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Escola de Pós-Graduação
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Escola de

Pós-Graduação

em Economia

da Fundação

Getúlio Vargas

Nº 768

ISSN 0104-8910

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Setembro de 2015

URL: <http://hdl.handle.net/10438/13886>

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Cavalcanti Ferreira, Pedro
Life Cycle Models, Heterogeneity of Initial
Assets, and Wealth Inequality/ Pedro Cavalcanti Ferreira,
Diego Braz Pereira Gomes - Rio de Janeiro : FGV,EPGE, 2015
37p. - (Ensaio Econômico; 768)

Inclui bibliografia.

CDD-330

Life Cycle Models, Heterogeneity of Initial Assets, and Wealth Inequality

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September 3, 2015

Abstract

Life cycle general equilibrium models with heterogeneous agents have a very hard time reproducing the American wealth distribution. A common assumption made in this literature is that all young adults enter the economy with no initial assets. In this article, we relax this assumption – not supported by the data - and evaluate the ability of an otherwise standard life cycle model to account for the U.S. wealth inequality. The new feature of the model is that agents enter the economy with assets drawn from an initial distribution of assets, which is estimated using a non-parametric method applied to data from the Survey of Consumer Finances. We found that heterogeneity with respect to initial wealth is key for this class of models to replicate the data. According to our results, American inequality can be explained almost entirely by the fact that some individuals are lucky enough to be born into wealth, while others are born with few or no assets.

*We would like to thank Carlos E. da Costa, Todd Schoellman, Cezar Santos, Felipe Iachan, and Romeu Gomes Filho for their comments. Ferreira acknowledges the financial support of CNPq/INCT and FAPERJ; Gomes, the financial support of CAPES.

1 Introduction

Wealth in the United States is highly concentrated and very unequally distributed. According to the Survey of Consumer Finances (SCF), in 2010, the richest 1% held one-third of the total wealth in the economy, the richest 20% held more than 80% of total wealth, and the Gini index of wealth was approximately 0.83.¹ In addition, this inequality has increased over time. In 1989, the richest 1% held approximately 30% of total wealth, and the Gini index was 0.79. These facts make redistribution of wealth a central issue in discussions of economic policy. The main tools used by economists and policymakers to perform *ex ante* policy evaluations include life cycle general equilibrium models with heterogeneous agents. It is therefore important that this class of models are able to reproduce the main features of the American distribution of wealth.

To the best of our knowledge, no life cycle general equilibrium model with heterogeneous agents has been able to reproduce the American wealth distribution. In addition to [Huggett \(1996\)](#), which is a standard model, other articles incorporated different mechanisms, such as transmission of physical and human capital ([De Nardi \(2004\)](#) and [De Nardi and Yang \(2014\)](#)), random inheritance ([Hendricks \(2007a\)](#)), discount rate heterogeneity ([Hendricks \(2007b\)](#)), housing decisions and tax incentives ([Cho and Francis \(2011\)](#)), idiosyncratic risk in out-of-pocket medical and nursing home expenses ([Kopecky and Koreshkova \(2014\)](#)), but in one way or another, these articles fell short of the observed wealth distribution.²

A common assumption made in this literature is that all young adults enter the economy with no initial assets.³ In other words, it is assumed that there is no heterogeneity with respect to initial wealth. This assumption, however, is rejected by the data. According to the SCF data, the average net worth of young adults aged between 20 and 25 in 2010 was close to \$24,000. The data also show that young adults are not equally

¹The measure of wealth that we consider is net worth, which includes all assets held by households (e.g., real estate, financial wealth, vehicles) net of all liabilities (e.g., mortgages and other debts). It is thus a comprehensive measure of most marketable wealth.

²To date, the most successful economic models of U.S. wealth inequality, such as those by [Krusell and Smith \(1998\)](#), [Quadri \(2000\)](#), [Li \(2002\)](#), [Castañeda et al. \(2003\)](#), [Meh \(2005\)](#), [Cagetti and De Nardi \(2006\)](#), [Kitao \(2008\)](#), and [Cagetti and De Nardi \(2009\)](#), do not use a realistic life cycle structure. These models use the dynastic framework or the stylized demographic structure from [Gertler \(1999\)](#), which was derived from the [Blanchard \(1985\)](#) model of perpetual youth.

³[Hendricks \(2007a\)](#) considered that young adults enter the economy with assets drawn from a non-uniform probability distribution. However, its model is in partial equilibrium.

distributed in relation to wealth. The average net worth of the 1% richest young adults aged between 20 and 25 in 2010 was \$1,433,000. This value is approximately 60 times greater than the average of the entire age group.

In this article, we relax the assumption of no heterogeneity with respect to initial wealth and evaluate the ability of an otherwise standard life cycle general equilibrium model with heterogeneous agents to account for U.S. wealth inequality. In this economy, agents differ by age, asset holdings, labor productivity, and average lifetime earnings, and they choose their consumption, labor time and asset holdings for the next period. Retirement is exogenous, income tax is progressive and there is uncertainty regarding the time of death and the next period's labor productivity. The new feature of the model is that agents enter the economy with assets drawn from an initial distribution of assets. After this first draw, the accumulation of assets is endogenous to the model.

This initial distribution of assets was estimated using a non-parametric method applied to the SCF data, which is an appropriate database to measure wealth inequality. As discussed by [Castañeda et al. \(2003\)](#) and [Cagetti and De Nardi \(2008\)](#), the SCF over-samples rich households, which is especially important given the high degree of wealth concentration observed in the data. Therefore, it provides a more accurate measure of wealth inequality and of total wealth holdings than, for instance, the Panel Study of Income Dynamics (PSID), which was used by [Hendricks \(2007a\)](#) in a partial equilibrium setting.

More importantly, the appropriate sample should not consist of only young adults.⁴ This is because such survey data only consider the wealth that is legally owned by the individual, but in practice, young adults have access to their families' wealth, which is generally accounted for by their parents or grandparents. As evidence that young adults have access to their families' wealth, in 2010, 26% of household heads aged between 20 and 25 years who owned some property did not have mortgages associated with the purchase of the primary residence.⁵ In the same year, 44% of young adults with some college experience had no outstanding education loans. It is very unlikely that these

⁴[Hendricks \(2007a\)](#) considered a sample of households with heads aged between 19 and 21.

⁵Properties include houses, mobile homes, ranches, farms, condominiums, etc. Regarding mortgages, we considered the first and junior liens on primary residence used to purchase the primary residence. These data are from the SCF.

young people had accumulated the resources to buy a property or to pay off student loans in such a short period. As we will argue in the next section, there is evidence that young adults use their family wealth.

Therefore, the wealth of young adults recorded in surveys is a lower bound of the wealth available to them, and using that bound to estimate the initial distribution of assets would underestimate actual wealth inequality. To prevent this, we estimated the initial distribution of assets using data for all households and assumed that young adults have full access to their families' wealth when they enter the economy. We subsequently relax this full access assumption and also consider cases where young adults access only fractions of their families' wealth.

