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A SIMPLE ANALYSIS OF THE U.S. EMISSION PERMITS AUCTIONS

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Abstract

We examine a stylized version of EPA auctions when agents know the list of values of sellers and buyers. Sellers and buyers behave strategically. We show that there are two types of equilibria: inefficient equilibria where no goods are traded and efficient equilibria where all exchange occurs at a uniform price. We also provide examples of the EPA auction game under incomplete information when the uniform price equilibrium holds and when it does not hold. When the uniform price equilibrium holds, sellers shade their bids up and buyers shade their bids down. In the example where the uniform price equilibrium does not hold, both buyers and sellers shade their bids down in an equilibrium. **JEL Classification:** D44, Q29. **Keywords:** Emission permits auctions; double auctions.

1. Introduction

The Clean Air Act Amendments of 1990 have created tradeable emission permits to control sulfur dioxide pollution. A permit allows the emission of up to one ton of sulfur dioxide by its holder. Any firm affected by the act can emit at most as many tons of sulfur dioxide as the number of permits held. These permits can be traded and to facilitate this process the Environmental Protection Agency (EPA) conducts annual auctions.

The EPA auction works as follows. Potential sellers submit sealed ask prices and potential buyers submit sealed bid prices. Buyers and sellers may make

multiple submissions. The bid prices are ranked from the highest to the lowest while ask prices are ranked from lowest to highest. No exchange occurs if the lowest ask price exceeds the highest bid price. If the lowest ask price does not exceed the highest bid price, these two bidders are matched and the buyer pays the seller his bid price in exchange for one permit. This matching process continues in ascending order of specified ask price until all bids are awarded, permits offered are sold out, or all remaining ask prices exceed remaining bid prices.

Research in the area of mechanisms for trading pollution rights has been concentrated in two areas: the optimal design of institutions,¹ and in the evaluation of existing permit systems.² We add to the latter literature.³

The complexity of double-auctioning games makes the calculation of equilibria difficult. To make auctioning games tractable some simplifying assumptions are made. For example, one side of the market behaves nonstrategically and each player on the other side of the market follows the same bidding function.⁴

In this paper we consider the case where both sellers and buyers behave strategically under full information about the actual distribution of buyers and sellers. We show that the Nash equilibria under full information are either inefficient with no trade or efficient where all trades occur at a uniform price.

We also provide examples of an EPA auction game with incomplete information. We give examples when the uniform price equilibrium holds and when it does not hold. When the uniform price equilibrium holds, some sellers actually shade their bids up and buyers shade their bids down in equilibrium. In the example where the uniform price equilibrium does not hold, buyers shade their bids down in equilibrium and sellers bid zero.

¹For example, Laffont and Tirole (1996) and Ledyard and Szakaly-Moore (1994).

²For example, Cason (1993, 1995) and Cason and Plott (1996).

³Our model is similar to the assignment game considered by Böhm-Bawerk (1891) and Shapley and Shubik (1972). Shapley and Shubik, however, consider the core as the solution concept.

⁴Satterthwaite and Williams (1989) consider a uniform price auction where buyers and sellers have unit demands/supplies. The rules of the auction and the assumption that the number of buyers equals the number of sellers leads the sellers to behave nonstrategically. Cason (1993) examines a stylized version of the EPA auction where buyers are non-strategic and there are more sellers than buyers.

2. The Model

The players consist of a finite number, $n \geq 1$, of sellers and a finite number, $m \geq 1$, of buyers. Let $N = \{1, \dots, n\}$ denote the set of sellers and $M = \{n+1, \dots, n+m\}$ denote the set of buyers. Each seller has one unit of an identical good and each buyer has no units of the good. Each player wants at most one unit of the good. In the sealed bid auction each buyer $j \in M$ independently proposes an offer b_j and each seller $i \in N$ independently proposes an ask price a_i . This happens simultaneously. The goods are exchanged in the following way. Order and label by superscripts the offers from highest to lowest, i.e., $b^1 \geq b^2 \geq \dots \geq b^m$, and the ask prices from lowest to highest, i.e., $a^1 \leq a^2 \leq \dots \leq a^n$. If there are ties, then the ordering is chosen by randomization giving each ordering in a tie the same probability of being chosen. Then we check to see if $a^1 \leq b^1$ holds. If it does not, then no exchange takes place. If it does, then we find the maximum integer k satisfying $a^k \leq b^k$. For $i \leq k$, the i^{th} buyer in the list pays his offer b^i to the i^{th} seller in the list in exchange for his unit of the good. Buyers and sellers who appear in a position after the k^{th} person in their list make no exchanges.

