1)) k^1)^{\alpha-1},$$

which represents $\partial_w V(w(s^1), s^1)$ as a function of $k^0(s^0)$ and $k^1(s^0)$ whenever $h(s^1) = 0$ and $\partial_w V(w(s^1), s^1) > 1$. We also define $\bar{k} := \frac{k^0 + k^1}{2}$, the average capital allocation across projects. We then analyze three possible, but mutually exclusive, characterizations of a solution to Program (27).

1. $\partial_w V(w(s_h^1), s_h^1) > \partial_w V(w(s_l^1), s_l^1) \geq 1$.

It follows from the first-order conditions with respect to financial slack that $h(s_l^1) = 0$, and with respect to capital that $k^0 > k^1$. Then, since $h(s^1) \geq 0$, $\partial_w V(w(s_h^1), s_h^1) \leq \varphi(k^0, k^1, s^1) \leq \varphi(\bar{k}, \bar{k}, s_h^1)$. Analogously, $\partial_w V(w(s_l^1), s_l^1) \geq \varphi(k^0, k^1, s_l^1) \geq \varphi(\bar{k}, \bar{k}, s_l^1)$. Combining both we get $\varphi(\bar{k}, \bar{k}, s_h^1) > \varphi(\bar{k}, \bar{k}, s_l^1)$ which simplifies to

$$(1 + \epsilon_h)^\rho ((1 + \epsilon_h) + (1 + \sigma \epsilon_h))^{\alpha-1} > (1 + \epsilon_l)^\rho ((1 + \epsilon_l) + (1 + \sigma \epsilon_l))^{\alpha-1}. \quad (30)$$

Additionally, $\partial_w V(w_0, s^0) > 1$ follows from 29 and, together with 28, implies that k^0, k^1 are strictly below the first-best level.

2. $\partial_w V(w(s_l^1), s_l^1) > \partial_w V(w(s_l^1), s_l^1) \geq 1$. All steps are analogous to the previous computations and we get

$$(1 + \epsilon_h)^\rho ((1 + \epsilon_h) + (1 + \sigma\epsilon_h))^{\alpha-1} < (1 + \epsilon_l)^\rho ((1 + \epsilon_l) + (1 + \sigma\epsilon_l))^{\alpha-1} \quad (31)$$

instead of (30).

3. $\partial_w V(w(s_h^1), s_h^1) = \partial_w V(w(s_l^1), s_l^1)$. In this case, there are two mutually exclusive sub-cases. In both of these sub-cases, we have $k^0 = k^1 = \bar{k}$ as a consequence of the capital investment Euler equation.

- (a) $\partial_w V(w_0, s^0) = \partial_w V(w(s_h^1), s_h^1) = \partial_w V(w(s_l^1), s_l^1)$. From 28, it follows that for $j = 0, 1$, $\alpha \bar{A}(k^j)^{\alpha-1} = \delta$, i.e., capital reaches the first-best level at $t = 0$.
- (b) $\partial_w V(w_0, s^0) > \partial_w V(w(s_h^1), s_h^1) = \partial_w V(w(s_l^1), s_l^1)$. It follows that $h(s_h^1) = h(s_l^1) = 0$. Then, $V(w(s^1), s^1) = \max\{1, \varphi(\bar{k}, \bar{k}, s^1)\}$ for $s^1 = s_h^1, s_l^1$, which requires

$$(1 + \epsilon_h)^\rho ((1 + \epsilon_h) + (1 + \sigma\epsilon_h))^{\alpha-1} = (1 + \epsilon_l)^\rho ((1 + \epsilon_l) + (1 + \sigma\epsilon_l))^{\alpha-1}. \quad (32)$$

Additionally, $\partial_w V(w_0, s^0) > \partial_w V(w(s^1), s^1)$ implies k^0, k^1 are below the first-best level, i.e., $\alpha \bar{A}(k^j)^{\alpha-1} > \delta$.

Together the cases above exhaust all possibilities for the behavior of a solution and provide a key condition for evaluating whether the marginal value of resources tends to be higher in s_h^1 or s_l^1 . We can define a useful function in

$$\psi_\sigma(\rho, \alpha) := \left(\frac{1 + \epsilon_h}{1 + \epsilon_l} \right)^\rho \left[\frac{(1 + \epsilon_l) + (1 + \sigma\epsilon_l)}{(1 + \epsilon_h) + (1 + \sigma\epsilon_h)} \right]^{1-\alpha} - 1,$$

which is a continuous function in $[0, 1]^2$ and is increasing in both ρ and α . It also satisfies $\psi(\rho = 0, \alpha) < 0$ and $\psi(\rho = 1, \alpha) > 0$ for all $\alpha \in [0, 1]$. Therefore, we can define implicitly $\psi(\bar{\rho}(\alpha), \alpha) = 0$, so that whenever $\rho > \bar{\rho}(\alpha)$ (respectively, $<$ or $=$), condition 30 is ensured (respectively, 31 or 32). Also, $\psi_\sigma(1 - \alpha, \alpha) > 0$ implies $\bar{\rho}(\alpha) < 1 - \alpha$. Last, as $\sigma \rightarrow 1$, $\psi_\sigma \rightarrow \left(\frac{1 + \epsilon_h}{1 + \epsilon_l} \right)^{\rho + \alpha - 1} - 1$ so $\bar{\rho}(\alpha) \rightarrow 1$.