We calculated the equilibrium of our model by first assuming that agents enter the economy with zero initial assets and then assuming that agents enter with assets drawn from the estimated distribution. As expected, our model was unable to reproduce American wealth inequality when all young adults enter the economy with no initial assets. However, when we use the estimated initial distribution of assets, our model was able to describe U.S. wealth inequality quite accurately. We reproduced the share of wealth going to the very top households – for instance, the richest 1% has 34.8% of the total wealth – and the wealth Gini coefficient very closely.⁶ Hence, according to our results, American inequality can be explained almost entirely by the fact that some individuals are lucky enough to be born into wealth, while others are born with few or no assets.

The remainder of the article is organized as follows. In the next section, we present some stylized facts and discuss wealth inequality of young adults. In Section 3, we describe our standard life cycle general equilibrium model. In Section 4, we present the model parameterization. In Section 5, we explain the non-parametric method used to estimate the initial distribution of assets. In Section 6, we report our findings and quantify the role played by the initial distribution of assets in accounting for U.S. wealth inequality. In Section 7, we perform a robustness analysis, and in Section 8, we offer some concluding comments.

⁶See Appendix B for a summary of the results reported in the literature.

2 Stylized Facts

Wealth inequality has been a feature of the American economy for a long time. Table 1 presents data from the Survey of Consumer Finances of the percentage of total net worth held by various wealth groups in different years. Although wealth concentration has increased since 1989 – when the wealth share of the richest 5% was 6 percentage points below that the share in 2010 – it is clear that the distribution was already very unequal. In recent years, the net worth held by the richest 1% of households is nearly 34% of the total wealth in the economy, while the top 5% share is slightly above 60%. The wealth Gini coefficient is above 0.80. Over the entire period, the median net worth held by the richest 10% of households was 69% of total wealth.

There are also wide differences in the wealth distribution among young adults. The data in Table 2 show that the average net worth of households with heads aged twenty to twenty-five years old, according to the SCF, was twenty-four thousand dollars in 2010, while that of one of the richest 1% (5%) of households was 1.4 million (400 thousand) dollars in the same year. From 1989 to 2010, the wealth of a typical young man in the richest 1% group was, on average, 61 times higher than average wealth of the age group. Over the entire period, the median net worth held by the richest 10% of households was 257 thousands dollars. Clearly, the distribution of wealth among young adults is also very concentrated. Moreover, the assumption commonly made in the literature that individuals start their working lives with zero initial assets contradicts the evidence.

Table 3 presents additional evidence of wealth inequality among young adults. In 2010, according to the SCF, of all heads of household aged 20 to 25 who owned property, 26% had no mortgage outstanding.⁷ In the same year, 44% household heads in the same age group with some college education had no education loans outstanding. Using data from the National Postsecondary Student Aid Study (NPSAS), Kantrowitz (2014) shows that the number of students who graduate from college without debt has been decreasing over the last 20 years, among other reasons, because of the steep increase in tuition, but 30% of students in 2012-13 still graduated without student loans. In addition, according to the Pew Research Center (2014), college-educated households with student

⁷By comparison, in Canada, according to the 2008 Survey of Household Spending, the mean age of persons without a mortgage was 62.

Table 1: Percentage of Net Worth Held by the Richest Groups (%)

Year	1%	5%	10%	20%	40%	60%	80%	Gini
1989	30.03	54.80	66.79	80.44	93.56	98.72	99.96	0.79
1992	29.98	54.35	66.86	79.92	93.01	98.45	99.93	0.78
1995	34.81	55.94	67.68	80.25	92.79	98.27	99.89	0.79
1998	33.50	57.01	68.41	81.31	93.44	98.59	99.93	0.79
2001	32.06	57.22	69.52	82.38	94.24	98.72	99.93	0.80
2004	33.29	57.10	69.20	82.70	94.45	98.80	99.93	0.80
2007	33.52	60.19	71.26	83.24	94.43	98.87	99.94	0.81
2010	33.80	60.49	73.93	86.22	96.03	99.28	99.97	0.83
Median	33.39	57.05	68.80	81.84	93.90	98.72	99.93	0.80

Source: Survey of Consumer Finances; Authors' analysis.

Table 2: Average Net Worth Held by the Richest Groups (Aged 20-25)

Year	1%	5%	10%	20%	40%	60%	80%	Avg.
1989	4,126	894	381	219	120	81	64	46
1992	815	403	250	156	90	61	46	35
1995	792	234	167	100	59	41	31	21
1998	1,436	308	198	115	63	44	33	22
2001	3,493	842	500	267	143	97	72	55
2004	1,013	418	265	155	89	60	45	33
2007	5,982	929	469	251	133	89	67	48
2010	1,433	398	217	126	69	47	35	24
Median	1,435	410	257	156	89	61	46	34

Source: Survey of Consumer Finances; Authors' analysis. Notes: All values are in thousands of 2010 dollars.

loans to repay have lower net worth than those with no student debt (\$8,700 and \$64,700, respectively).

Given the young age of these persons, and consequently, the short span of time in the labor market, it is highly unlikely that they could finance their educations and houses themselves. In fact, [Kantrowitz \(2011\)](#) shows that 56% of upper-income undergraduate

Table 3: Percentage of Young Adults Aged between 20 and 25 (%)

	1989	1992	1995	1998	2001	2004	2007	2010	Med.
No mortgages ^a	–	–	26.7	26.7	27.0	17.1	13.8	26.0	26.4
No education loans ^b	57.8	49.5	58.2	55.0	53.1	51.0	52.4	44.0	52.7

Source: Survey of Consumer Finances; Authors’ analysis. Notes: ^a Head of household aged 20–25, owns a property but has no mortgage on the primary residence. ^b Head of household aged 20–25, has some college but no education loans.

students graduated with no debt in 2007 and 2008, compared with 36% of low-income students, and students whose parents have advanced degrees are more likely to graduate without debt, probably because their parents have higher average incomes. More importantly, more than two-thirds of students who graduated without debt receive help paying for tuition and fees from their parents.

In many cases, parents take out student loans themselves, the most common option being the Parent PLUS Loan. More than 3 million Parent PLUS borrowers owe nearly \$62 billion, or approximately \$20,000 per borrower, according to the Department of Education.⁸ It seems clear that there is not only high wealth inequality when comparing their own assets but also in the number of young adults who have access to their parents’ wealth (or borrowing capacity), so the numbers in the SCF database underestimate true inequality of wealth among young people.

3 Model Economy

We built a life cycle general equilibrium model in the tradition of [İmrohoroglu et al. \(1995\)](#) and [Huggett \(1996\)](#). The benchmark economy consists of a large number of heterogeneous agents, a competitive production sector, and a government with a commitment technology. The model is rich enough for agents to have retirement, precautionary, and lifetime uncertainty savings motives. Time is discrete, and one model period is a year. All shocks are independent among agents, and consequently, there is no uncertainty over the aggregate variables, although there is uncertainty at the individual level. We describe

⁸Note, however, that interest rates for the Parent PLUS program are higher than those of direct student loan programs, such as Federal Stafford Loans. Hence, Parent PLUS figures underestimate the total stock of parent debt, as they can borrow in their children’s names to take advantage of lower Stafford Loan rates, for instance.

the features of the model below.