We will assume that the true value of each buyer and each seller is common knowledge. To make the game interesting we assume that $v_j > s_i$ for some $j \in M$ and some $i \in N$ where v_j denotes buyer j 's valuation and s_i seller i 's valuation. We denote the strategy set of buyer j by B_j and the strategy set of seller i by A_i and define $B_j = A_i = \mathbb{R}^+$ for all $j \in M$ and $i \in N$. We only consider pure strategies. Let $A = A_1 \times \dots \times A_n$ and $B = B_1 \times \dots \times B_m$. We call $(a, b) = (a_1, \dots, a_n, b_{n+1}, \dots, b_{n+m}) \in A \times B$ a *strategy combination*. Let $a_{-i} = (a_1, \dots, a_{i-1}, a_{i+1}, \dots, a_n)$ and let $b_{-i} = (b_1, \dots, b_{i-1}, b_{i+1}, \dots, b_n)$. For strategy combination (a, b) let $q_{ij}(a, b)$ denote the probability that seller i is matched with buyer j when strategy combination (a, b) is used. For a set $\mathcal{A}$ we denote the cardinality of $\mathcal{A}$ by $\#\mathcal{A}$. Then for any strategy combination (a, b) , any $i \in N$, and any $j \in M$ the probability $q_{ij}(a, b)$ is computed as follows:

$$q_{ij}(a, b) = \begin{cases} b_j < a_i, \text{ or} \\ 0 \text{ if } \begin{cases} \#\{j' \in M : b_{j'} > b_j\} \geq \#\{i' \in N : a_{i'} \leq a_i\}, \text{ or} \\ \#\{j' \in M : b_{j'} \geq b_j\} \leq \#\{i' \in N : a_{i'} < a_i\}; \end{cases} \\ 1/r_{ij} \text{ otherwise, where } r_{ij} = \max\{\#\{j' \in M : b_{j'} = b_j\}, \#\{i' \in N : a_{i'} = a_i\}\}. \end{cases}$$

The formula for $q_{ij}(a, b)$ says that seller i and buyer j will not be matched, i.e., $q_{ij}(a, b) = 0$, if: i) j 's bid is less than i 's ask price, or ii) there are at least as

many buyers with bids above b_j as there are sellers with asks below or equal to a_i , or iii) there are at least as many sellers with asks below a_i as there are buyers with bids above or equal to b_j . Under i), no match can occur by definition. Under ii), seller i is definitely matched with a buyer bidding more than b_j , and under iii), buyer j is definitely matched with some seller asking less than a_i . Seller i and buyer j will be matched with positive probability, i.e., $q_{ij}(a, b) > 0$ otherwise. In this case $q_{ij}(a, b)$ equals 1 divided by either the number of buyers bidding b_j or the number of sellers asking a_i whichever is greater.

The payoff function $u_i : A \times B \rightarrow \mathbb{R}$ of seller i is defined by: $u_i(a, b) = \sum_{j \in M} q_{ij}(a, b)(b_j - s_i)$. Similarly, the payoff function $u_j : A \times B \rightarrow \mathbb{R}$ of buyer j is defined by: $u_j(a, b) = \sum_{i \in N} q_{ij}(a, b)(v_j - b_j)$.

A strategy combination (a, b) is a *Nash equilibrium* if and only if 1) for each buyer $j \in M$, $u_j(a, b) \geq u_j(a, b_{-j}, b'_j)$ for all $b'_j \in B_j$, and 2) for each seller $i \in N$, $u_i(a, b) \geq u_i(a_{-i}, a'_i, b)$ for all $a'_i \in A_i$.