Proofs from Section 3.2

We have $\frac{\partial V(w(s^1), \gamma, s^1)}{\partial w(s^1)} = \min \left\{ \frac{A \left(\frac{w_1(s^1)}{\varphi(\gamma, s^1)} \right)^{\alpha-1} + (1 - \theta(\gamma, s^2))}{\varphi(\gamma, s^1)}, 1 \right\}$ for $s^1 = s_h^1, s_l^1$ and s^2 being its

unique successor. Given that $\varphi(\gamma, s^1) = 1 - \theta_2(\gamma, s^2)$, it simplifies to $\frac{\partial V(w(s^1), \gamma, s^1)}{\partial w(s^1)} = 1 + A[w_1(s^1)]^{\alpha-1} [\varphi(\gamma, s^1)]^{-\alpha} > 1$. This expression is decreasing in both $\varphi(\gamma, s^1)$ and $w_1(s^1)$. As a consequence, it is increasing in $\theta(\gamma, s^2)$.

Lemma 2. *In the environment described in example 2, $\frac{\partial V(w(s_h^1), \gamma, s_h^1)}{\partial w(s_h^1)} > \frac{\partial V(w(s_l^1), \gamma, s_l^1)}{\partial w(s_l^1)} > 1$.*

Proof. There are two cases to consider: $h(s_h^1) = 0$ and $h(s_h^1) > 0$. In the former, $w_1(s_h^1) = \frac{A}{\alpha} k_0^\alpha + (1 - \theta_0(\gamma, s^0)) k_0 \leq w_1(s_l^1)$. Then, $\frac{\partial V(w(s_h^1), \gamma, s_h^1)}{\partial w(s_h^1)} = 1 + A[w_1(s_h^1)]^{\alpha-1} [\wp(\gamma, s_h^1)]^{-\alpha} \geq 1 + A[w_1(s_l^1)]^{\alpha-1} [\wp(\gamma, s_h^1)]^{-\alpha} > 1 + A[w_1(s_l^1)]^{\alpha-1} [\wp(\gamma, s_l^1)]^{-\alpha} = \frac{\partial V(w(s_l^1), \gamma, s_l^1)}{\partial w(s_l^1)}$. In the latter, we need $\frac{\partial V(w(s_h^1), \gamma, s_h^1)}{\partial w(s_h^1)} = \frac{\partial V(w_0, \gamma, s^0)}{\partial w_0} \geq \frac{\partial V(w(s_l^1), \gamma, s_l^1)}{\partial w(s_l^1)}$. If equality between all three multipliers were to happen, the investment Euler equation would establish that $\frac{\partial V(w_0, \gamma, s^0)}{\partial w_0} = (1 + A[w_0]^{\alpha-1} [\wp_0]^{-\alpha}) E \left[\frac{\partial V(w(s_h^1), \gamma, s_h^1)}{\partial w(s_h^1)} \right] \implies 1 = 1 + A[w_0]^{\alpha-1} [\wp_0]^{-\alpha} > 1$ reaching a contradiction. $\square$

Proof of Lemma 1. Both statements in Lemma (1) follow from the lemma above. The first one is immediate. For the second, we argue that since $\frac{\partial V(w(s^1), \gamma, s^1)}{\partial w(s^1)} > 0$ for $s^1 = s_h^1, s_l^1$, we get the result that firms resort to maximal leverage at $t = 1$ and $k_2(s_h^1) \geq \frac{\frac{A}{\alpha} k_1^\alpha(s_0) + (1 - \theta_0) k_1(s_0)}{\wp(\gamma, s_h^1)} > \frac{\frac{A}{\alpha} k_1^\alpha(s_0) + (1 - \theta_0) k_1(s_0)}{\wp(\gamma, s_l^1)} = k_2(s_l^1)$ where the last equality follows from the fact that $\frac{\partial V(w_0, \gamma, s^0)}{\partial w_0} \geq \frac{\partial V(w(s_h^1), \gamma, s_h^1)}{\partial w(s_h^1)} > \frac{\partial V(w(s_l^1), \gamma, s_l^1)}{\partial w(s_l^1)}$, which ensures that $h(s_l^1) = 0$. $\square$

Proof of Proposition 3. When changes occur with respect to $\theta(\gamma, s^2)$ only, projects are evaluated according to

$$\frac{\partial V(w_0, \gamma, s^0)}{\partial \gamma} = \beta E \left[\left(\frac{\partial V(w(s^1), \gamma, s^1)}{\partial w(s^1)} - \frac{\beta}{R} \frac{\partial V(w(s^2), \gamma, s^2)}{\partial w(s^2)} \right) \frac{\partial \theta(\gamma, s^2)}{\partial \gamma} k_2(s^1) \right],$$

which simplifies further, given that $\beta = R = \frac{\partial V(w(s^2), \gamma, s^2)}{\partial w(s^2)} = 1$. Additionally, we describe projects ordered as mean-preserving spreads, so we can write $\theta(\gamma, s^2) = \theta(\underline{\gamma}, s^2) + g(\gamma) \Delta(s^2)$ where $E[\Delta] = 0$ and $\Delta(s^2 = s_h) > 0 > \Delta(s^2 = s_l)$, and $g(\gamma)$ is an increasing differentiable function. Then,

$$\frac{\partial V(w_0, \gamma, s^0)}{\partial \gamma} = g'(\gamma) E \left[\left(\frac{\partial V(w(s^1), \gamma, s^1)}{\partial w(s^1)} - 1 \right) k_2(s^1) \Delta(s^2) \right].$$

Given both statements in Lemma 1, we can ensure this term has a positive sign and rewrite it according to the statement of the proposition. $\square$

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