3.1 Demography

The economy is populated by agents with age $j \in \{1, \dots, J\}$. The population grows exogenously at a constant rate η . Agents face exogenous uncertainty regarding the age of death. Conditional on being alive at age j , the probability of surviving to age $(j+1)$ is given by Π_j . All agents enter in the economy with age $j = 1$, which means that $\Pi_0 = 1$. Moreover, agents die with certainty at the end of age J , which means that $\Pi_J = 0$. The share of agents of age j is given by μ_j and can be recursively defined as

$$\mu_{j+1} = \frac{\Pi_j}{1 + \eta} \mu_j.$$

3.2 Preferences

In each period of life, agents are endowed with ℓ units of time, which can be split between labor and leisure. The choice of labor time is given by $l \in \mathcal{L}$, where the set of labor times $\mathcal{L}$ is finite. Agents enjoy utility from consumption, leisure, and accidental bequests, and they maximize the discounted expected utility throughout their lives. The intertemporal discount factor is given by β . There is a cost of working, which is treated as a loss of leisure. The period utility function over consumption and leisure is given by

$$u(c, l) = \frac{\left[c^\gamma (\ell - l - \phi \mathbf{1}_{\{l > 0\}})^{1-\gamma} \right]^{1-\sigma}}{1 - \sigma},$$

where c is the consumption, γ is the share of consumption in utility, σ is the risk aversion parameter, ϕ is the time cost of work, and $\mathbf{1}_{\{\cdot\}}$ is an indicator function that maps to one if its argument is true. The term in parentheses represents leisure time. The “warm-glow” utility from leaving accidental bequests is given by

$$u^B(a') = \psi_1 \frac{(\psi_2 + a')^{\gamma(1-\sigma)}}{1 - \sigma},$$

where ψ_1 represents the weight on the utility from bequeathing and ψ_2 affects its curvature.

3.3 Asset Market

Agents can acquire a one-period riskless asset in each period of their lives. We assume that this asset provides claims to capital used in the production sector. Current asset holdings are denoted as $a \in \mathcal{A}$, where the set of assets $\mathcal{A}$ is finite. Agents enter the economy with an endowment of assets drawn from the distribution $\Omega(a)$. The riskless rate of return on asset holdings is denoted by r . Agents are not allowed to incur debt at any age, so that the amount of assets carried over from age j to $(j + 1)$ is such that $a' \geq 0$, where a' is next period's asset holdings. For simplicity, we assume that all assets left by the deceased are collected by the government and distributed to the live agents as a lump-sum bequest transfer B .

3.4 Labor Productivity

In each period of life, agents receive an idiosyncratic labor productivity shock that is revealed at the beginning of the period. We denote this shock as $z \in \mathcal{Z}$, where the set of labor productivities $\mathcal{Z}$ is finite. This productivity shock follows a first-order Markov process. Conditional on having a productivity z , the probability of having next period's productivity z' is given by $\Gamma(z, z')$. The invariant distribution of this Markov process is given by $\bar{\Gamma}(z)$. Workers receive a wage rate w measured in efficiency units, which implies that the labor income of a worker who supplies l to the labor market is given by $y = wzl$.

3.5 Social Security

Once agents reach the retirement age R , they automatically stop working and start receiving a Social Security benefit. This benefit depends on the average lifetime earnings of an agent, which is calculated by taking into account individual earnings up to age $(R - 1)$.⁹ We denote the average lifetime earnings as $x \in \mathcal{X}$, where the set of average

⁹According to Social Security legislation, the average lifetime earnings should be calculated by taking into account the 35 years with the highest individual earnings up to the *Earliest Retirement Age*. For simplicity, we consider the entire history of earnings because it is difficult to identify the 35 highest earnings years when solving the model.

lifetime earnings $\mathcal{X}$ is finite. It can be recursively defined as

$$x' = \begin{cases} \frac{x(j-1) + \min\{y, y^{SS}\}}{j} & \text{if } j < R, \\ x & \text{if } j \geq R, \end{cases}$$

where y^{SS} is the *Social Security Wage Base* (SSWB), which is the maximum earned gross income to which the Social Security tax applies.

Let $b(x)$ be the Social Security benefit function, which corresponds to the *Primary Insurance Amount* (PIA).¹⁰ This benefit is calculated as a piecewise linear function, which in accordance with the rules of the U.S. Social Security system, is given by

$$b(x) = \begin{cases} \theta_1 x & \text{if } x \leq x_1, \\ \theta_1 x_1 + \theta_2(x - x_1) & \text{if } x_1 < x \leq x_2, \\ \theta_1 x_1 + \theta_2(x_2 - x_1) + \theta_3(x - x_2) & \text{if } x_2 < x \leq y^{SS}, \end{cases}$$

where $\{x_1, x_2\}$ are the bend points of the function and the parameters $\{\theta_1, \theta_2, \theta_3\}$ satisfy $0 \leq \theta_3 < \theta_2 < \theta_1$.

3.6 Government

Government revenues are provided from income, consumption, and Social Security taxes and are used to finance Social Security benefits and government expenditures G . The income tax paid by all agents is given by $\tau^Y(y^T)$, which is a progressive function of taxable income y^T . Taxable income is based on labor income and asset income and is given by $y^T = y + ra$. The Social Security benefit is not subject to income tax. For the progressive income tax function, we follow the 2010 Internal Revenue Service (IRS) rules

¹⁰This is the benefit to which individuals are entitled at the *Full Retirement Age*. Its value is neither reduced for early retirement nor increased for delayed retirement.

and assume that

$$\tau^Y(y^T) = \begin{cases} \tau_1 y^T & \text{if } y^T \leq y_1, \\ \tau_1 y_1 + \tau_2 (y^T - y_1) & \text{if } y_1 < y^T \leq y_2, \\ \tau_1 y_1 + \tau_2 (y_2 - y_1) + \tau_3 (y^T - y_2) & \text{if } y_2 < y^T \leq y_3, \\ \tau_1 y_1 + \sum_{n=2}^3 \tau_n (y_n - y_{n-1}) + \tau_4 (y^T - y_3) & \text{if } y_3 < y^T \leq y_4, \\ \tau_1 y_1 + \sum_{n=2}^4 \tau_n (y_n - y_{n-1}) + \tau_5 (y^T - y_4) & \text{if } y_4 < y^T \leq y_5, \\ \tau_1 y_1 + \sum_{n=2}^5 \tau_n (y_n - y_{n-1}) + \tau_6 (y^T - y_5) & \text{if } y_5 < y^T, \end{cases}$$

where $\{\tau_1, \tau_2, \tau_3, \tau_4, \tau_5, \tau_6\}$ are the marginal income tax rates, and $\{y_1, y_2, y_3, y_4, y_5\}$ are the income brackets.¹¹ The consumption tax rate paid by all agents is given by τ^C , which is proportional and levied directly on consumption. The Social Security tax rate paid by workers is given by τ^{SS} , which is proportional and levied on the minimum of labor income and Social Security wage base. We can formalize Social Security taxes as

$$T^{SS} = \tau^{SS} \min \{y, y^{SS}\}.$$

For ease of notation, we define the total taxes paid by agents, excluding the consumption tax:

$$T = \tau^Y(y^T) + T^{SS}.$$

3.7 Production Sector

We assume that there is a representative firm that acts competitively and produces a single consumption good. This firm maximizes profits by renting capital and labor from agents and paying an interest rate r and a wage rate w for these factors, respectively. The production function is specified as a Cobb-Douglas function, which is given by $F(K, L) = AK^\alpha L^{1-\alpha}$, where K and L are the aggregate capital and labor inputs, A