We say that a strategy combination (a, b) is *efficient* if and only if (a, b) leads to an allocation where the players with the highest values each have one unit of the good and the other players have none.⁵

3. Main Results

Next we show that in every Nash equilibrium all buyers who obtain goods submit the same bids.

Proposition 1: If (a, b) is a Nash equilibrium, then the strategies of the buyers who obtain goods are identical, i.e., $b^1 = b^2 = \dots = b^k$.

Proof: We prove the contrapositive. Suppose that (a, b) is a strategy combination and for some buyers j and j' who obtained goods at the auction $b_j \neq b_{j'}$. Without loss of generality assume that $b_j > b_{j'}$. Since j' got an object in the auction, any bid above $b_{j'}$ would obtain an object with certainty. If j chose $b_j^* = \frac{b_j + b_{j'}}{2}$, then he would get $u_j(a, b_{-j}, b_j^*) = v_j - \frac{b_j + b_{j'}}{2} > v_j - b_j = u_j(a, b)$. But then (a, b) is not a Nash equilibrium. $\square$

⁵Note that every game will have Nash equilibria that are inefficient. For example consider the game where $n = 2$ sellers value the good at \$.5 and $m = 2$ buyers value the good at \$1. Efficiency here requires that all goods are exchanged from sellers to buyers. However, no goods are exchanged in the Nash equilibrium where all buyers offer \$.25 and all sellers ask for \$.2.

Proposition 1 implies that in equilibrium all exchanges will take place at a uniform price. In Proposition 2 we show that every Nash equilibrium involving some exchange is efficient. We will need the following Lemma.

Lemma 1.1: Let (a, b) be a strategy combination under which all exchanges take place at the uniform price p . Further suppose that one of the following is true: 1) $s_x < p$ for some seller x who didn't make an exchange at the auction; 2) $v_y > p$ for some buyer y who didn't make an exchange at the auction; or 3) $u_x(a, b) < 0$ for some seller x or $u_y(a, b) < 0$ for some buyer y . Then (a, b) is not a Nash equilibrium.

Proof: Consider 1). Since seller x did not receive a good at the auction it must be true that $a_x \geq s^k$. If $a_x > s^k$, then $u_x(a, b) = 0$. If $a_x = s^k$, then $u_x(a, b) = \alpha(p - s_x)$, where $\alpha = \sum_{j \in M} q_{xj}(a, b) < 1$. If seller x changed to $a'_x = 0$, then $u_x(a_{-x}, a'_x, b) = p - s_x > \alpha(p - s_x) > 0$.

Consider 2). Since buyer y did not obtain an object at the auction it must be true that $b_y \leq p$. If $b_y < p$, then $u_y(a, b) = 0$. If $b_y = p$, then $u_y(a, b) \leq \alpha(v_y - p)$, where $\alpha = \sum_{i \in N} q_{iy}(a, b) < 1$. If buyer y chose $b'_y = p + (1 - \alpha)(v_y - p)/2$, then $u_y(a, b_{-y}, b'_y) = (1 + \alpha)(v_y - p)/2 > \alpha(v_y - p) > 0$.

Consider 3). In either case (a, b) cannot be a Nash equilibrium since each buyer and seller can always ensure a payoff of zero by bidding his true value. $\square$

Proposition 2: If (a, b) is a Nash equilibrium where some goods are exchanged then (a, b) is efficient.

Proof: We prove this proposition by contradiction. Suppose that (a, b) is an inefficient Nash equilibrium where some goods are exchanged. It follows from Proposition 1 that $b^1 = \dots = b^k = p$, i.e., all goods are exchanged at a uniform price p . We argue that one of the three suppositions of Lemma 1.1 must be true which contradicts that (a, b) is a Nash equilibrium.

The auction always ensures that each player ends up with at most one unit of the good. Hence if (a, b) is inefficient it must be because one of the players with the highest values did not end up with a permit. This means that one

of the following three things is true: *i*) for some seller $i \in N$ and some buyer $j \in M$ we have $s_i < v_j$ and neither i nor j made an exchange at the auction; *ii*) for two buyers $j, j' \in B$ we have $v_j > v_{j'}$ and j' got one unit in the auction and j got no units in the auction; *iii*) for two sellers $i, i' \in S$ we have $s_i < s_{i'}$ and i' sold his unit in the auction and i did not.