¹¹To the best of our knowledge, every article in the quantitative macroeconomic literature that considers a progressive income tax function uses the functional form estimated by [Gouveia and Strauss \(1994\)](#). This functional form is very convenient because it is differentiable, which facilitates solving the optimization problem when the choice of labor time is endogenous. However, we adopted a more realistic approach and considered the official functional form defined by the IRS.

is the total factor productivity, and α is the share of capital in the output. Capital is assumed to depreciate at a rate δ each period. We can write the problem of the firm as

$$\max_{K,L} F(K, L) - (r + \delta)K - wL.$$

3.8 Agents' Problem

There are two groups of heterogeneous agents, workers and retirees. Workers are those aged $(R - 1)$ and younger, while retirees are those aged R and older. Let $\mathcal{S}^W$ and $\mathcal{S}^R$ be the state spaces of workers and retirees, respectively. The state vector of a worker is given by $s_W = (j, z, a, x) \in \mathcal{S}^W$, and its value function is given by $V^W : \mathcal{S}^W \rightarrow \mathbb{R}$. Similarly, the state vector of a retiree is given by $s_R = (j, a, x) \in \mathcal{S}^R$, and its value function is given by $V^R : \mathcal{S}^R \rightarrow \mathbb{R}$. The agents' problem must be solved separately for specific age groups. Therefore, we describe the problem by dividing it between the age groups.

For $j \in \{2, \dots, R - 2\}$, agents are workers in the current period and will remain workers in the next period. They choose their consumption, labor time, and the next period's asset holdings. Their problem can be recursively defined as

$$V^W(s_W) = \max_{(c,l,a')} \left\{ u(c, l) + \beta \left[\Pi_j E V^W(s'_W) + (1 - \Pi_j) u^B(a') \right] \right\}$$

subject to

$$(1 + \tau^C) c + a' + T = y + (1 + r)a + B,$$

$$c \geq 0, \quad l \in \mathcal{L}, \quad a' \geq 0.$$

For $j = 1$, the only difference in relation to the above problem is in the budget constraint. In this first period of their life, agents are “born” with initial assets drawn from an exogenous distribution, and of course, receive no interest from the previous period's assets. Everything else is the same. Their budget constraint is given by

$$(1 + \tau^C) c + a' + T = y + a + B.$$

For $j = (R - 1)$, the only difference is in the Bellman equation. Because these agents will be retirees in the next period, they take into account the value function of retirees in the next period rather than the expected value function of workers.

For $j \in \{R, \dots, J - 1\}$, agents are retirees in the current period and will remain retirees in the next period. They choose their consumption and the next period's asset holdings. They no longer choose labor time because they start receiving Social Security benefits in the current period. Their problem can be recursively defined as

$$V^R(s_R) = \max_{(c, a')} \left\{ u(c, 0) + \beta \left[\Pi_j V^R(s'_R) + (1 - \Pi_j) u^B(a') \right] \right\}$$

subject to

$$(1 + \tau^C) c + a' + T = b(x) + (1 + r)a + B,$$

$$c \geq 0, \quad a' \geq 0.$$

For $j = J$, the only difference in relation to the above problem is in the Bellman equation. Because these agents will be dead in the next period, they no longer take into account the value function of retirees in the next period, considering only the utility from leaving accidental bequests.

After solving the above problems, we obtain policy functions for the control variables. From now on, for ease of notation, we will consider $s \in \mathcal{S}$ as a generic state vector in a generic state space. Therefore, if we are considering workers, we have that $s \equiv s_W$ and $\mathcal{S} \equiv \mathcal{S}^W$. Similarly, if we are considering retirees, we have that $s \equiv s_R$ and $\mathcal{S} \equiv \mathcal{S}^R$. Therefore, we can define the policy function of consumption as $c : \mathcal{S} \rightarrow \mathbb{R}_+$, the policy function of labor time as $l : \mathcal{S} \rightarrow \mathcal{L}$, and the policy function of next period's asset holdings as $a' : \mathcal{S} \rightarrow \mathbb{R}_+$.

3.9 Agents' Distribution

The stationary distribution of agents among the states is described by a probability distribution function $\lambda : \mathcal{S} \rightarrow [0, 1]$. This distribution depends on the policy functions and the exogenous stochastic processes. In this section, when used in the same equation, variables with primes denote the current period, and variables without primes denote

the previous period. To describe how the distribution is constructed, we must divide it among the age groups.

For $j = 1$, agents have just entered the economy, and there is no transition. Their distribution depends only on the share of age groups, the initial distribution of assets, and the invariant distribution of labor productivity, which can be defined as

$$\lambda(1, a, z) = \mu_1 \Omega(a) \bar{\Gamma}(z).$$

The above expression makes explicit the main difference between our model and the rest of the literature, which is the initial draw of an asset from a distribution.

For $j \in \{2, \dots, R-1\}$, workers in the previous period remain workers in the current period. Their distribution depends on the population growth rate, the policy function of next period's asset holdings, the endogenous transition of average lifetime earnings, the survival probabilities, and the transition of labor productivities, which can be recursively defined as

$$\lambda(s') = \frac{1}{1+\eta} \sum_s \mathbf{1}_{\{a'(s)=a'\}} \mathbf{1}_{\{x'(s)=x'\}} \Pi_{j-1} \Gamma(z, z') \lambda(s).$$

For $j = R$, the only difference in relation to the above expression is that we no longer consider the transition of labor productivities because agents become retirees at this age.