Consider *i*). If $s_i < p$, then we have 1) of Lemma 1.1 with $x = i$. If $s_i \geq p$, then we have 2) of Lemma 1.1 with $y = j$. Consider *ii*). If $v_{j'} \geq p$, then we have 2) of Lemma 1.1 with $y = j$. If $v_{j'} < p$, then we have 3) of Lemma 1.1 with $y = j'$. Consider *iii*). If $s_{i'} \leq p$, then we have 1) of Lemma 1.1 with $x = i$. If $s_{i'} > p$, then we have 3) of Lemma 1.1 with $x = i'$. $\square$

Propositions 1 and 2 allow us to break the set of Nash equilibria into two classes: inefficient equilibria characterized by no exchange since buyers offer too little and sellers ask too much; and efficient equilibria characterized by a uniform price.

4. Do uniform-price equilibria occur with incomplete information?

In this section we look at two examples of games of incomplete information with independent private values. In the first example we construct a Nash equilibrium under which all exchanges occur at a uniform price with probability one. In the second example, we construct a Nash equilibrium that does not involve a uniform price.

Example 1: There are m buyers and n sellers, with $m = n$. Values are independently distributed. Each seller's value is uniformly distributed on the interval $[0,1]$ and each buyer's value is uniformly distributed on the interval $[2,3]$. Each player knows his own value but only the distributions of the remaining players values. All buyers and sellers are risk neutral.

Let's see that a strategy combination requiring every player to submit a price $p = \frac{3}{2}$ regardless of their values constitutes a Nash equilibrium. Consider an arbitrary buyer j with value v_j who expects that the other buyers and sellers will submit $\frac{3}{2}$. If he submits a bid below $\frac{3}{2}$ his payoff is zero

since he is not matched. If he bids at least $\frac{3}{2}$, his payoff is $v_j - \frac{3}{2}$, since he will definitely be matched. His payoff is maximized by setting $b_j = \frac{3}{2}$, and $v_j \geq 2$. A similar argument holds for an arbitrary seller.

Notice that if there is any dispersion of bids in the above example, then sellers would have an incentive to shade their bids down to zero. However, this cannot be supported by Nash play, since buyers would then jump down to zero as well.

A close examination of example 1 provides us with two features that seem crucial for the uniform-price equilibrium to hold in the incomplete information framework: 1) the number of buyers equals the number of sellers and, 2) all buyers' values are strictly greater than all sellers' values. These two assumptions allow the players to "focus" on any price p that lies between the minimum possible valuation for a buyer and maximum possible valuation for a seller as a Nash equilibrium. In the next example, we modify the second feature, namely, we consider the case where there are more buyers than sellers ($m > n$).

Example 2: Everything is the same as in example 1 except $m > n$. The strategies of example 1 require each player to submit the same price p regardless of their true values. We show that this cannot be a Nash equilibrium. If $p \in [1, 2]$, as in example 1, a particular buyer gets an object with probability $\frac{n}{m}$. Thus, buyer j would get an expected payoff of $\frac{n}{m}(v_j - p)$. But by increasing her bid by an arbitrarily small amount ε , her expected payoff becomes $(v_j - p - \varepsilon)$. For any finite number of buyers we can always find sufficiently small ε to make her increase her bid by this much. If $p \in [0, 1)$ then a seller with value of 1 could obtain a higher expected payoff from submitting 1 rather than p . If $p > 2$ then a buyer with a value of 2 could obtain a higher expected payoff by submitting a bid of 2.

It can be verified that the following strategy profile is an equilibrium:

$$\begin{aligned} b_j^*(v_j) &= \frac{1}{\Gamma(v_j)} \int_2^{v_j} y \Gamma'(y) dy, \quad \forall j \in M \\ a_i^*(s_i) &= 0, \quad \forall i \in N \end{aligned}$$

Where $\Gamma(v_j) = \sum_{i=1}^n \binom{m-1}{m-i} [v_j]^{m-i} [1 - v_j]^{i-1}$ is the probability that v_j is one of the n highest values.