For $j \in \{R+1, \dots, J\}$, retirees in the previous period remain retirees in the current period. The endogenous transition of average lifetime earnings is trivial and need not be taken into account. Their distribution depends on the population growth rate, the policy function of next period's asset holdings, and the survival probabilities, which can be recursively defined as

$$\lambda(s') = \frac{1}{1+\eta} \sum_s \mathbf{1}_{\{a'(s)=a'\}} \Pi_{j-1} \lambda(s).$$

3.10 Equilibrium Definition

A stationary recursive competitive equilibrium for this economy consists of value functions $\{V^W, V^R\}$, policy functions $\{c, l, a'\}$, factor prices $\{r, w\}$, a consumption tax rate τ^C , a lump-sum bequest transfer B , and a stationary distribution of agents λ such that the following conditions are satisfied:

1. The value functions $\{V^W, V^R\}$ and the policy functions $\{c, l, a'\}$ solve the agents' problem.
2. Factor prices $\{r, w\}$ are determined competitively in the production sector, that is,

$$r = F_K(K, L) - \delta \quad \text{and} \quad w = F_L(K, L).$$

3. The asset, labor, and consumption goods markets clear, that is,

$$K' = \sum_s a'(s) \lambda(s),$$

$$K = \frac{K'}{1 + \eta},$$

$$L = \sum_s z l(s) \lambda(s),$$

$$C + K' + G = F(K, L) + (1 - \delta)K + E,$$

where the aggregate consumption C and the aggregate initial endowment of assets E are, respectively, given by

$$C = \sum_s c(s) \lambda(s),$$

$$E = \sum_{j=1}^s a \lambda(s).$$

4. The consumption tax rate τ^C balances the government budget, that is,

$$G + \sum_s b(x) \lambda(s) = \tau^C C + \sum_s T(s) \lambda(s) + \sum_s \frac{(1 - \Pi_j) \tau^Y (r a'(s))}{1 + \eta} \lambda(s).$$

5. The lump-sum bequest transfer B is equal to the amount of assets left by the deceased, that is,

$$B = \sum_s \frac{(1 - \Pi_j) [(1 + r) a'(s) - \tau^Y (r a'(s))]}{1 + \eta} \lambda(s).$$

4 Model Parameterization

We parameterized the model using several different data sources. Two sources of micro data were used, the Survey of Consumer Finances (SCF)¹² and the Medical Expenditure Panel Survey (MEPS).¹³ Our main source of macro data is the [Council of Economic Advisers \(2013\)](#). Other sources of data were used and will be appropriately cited. Below we describe in detail how each parameter was estimated.

4.1 Demography

In this economy, a period corresponds to one year. We assumed that agents enter the economy at age 20 ($j = 1$) and can survive to a maximum age of 100 ($J = 81$). We set the population growth rate η so that the fraction of agents aged 65 and over equaled 12.55% in equilibrium. This target was calculated using population data from Table B-34 of the [Council of Economic Advisers \(2013\)](#).¹⁴ The final value of η is 2.62%. The survival probabilities Π_j were calculated using data from Table 6 of [Bell and Miller \(2005\)](#). We considered the calendar year 2010 and used the average of males and females. The final values are presented in Figure 1.

Figure 1: Survival Probabilities

¹²See <http://www.federalreserve.gov/econresdata/scf/scfindex.htm>.

¹³See <http://meps.ahrq.gov/mepsweb/>.

¹⁴We used the average of the fractions from 1996 to 2010.

4.2 Preferences

The time endowment ℓ was set to 8,760 hours, which is the total number of hours per year (considering 365 days per year and 24 hours per day). To construct the set of labor times $\mathcal{L}$, we assumed that a worker can work from zero to 18 hours per day, but the number of hours can only vary discretely by multiples of two, that is, a worker can work zero, two, four hours and so on but cannot work five or four and a half hours a day. We considered 252 working days per year, so the maximum number of hours/year is 4,536. The values of the grid of hours are presented in Table 4. We set the intertemporal discount factor β so that the capital-output ratio of our model equaled 3.02 in equilibrium. This target was calculated using output data from the [Council of Economic Advisers \(2013\)](#) and capital data from [Feenstra et al. \(2013\)](#).¹⁵ The final value of β in the benchmark calibration is 0.7641.

Table 4: Set of Labor Times

1	2	3	4	5	6	7	8	9	10
0	504	1,008	1,512	2,016	2,520	3,024	3,528	4,032	4,536

The share of consumption in utility γ was set to 0.36, and the risk aversion parameter σ was set to 3. Both values were taken from [Nishiyama and Smetters \(2014\)](#). We set the time cost of work ϕ so that the average work hours in our model equaled 1,764 in equilibrium. This target was calculated using data from Table B-47 in [Council of Economic Advisers \(2013\)](#).¹⁶ The final value of ϕ in the benchmark calibration is 1,900 hours. The parameters of the bequest utility function were taken from [French \(2005\)](#). The weight on the bequest utility ψ_1 was set to 0.037 and was taken from the fourth specification of Table 2. The curvature parameter ψ_2 was set to \$400,000.

¹⁵We used the average of the ratios from 1996 to 2010.

¹⁶We used data for average weekly hours of the total private workforce and used 52 weeks per year to annualize these values. We used the average annualized values from 1996 to 2010.

4.3 Labor Productivity

We used the MEPS database to estimate the set of labor productivities $\mathcal{Z}$ and the transition probabilities $\Gamma(\cdot)$.¹⁷ To calculate the values of the productivities we used cross-sectional data from 1996 to 2010. For each cross-section, we considered only individuals aged between 20 and 64 with strictly positive sample weights and strictly positive wage incomes. All wages were converted to 2010 dollars using the annual CPI for all items. We first calculated the weighted average of annual wages for the whole sample, which is \$43,276.31 in 2010 dollars. Next, we approximated the distribution of wages using a histogram with bins corresponding to the 1st–25th, 26th–50th, 51st–75th, 76th–95th, and 96th–100th percentiles.¹⁸ Within each bin, we calculated the weighted average wage using the sample weights. Our labor productivities were then calculated as the ratios of these averages to the average for the whole sample.

Table 5: Labor Productivities, Transition Probabilities and Invariant Distribution

	z_1	z_2	z_3	z_4	z_5	Prods.	Inv.
z_1	0.5998	0.2615	0.0931	0.0384	0.0073	0.2453	0.2014
z_2	0.2005	0.4922	0.2286	0.0679	0.0107	0.6198	0.2440
z_3	0.0772	0.1944	0.5322	0.1764	0.0199	1.0309	0.2631
z_4	0.0424	0.0754	0.1896	0.6068	0.0860	1.7342	0.2281
z_5	0.0271	0.0458	0.0837	0.2994	0.5439	3.5826	0.0634

To calculate the transition probabilities, we used a non-parametric method and data for annual panels from 1996–1997 to 2009–2010.¹⁹ For each panel, we considered only individuals aged between 20 and 64 with strictly positive wage incomes in both years.

¹⁷As we want to estimate annual transitions, it is appropriate to use the MEPS database because it consists of annual panels and, therefore, no adjustment needs to be performed because of the frequency of the panel data.

¹⁸This approximation was used to capture the long right tail of the wage distribution.

¹⁹The primary method of estimating transitions of productivities adopted in the literature is the discretization of a lognormal AR(1) process into a Markov chain. However, using a large panel data set of earnings histories drawn from U.S. administrative records, [Guvenen et al. \(2015\)](#) found that earnings shocks display substantial deviations from lognormality. Thus, given the large amount of data provided by the MEPS database, we preferred to adopt a non-parametric approach.