Notice that buyers and sellers use symmetric strategies. The symmetric seller's strategy informs a seller to ask zero regardless of his valuation. The symmetric buyer's strategy informs a buyer to bid the expected value of the n^{th} highest buyer conditional on the n^{th} highest buyer being below her. Notice that the buyer's strategy is an increasing function of his true value. For any seller i , bidding anything different from zero will guarantee him to be matched with the n^{th} highest bid or no bid. By bidding zero, he has a $1/n$ chance of being matched with each of the n highest bids. Therefore, asking zero (the lower bound) maximizes his expected payoff. A buyer will behave as in a multi-unit auction where n identical units are sold at a zero reservation value yielding the above equilibrium buyers' strategies.

In example 2, the number of buyers being greater than the number of sellers was crucial to obtain the equilibrium described. Competition amongst buyers forced all final prices to be in the buyers support. Given this result, one might be tempted to conjecture that if the number of sellers is greater than the number of buyers, then we can find an equilibrium where sellers shade their bids and buyers enter identical price bids. However, this is not true. The reason stems from the different treatment of buyers and sellers in the EPA auction. We conjecture that if an equilibrium exists in this case it will involve non-uniform price bids that all lie in the sellers support. Intuitively, if buyers submit bid prices above the sellers support, all sellers would compete for the highest price by asking zero. This cannot be an equilibrium since buyers would have an incentive to bid zero as well.

5. Conclusion

In this paper we examine the outcome of a stylized version of the EPA auctions with complete information. We show that there are either inefficient equilibria (where no goods are exchanged) or efficient equilibria (where all possible gains from trades are realized). The efficient equilibria have the property that all trades occur at a uniform price.

We provided an example of the EPA auctions game under incomplete information where a uniform-price equilibrium holds. This example shows that it is possible to have sellers shading their bids up when both buyers and sellers behave strategically, the number of buyers is equal to the number of sellers and the supports of values do not intersect. We also provided an example under incomplete

information where a Nash equilibrium involves both buyers and sellers shading their bids down. In general, it seems that the types of equilibria that will arise in the incomplete information case depend on 1) the number of buyers relative to the number of sellers and 2) whether or not the buyers and sellers supports intersect. Clearly more research is needed to characterize the equilibria of these games under incomplete information. Additional work is also needed to generalize our analysis to the case where buyers and sellers have multi-unit trading capacities and to incorporate the existence of a secondary market.

References

- [1] Böhm-Bawerk, E. von, 1923, **Positive Theory of Capital** (translated by W. Smart). G. E. Steckert, New York (original publication 1891).
- [2] Cason, T. N., 1993, "Seller Incentive Properties of EPA's Emission Trading Auctions," **Journal of Environmental Economics and Management** 25(2), 177-195.
- [3] Cason, T. N., 1995, "An Experimental Investigation of the Seller Incentives in EPA's Emission Trading Auction," **American Economic Review** 85(4), 905-922.
- [4] Cason, T. N. and C. R. Plott, 1996, "EPA's New Emissions Trading Mechanism: A Laboratory Experiment," **Journal of Environmental Economics and Management** 30(2), 133-160.
- [5] Laffont, J.-J. and J. Tirole, 1996, "Pollution Permits and Compliance Strategies," **Journal of Public Economics** 62(1-2), 85-125.
- [6] Ledyard, J. O. and K. Szakaly-Moore, 1994, "Designing Organizations for Trading Pollution Rights," **Journal of Economic Behavior and Organization** 25, 167-196.
- [7] Satterthwaite, M.A. and S.R. Williams, 1989, "The Rate of Convergence to Efficiency in the Buyer's Bid Double Auction as the Market Becomes Large", **Review of Economic Studies** 56(4), 477-498.
- [8] Shapley, L.S., and M. Shubik, 1972, "The Assignment Game I: The Core", **International Journal of Game Theory** 1, 111-130.

TEXTOS PARA DISCUSSÃO DO CERES

PINHEIRO, Armando Castelar. Infra-Estrutura, Crescimento e a Reforma do Estado. Rio de Janeiro: CERES/EPGE/FGV, 1997. Texto para Discussão n.º 001, Junho/97.

CYSNE, Rubens Penha e FARIA, Lauro Flávio Vieira de. Comments on the Brazilian Financial System. Rio de Janeiro: CERES/EPGE/FGV, 1997. Texto para Discussão n.º 002, Julho/97.

MARINHO, Alexandre, RESENDE, Marcelo e FAÇANHA, Luis Otavio. Brazilian Federal Universities: Relative Efficiency Evaluation and Data Envelopment Analysis. Rio de Janeiro: CERES/EPGE/FGV, 1997. Texto para Discussão n.º 003, Agosto/97.

CYSNE, Rubens Penha. Comentários sobre Defesa da Concorrência no Brasil. Rio de Janeiro: CERES/EPGE/FGV, 1997. Texto para Discussão n.º 004, Setembro/97.

PASTORE, Affonso Celso. Senhoriação e Inflação: O Caso Brasileiro. Rio de Janeiro: CERES/EPGE/FGV, 1997. Texto para Discussão n.º 005, Setembro de 1997.

MATTOS, César Costa Alves de. The Recent Evolution of Competition Policy in Brazil: an Incomplete Transition. Rio de Janeiro: CERES/EPGE/FGV, 1997. Texto para Discussão n.º 006, Outubro de 1997.

PASTORE, Affonso Celso, PINOTTI, Maria Cristina. Inflação e Estabilização: Algumas Lições da Experiência Brasileira. Rio de Janeiro: CERES/EPGE/FGV, 1997. Texto para Discussão n.º 007, Novembro de 1997.

PESSÔA, Samuel de Abreu. O Impacto da Redução do Custo Brasil sobre a Defasagem Cambial. Rio de Janeiro: CERES/EPGE/FGV, 1998. Texto para Discussão n.º 008, Maio de 1998.

PINHEIRO, Armando Castelar. A Reforma do Judiciário: Uma Análise Econômica. Rio de Janeiro: CERES/EPGE/FGV, 1998. Texto para Discussão n.º 009, Agosto de 1998.

HOLANDA, Marcos Costa. Gastos com Pessoal em Contexto de Ajuste Fiscal, o Caso dos Estados do Ceará, Maranhão e Piauí. Rio de Janeiro: CERES/EPGE/FGV, 1998. Texto para Discussão n.º 010, Agosto de 1998.

CYSNE, Rubens Penha. Aspectos Macro e Microeconômicos das Reformas Brasileiras. Rio de Janeiro: CERES/EPGE/FGV, 1998. Texto para Discussão n.º 011, Agosto de 1998.

PESSOA, Samuel de Abreu. Ajustamento de uma economia após uma elevação da produtividade. Rio de Janeiro: CERES/EPGE/FGV, 1998. Texto para Discussão n.º 012, Agosto de 1998.

VELOSO, João Paulo dos Reis. Brasil. Um Projeto para o Século XXI - O Modelo de "Especializações Avançadas". Rio de Janeiro: CERES/EPGE/FGV, 1998. Texto para Discussão n.º 013, Agosto de 1998.

BAUMANN, Renato. As Tarifas de Importação no Plano Real. Rio de Janeiro: CERES/EPGE/FGV, 1998. Texto para Discussão n.º 014, Agosto de 1998.

YOSHINO, Joe Akira. Regulação do Sistema Financeiro Brasileiro. Rio de Janeiro: CERES/EPGE/FGV, 1998. Texto para Discussão n.º 015, Outubro de 1998.

BAUMANN, Renato. Foreign Investment In Brazil and The International Financial Markets. Rio de Janeiro: CERES/EPGE/FGV, 1998. Texto para Discussão n.º 016, Outubro de 1998.

KLINE, Jeffrey J., MENEZES, Flavio M.. A Simple Analysis of the U.S. Emission Permits Auctions. Rio de Janeiro: CERES/EPGE/FGV, 1999. Texto para Discussão n.º 017, Janeiro de 1999.

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