Constructing the transition matrix requires simply calculating the weighted fractions of individuals who made the transition from a state z in the first year to a state z' in the second year.²⁰ The values of the productivities, transition probabilities, and invariant distribution are presented in Table 5.

4.4 Social Security

The retirement age R was set to 46, which corresponds to 65 years old in the real world. The other parameters were collected from the official Social Security Website. The Social Security Wage Base y^{SS} was set to \$106,800, which is the official value for 2010.²¹ The bend points of the benefit function $\{x_1, x_2\}$ were set to \$9,132 and \$55,032, respectively. These are the official annualized values for 2010.²² The parameters $\{\theta_1, \theta_2, \theta_3\}$ were set to their official values, which are 90%, 32%, and 15%, respectively.²³ To estimate the set of average lifetime earnings $\mathcal{X}$, we first formed a set containing the value zero, the two bend points of the benefit function, and the Social Security Wage Base. Between each set of points, we added two more equally spaced points to form a final set of 10 elements. The final values are presented in Table 6.

Table 6: Set of Average Lifetime Earnings

1	2	3	4	5	6	7	8	9	10
0	3,044	6,088	9,132	24,432	39,732	55,032	72,288	89,544	106,800

4.5 Government

Government expenditures G was set to represent 18.77% of total output in equilibrium. This target was calculated using output and government expenditure data from Table B-1 from the Council of Economic Advisers (2013).²⁴ The marginal income tax rates and income brackets were collected from the Tax Foundation (2013). The

²⁰The weighted fractions were calculated using the longitudinal sample weights.

²¹See <http://www.ssa.gov/oact/cola/cbb.html>.

²²See <http://www.ssa.gov/oact/cola/bendpoints.html>.

²³See <http://www.ssa.gov/oact/cola/piaformula.html>.

²⁴We used the average of the ratios from 1996 to 2010.

official values for the marginal income tax rates in 2010 were $\{\tau_1, \tau_2, \tau_3, \tau_4, \tau_5, \tau_6\} = \{10\%, 15\%, 25\%, 28\%, 33\%, 35\%\}$. For the income brackets, we took the average of the values corresponding to single, married filing jointly or qualified widow(er), married filing separately, and head of household filing statuses. The final values are $\{y_1, y_2, y_3, y_4, y_5\} = \{\$11,362.50, \$45,387.50, \$101,500.00, \$169,068.75, \$326,943.75\}$. The Social Security tax rate τ^{SS} was collected from the [Social Security Administration \(2014\)](#). We added the contributions of the employee and the employer, yielding a value of 12.4%.

4.6 Production Sector

We set total factor productivity A so that output per capita in our model equaled \$44,855 in equilibrium. This target was calculated using output and population data from [Council of Economic Advisers \(2013\)](#).²⁵ The final value of A in the benchmark calibration is 6.07289. The share of capital in the output α was set to 0.33 and the depreciation rate δ was set to 0.07. These values are within ranges commonly used in the literature.

5 Initial Distribution of Assets

In this section, we explain the method used to estimate the initial distribution of assets. We used data from the SCF database and pooled all the cross-sectional data from 1989, 1992, 1995, 1998, 2001, 2004, 2007, and 2010. We considered only the records of household heads aged 20 and over. In our model, assets were identified from this database as the net worth, and these values were converted to 2010 dollars using the annual CPI for all items.²⁶ The idea behind the non-parametric method is as follows. We first sort the sample in ascending order by net worth. Then, we divided the ordered sample into groups with the same total net worth. The support of the distribution was formed by the average net worth of each group and the probabilities were the fraction of individuals in each group.²⁷

Let i denote a typical individual in the sample, n be the size of the sample, x_i be the

²⁵We converted the output values to 2010 dollars using the GDP implicit price deflator provided by the same source. We used the average of output per capita from 1996 to 2010.

²⁶In the model, assets can only take positive values, so we converted negative net worth values to zero. This is the standard procedure in the literature.

²⁷The notation used in the formalization is not related to the notation used in our model.

net worth of individual i , and w_i be the sample weight associated with individual i . Note that we need only two variables from the database, the net worth and the sample weight. Because the first step is to sort the data in ascending order by net worth, we assume hereafter that $x_1 \leq x_2 \leq \dots \leq x_n$. The total net worth associated with individual i is given by $X_i = w_i x_i$. This variable is a population estimate of the total net worth held by individuals with the same characteristics as individual i . Let S_i be the cumulated total net worth up to individual i . Therefore, we have that $S_1 = X_1$ and $S_i = S_{i-1} + X_i$ for all $i > 1$.

We want to divide the ordered sample into groups with the same total net worth. However, we also want to include the zero value in the support of the distribution. Therefore, we create a group comprising all individuals with zero net worth and m groups that have the same total net worth, so the support of the distribution contains $(m + 1)$ elements. Let j denote a typical group. We assume that $j = 0$ is the group in which all individuals have zero net worth. Let Y be the constant total net worth of the m groups. This value is equal to the sum of total net worth of the whole sample divided by the number of groups, that is,

$$Y = \frac{1}{m} \sum_{i=1}^n X_i = \frac{S_n}{m}.$$

Let I_j be the interval that specifies the lower and upper limits of the cumulated total net worth of group j . Because the sample is ordered by net worth and each group has the same Y of total net worth, we can define these intervals as $I_0 = [0, 0]$ and $I_j = ((j - 1)Y, jY]$ for all $j > 0$. Therefore, the estimated number of individuals in each group is given by

$$N_j = \sum_{i=1}^n w_i \mathbf{1}_{\{S_i \in I_j\}}.$$

Let A_j be the average net worth of group j , with $A_j = Y/N_j$ for all $j > 0$ and $A_0 = 0$ for $j = 0$. Let P_j be the probability associated with the element A_j . These probabilities

Figure 2: Observed and Estimated Lorenz Curves

are the fractions of individuals in each group and are given by

$$P_j = \frac{N_j}{\sum_{k=0}^m N_k}.$$

We used the above method to estimate a distribution with a support containing 100 elements, which means that we set $m = 99$. Making the connection with our model, the set of assets $\mathcal{A}$ will be formed by the elements A_j and the initial distribution of asset $\Omega(\cdot)$ will be formed by the elements P_j . This non-parametric method has three desirable characteristics. First, it is not necessary to make any assumptions about the data generating process of the actual distribution. Second, as seen in Figure 2, the estimated distribution preserves the inequality observed in the data. Third, the first moment of the estimated distribution is equal to the first population moment. The proposition below formally states this last result.

Proposition 1. *The first moment of the estimated distribution is equal to the first pop-*

ulation moment.

Proof. We depart from the first moment of the estimated distribution and show that it is equal to the first population moment. For ease of notation, set

$$N = \sum_{j=0}^m N_j = \sum_{j=0}^m \sum_{i=1}^n w_i \mathbf{1}_{\{S_i \in I_j\}} = \sum_{i=1}^n w_i \sum_{j=0}^m \mathbf{1}_{\{S_i \in I_j\}} = \sum_{i=1}^n w_i.$$

The last equality above reflects that the intervals I_j are disjoint, which implies that each term S_i belongs to only one interval I_j . Therefore, we have that $P_j = N_j/N$. We can write the first moment of the estimated distribution as

$$\sum_{j=0}^m P_j A_j = \sum_{j=1}^m \frac{N_j}{N} \frac{Y}{N_j} = \frac{mY}{N} = \frac{1}{N} \sum_{i=1}^n X_i = \frac{1}{N} \sum_{i=1}^n w_i x_i.$$

The last term of the above expression is exactly the first population moment. *Q.E.D.*

6 Results

In this section, we quantify the role played by the initial distribution of assets in accounting for U.S. wealth inequality. Table 7 presents our main results. Our model economy (the “benchmark” line) describes for U.S. wealth inequality quite accurately. The Gini indexes of the model and the data are practically the same, and the simulated share of wealth held by the richest 1%, 5%, and 10% of the distribution is never more than 1.5 percentage points from the actual share. Compared to the results in the literature summarized in the Appendix B, the outcome of this experiment is one of the closest to the data. Although many studies have matched the Gini, most had difficulty replicating the share of wealth held by the richest one or five percent. Castañeda et al. (2003), for instance, replicates the share of the top 1% but misses the share of the top 5% by nine percentage points.

In Table 7, for the sake of comparison and to better understand our results, we also present the outcome of a simulation in which we calculated the equilibrium of our model considering that all agents enter the economy with zero initial assets, the usual assumption in the literature.²⁸ As expected, the model considerably underestimates the

²⁸We also simulated the model assuming that agents enter the economy with net worth drawn from a

Table 7: Main Results

	1%	5%	10%	20%	40%	60%	80%	Gini
Benchmark	34.86	55.64	67.30	80.77	93.53	98.49	99.95	0.79
Zero initial assets	8.66	28.75	45.08	66.07	87.22	96.79	99.54	0.64
Data	33.39	57.05	68.80	81.84	93.90	98.72	99.93	0.80

Notes: The data values are the medians of each year according to Table 1.

share of wealth held by the richest portion of the population. From the richest 1% to the richest 10%, the absolute error of the model in relation to the data exceeds twenty percentage points, a sizeable underestimation.

Although this last experiment cannot account for the wealth share of the richest groups, it is important to stress that even when starting from perfect asset equality, the model is still able to generate high inequality: the wealth Gini is 0.64, far from the perfect zero that we started with by construction. This is because income shocks affect agents unevenly, leading to income inequality, which in turn leads to wealth inequality. However, these earnings shocks are not dispersed enough to allow the model reproduce observed wealth inequality. There are not enough incentives in this case for high-income agents to save and accumulate wealth at the same pace and in the same amounts we observe in the data. Consequently, individuals will remain more equal, in terms of assets, throughout the life cycle in this case.

In our benchmark simulation, we assumed that young people have access to a larger pool of assets than those in their own names. In this sense, the initial asset distribution has enough disparity to partially offset the low incentive to save. There is plenty evidence, as discussed in Section 2, to support this hypothesis. One assumption that, of course, is not certain is the “correct” initial distribution or whether our strategy is close to reality. We do know that it matches the data, but how far would we be had we assumed that instead of full access to their family wealth, young people could only use a fraction of it? In Table 8, we repeated the main exercise assuming that young agents have access only

non-parametric distribution estimated only for the young adult sample. The results are consistent with the findings of Hendricks (2007a), underestimating wealth inequality and the share wealth held by the richest groups

Table 8: Results Considering Fractions of Initial Assets

Fractions	1%	5%	10%	20%	40%	Gini
0%	8.66	28.75	45.08	66.07	87.22	0.64
25%	14.23	33.75	48.68	67.63	87.12	0.66
50%	22.49	41.77	55.25	72.04	89.22	0.70
75%	29.35	49.14	61.59	76.62	91.55	0.75
100%	34.86	55.64	67.30	80.77	93.53	0.79
Data	33.39	57.05	68.80	81.84	93.90	0.80

Notes: The data values are the medians of each year according to Table 1.

to a fraction of the net worth from which they draw.²⁹

As one might expect, the smaller the fraction of assets to which young agents have access and that they can use, the more closely we replicate the standard result in the literature. In the limit, the 0% line, we are back to the model in which all agents start their lives with no assets. However, for higher fractions of assets, the model is still able to explain a large share of wealth inequality. Even when we halved the assets drawn by each individual (the 50% line), the model explains two-thirds of the wealth share held by the richest 1% and 73% of the richest 5%, still a close result.

The experiments in Table 8 provide an additional insight. When we multiply the support of the initial distribution of assets by a fraction, we do not change the inequality measures.³⁰ This means that in each of these experiments, the concentration of the top richest groups and the Gini index of the initial distribution are exactly the same.³¹ However, these same measures for the aggregate distribution of the whole economy increase with the multiplied fraction. This shows us that the inequality of the initial distribution is not the only important factor for the model to replicate the observed inequality and that the absolute values of the support matter. As all young adults start their working lives poorer, the wealth distribution of the economy becomes more equal, even when the

²⁹We also calculated all the results in this section considering a joint initial distribution of assets and labor productivity. The results are practically the same, showing that our assumption of the independence of these two initial distributions is not critical.

³⁰Except for the 0% line, where we obtain perfect equality.

³¹The only measures that change are the means and standard deviations, which vary in the same proportion as the multiplied fraction. For example, the mean of the 50% line is one-half the mean of the 100% line. The skewness and kurtosis also remain unchanged.

Figure 3: Variations in Inequality Measures

inequality of the initial distribution is as high as in the data. Figure 3 illustrates this by plotting the inequality measures of the aggregate and initial distributions of assets as a function of the mean of the initial distribution of assets. As the mean increases, the inequality measures of the aggregate distribution increase, while the measures of the initial distribution remain constant.

Our life cycle model is consistent with U.S. macroeconomic data, replicating some relevant U.S. macroeconomic variables quite closely. In Table 9, we present a comparison between some macroeconomic variables generated by the model and their corresponding values in the data. In our benchmark simulation, we matched output per capita, average work hours and capital per capita very closely. In the last case, the accuracy is due to the targeting of the output per capita and capital-output ratio. The share of government expenditures related to output was also targeted, implying very accurate government expenditures per capita, differing by only \$21. Without using any targeting strategy, the

Table 9: Observed and Estimated Macroeconomic Variables

	Data [*]	Benchmark
Output	44,855	44,855
Capital	135,454	135,313
Average work hours	1,764	1,764
Government expenditures	8,440	8,418
Social Security benefit	1,920	1,879
Interest rate	3.39%	3.94%
Hourly wage rate	17.99	18.65

^{*} Sources: [Council of Economic Advisers \(2013\)](#) and [Feenstra et al. \(2013\)](#). Financial values are in per capita terms. Averages from 1996 to 2010.

model was able to replicate the Social Security benefit per capita very closely, differing from data by only 2.2% (or 41 dollars). Factor prices were also replicated without targeting, with the simulated interest rate being only 0.55% higher than the data and the hourly wage only \$0.67 away from the observed hourly wage.

7 Robustness Analysis

In this section, we perform a robustness analysis to test the importance of some features of our model in explaining U.S. wealth inequality. To do this, we performed three new experiments. All the experiments assumed that agents enter the economy with assets drawn from the non-parametric distribution estimated from the whole sample. However, the parameters used to match moments of data were recalculated in each experiment. In the first, we disregard the bequest utility u^B , setting ψ_1 , the weight on the utility from bequeathing, to zero. In the second, we disregard the time cost of work, setting the parameter ϕ to zero. In the third, we disregard the labor choice, forcing all agents to work the same number of hours $\bar{l}$. We chose the value of $\bar{l}$ so that the average hours worked in this experiment was equal to 1,764 in equilibrium. The final value of $\bar{l}$ was equal to 1,843.09 hours.³²

These parameters and assumptions can have sizable effects on inequality. For in-

³²As we are already matching the average work hours in this experiment, the time cost of work is no longer needed, and we set its value to zero.

Table 10: Robustness Analysis

	1%	5%	10%	20%	40%	60%	80%	Gini
No bequest utility	35.93	57.21	67.43	79.41	92.18	97.98	99.93	0.78
No time cost of work	35.08	56.03	67.89	81.43	94.27	98.64	99.99	0.80
No labor choice	35.49	56.73	68.45	81.63	94.21	98.76	100.0	0.80
Data	33.39	57.05	68.80	81.84	93.90	98.72	99.93	0.80

Notes: The data values are the medians of each year according to Table 1.

stance, we assumed that assets left by the deceased are collected by the government and equally distributed to the surviving agents as lump-sum bequest transfers. With no utility from bequeathing, there is less incentive to accumulate wealth, so agents save less and consequently bequest transfers are smaller. Redistribution of wealth would decrease, increasing inequality.

From Table 10, we can see that these features alone are not critical to explaining U.S. wealth inequality. Labor market features have only a very small impact in this case, with no effect on the wealth Gini and only a marginal increase in the errors in the estimation of the wealth share held by the richest groups. The impact of removing bequests from the utility function is a bit larger, but the model still closely matches the data. It seems that, in this economy, the initial distribution of assets has the greatest effect on wealth inequality.

8 Conclusion

In this article, we built a standard life cycle general equilibrium model with heterogeneous agents to explain U.S. wealth inequality. The main feature that differentiates our model from the literature is that agents enter the economy with assets drawn from an initial distribution of assets, so that there is heterogeneity with respect to initial wealth. We compared the wealth distribution generated by our model with the distribution generated by a model that assumes that all agents enter the economy with zero initial assets, an assumption commonly adopted in the literature.

We found that a properly estimated initial distribution of assets is key for a standard

life cycle model that replicates U.S. wealth inequality. In addition, we concluded that both inequality in the initial distribution and the absolute values of the support of the distribution are important for the model to replicate the inequality observed in the data. Hence, for this class of models to replicate the data, it needs to assume “enough” initial inequality along with an appropriate initial level of wealth. Another possible mechanism, considered in [Castañeda et al. \(2003\)](#), would be to assume a labor productivity process with large dispersion.

Appendix A Summary of Parameters

Table 11: Summary of Parameters

Parameter	Value	Reference
Demography		
J	81	Maximum age equals 100 years old
η	2.62%	Share of pop. aged 65 and over equals 12.55%
Preferences		
ℓ	8,760	24 hours per day and 365 days per year
β	0.7641	Capital-output ratio equals 3.02
γ	0.36	Nishiyama and Smetters (2014)
σ	3	Nishiyama and Smetters (2014)
ϕ	1,900	Average work hours equals 1,764
ψ_1	0.037	French (2005)
ψ_2	\$400,000	French (2005)
Social Security		
R	46	Retirement age equals 65 years old
y^{SS}	\$106,800	Official Social Security Website
x_1	\$9,132	Official Social Security Website
x_2	\$55,032	Official Social Security Website
θ_1	90%	Official Social Security Website
θ_2	32%	Official Social Security Website
θ_3	15%	Official Social Security Website
Government		
G	\$8,418	Government-output ratio equals 18.77%
τ_1	10%	Tax Foundation (2013)
τ_2	15%	Tax Foundation (2013)
τ_3	25%	Tax Foundation (2013)

Table 11: Summary of Parameters

Parameter	Value	Reference
τ_4	28%	Tax Foundation (2013)
τ_5	33%	Tax Foundation (2013)
τ_6	35%	Tax Foundation (2013)
y_1	\$11,362.50	Tax Foundation (2013)
y_2	\$45,387.50	Tax Foundation (2013)
y_3	\$101,500.00	Tax Foundation (2013)
y_4	\$169,068.75	Tax Foundation (2013)
y_5	\$326,943.75	Tax Foundation (2013)
τ^{SS}	12.4%	Social Security Administration (2014)
Production Sector		
A	6.07289	Output per capita equals \$44,855
α	0.33	Value commonly used in the literature
δ	0.07	Value commonly used in the literature

Appendix B Results in the Literature

Table 12: Results in the Literature

Article / Data	Table	1%	5%	10%	20%	40%	60%	Gini
Data	–	33.4	57.0	68.8	81.8	93.9	98.7	0.80
Huggett (1996)	3	11.8	35.6	–	75.5	–	–	0.76
Krusell and Smith (1998)	1	24.0	55.0	73.0	88.0	–	–	0.82
Quadrini (2000)	XI	24.9	45.8	57.1	73.2	–	–	0.74
Floden and Lindé (2001)	VI	8.6	–	44.5	65.2	–	–	0.65
Li (2002)	3	20.3	46.9	59.4	70.3	–	–	–
Nishiyama (2002)	IV	19.4	41.2	55.7	74.4	93.3	99.6	0.74
Díaz et al. (2003)	10	13.7	50.6	76.6	–	–	–	–
Castañeda et al. (2003)	7	29.9	48.1	65.0	–	–	–	0.79
De Nardi (2004)	5	18.0	42.0	–	79.0	95.0	100.0	0.76
Meh (2005)	8	33.6	55.2	64.7	76.6	–	–	0.76
Cagetti and De Nardi (2006)	6	31.0	60.0	–	83.0	94.0	–	0.80
Hendricks (2007a)	9	11.7	35.2	51.4	71.8	91.1	98.3	0.70
Hendricks (2007b)	4	14.1	40.6	59.3	80.3	96.5	99.8	0.77
Kitao (2008)	3	35.4	58.4	66.0	80.9	96.1	99.9	0.80
Cagetti and De Nardi (2009)	3	30.0	60.0	–	85.0	95.0	–	0.82
Cho and Francis (2011)	7	–	24.3	40.8	63.4	–	–	0.63
De Nardi and Yang (2014)	4	14.8	42.2	59.7	78.5	94.5	99.3	0.76
Kopecky and Koreshkova (2014)	8	18.0	51.4	70.4	–	–	–	–

Notes: The data values are the medians for each year according to Table 1.

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